Beyond-Hermitian Quantum Physics I

-An Introduction to New Frontiers in Open Quantum Physics-

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Exploration of what's new in open quantum physics should start from revisiting the fundamental premise in closed quantum physics.

The most fundamental premise in closed quantum physics — Hermiticity

- **unitary dynamics**
- **real-valuedness of the eigenspectrum**
- **orthogonality of eigenstates**
- reciprocity ($\boldsymbol{O}^t$) = time reversal ($\boldsymbol{O}^*$)

"Beyond-Hermitian" implies removal of one or more of these fundamental constraints.

The key question:
What new potentials can be liberated therefrom?

Some ideas

- **nonunitary dynamics / *PT*-symmetry**

→ surpass the Lieb-Robinson bound

PRL 120, 185301 (2018)

The Lieb-Robinson bound sets
a theoretical upper limit on the speed at which information can propagate in non-relativistic quantum systems.

E. Lieb and D. Robinson, Commun. Math. Phys. **28**, 251 (1972)

Some ideas

- **nonunitary dynamics / *PT*-symmetry**

  → surpass the Lieb-Robinson bound

  PRL 120, 185301 (2018)

- **complex eigenspectrum**

  → intermediate-state eng., esp. magnetism reversal

  PRL 124, 147203 (2020)

Quantum magnetism

arises from the second-order perturbation theory in which the intermediate-state energy appears in the denominator. If this energy is complex, physics will change dramatically.

Some ideas

- **nonunitary dynamics / *PT*-symmetry**
  → **surpass the Lieb-Robinson bound**
  PRL 120, 185301 (2018)
- **complex eigenspectrum**
  → **intermediate-state eng., esp. magnetism reversal**
  PRL 124, 147203 (2020)
- **nonorthogonality of eigenstates**
  → **continuous QPT without gap closing**
  PRL 125, 260601 (2020)
- reciprocity ($\boldsymbol{O}^t$) ≠ time reversal ($\boldsymbol{O}^*$)
  → **unification and ramification of topological phases**
  Nat. Commun. 10, 297 (2019)
  PRX 8, 031079 (2018)
  PRX 9, 041015 (2020)

10 → 38

Some ideas

- **non**unitary dynamics / *PT*-symmetry

  → surpass the Lieb-Robinson bound

  PRL 120, 185301 (2018)

- **complex** eigenspectrum

  → intermediate-state e… …tism reversal

  …

- **non**orthog… …states

  → conti… …out gap closing

  PRL 125, 260601 (2020)

- … …ty (O^t) ≠ time reversal (O^*)

  → u…fication and ramification of topological phases

  Nat. Commun. 10, 297 (2019)

  PRX 8, 031079 (2018)

  PRX 9, 041015 (2020)

There are many unexplored topological phases hidden in open quantum systems!

10 → 38

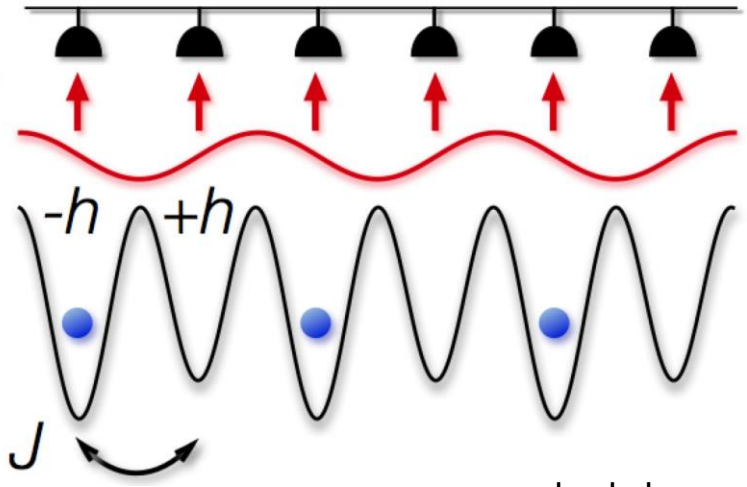
Nonunitary dynamics/*PT*-symmetry: surpassing the Lieb-Robinson bound

Solvable model: fermions under periodic dissipation

site-resolved detection of atom number

periodic dissipative potential (one-body loss)

half-filled free fermions in a superlattice



one-body loss

effective Hamiltonian: $\hat{H}_{\rm eff} = \hat{H}_{\rm PT} - 2i\gamma\hat{N}$

PT-symmetric Hamiltonian:

$$\hat{H}_{\rm PT} = -\sum_{l=0}^{L-1}\left[\left(J+(-1)^l i\gamma\right)\left(\hat{c}_{l+1}^\dagger\hat{c}_l+\hat{c}_l^\dagger\hat{c}_{l+1}\right)+(-1)^l h\hat{c}_l^\dagger\hat{c}_l\right]$$

jump term: $\mathcal{J}[\hat{\rho}] = 2\gamma\sum_l[2\hat{c}_l\hat{\rho}\hat{c}_l^\dagger + (-1)^l(\hat{c}_l\hat{\rho}\hat{c}_{l+1}^\dagger + \hat{c}_{l+1}\hat{\rho}\hat{c}_l^\dagger)]$

Lindblad equation (ensemble level)

$$\frac{d\hat{\rho}(t)}{dt} = -i\left(\hat{H}_{\rm eff}\hat{\rho} - \hat{\rho}\hat{H}_{\rm eff}^\dagger\right) + \mathcal{J}[\hat{\rho}]$$

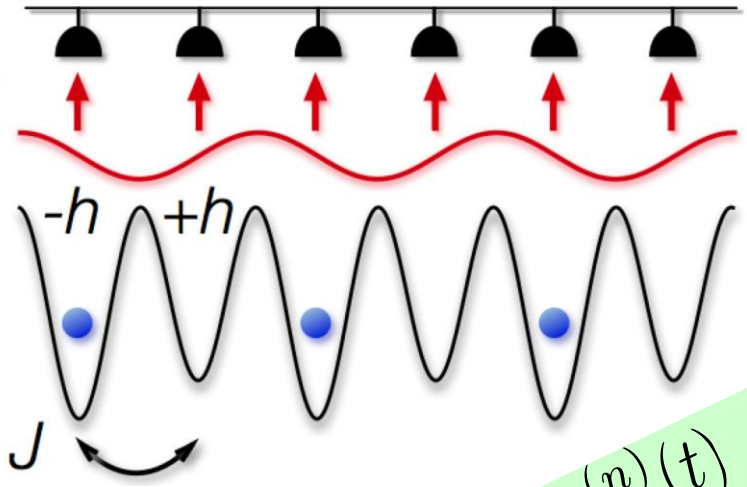
Y. Ashida and MU, PRL **120**, 185301 (2018)

Solvable model: fermions under periodic dissipation

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effective Hamiltonian: $\hat{H}_{\rm eff} = \hat{H}_{\rm PT} - 2i\gamma \hat{N}$ one-

PT-symmetric Hamiltonian: $\hat{H}_{\rm PT} = -\sum^{L-1}$ $\left[(-1)^l i\gamma \right)\left(\hat{c}_{l+1}^\dagger \hat{c}_l + \hat{c}_l^\dagger \hat{c}_{l+1} \right) + (-1)^l h \hat{c}_l^\dagger \hat{c}_l \Big]$

jump: $\mathcal{J}[\hat{\rho}] = 2\gamma \sum_l [2\hat{c}_l \hat{\rho} \hat{c}_l^\dagger + (-1)^l (\hat{c}_l \hat{\rho} \hat{c}_{l+1}^\dagger + \hat{c}_{l+1} \hat{\rho} \hat{c}_l^\dagger)]$

Lindblad equation (ensemble level) $\dfrac{d\hat{\rho}(t)}{dt} = -i\left(\hat{H}_{\rm eff}\hat{\rho} - \hat{\rho}\hat{H}_{\rm eff}^\dagger \right) + \mathcal{J}[\hat{\rho}]$

Exact solution for full-counting dynamics $\hat{\rho}^{(n)}(t)$ can be obtained!

Y. Ashida and MU, PRL **120**, 185301 (2018)

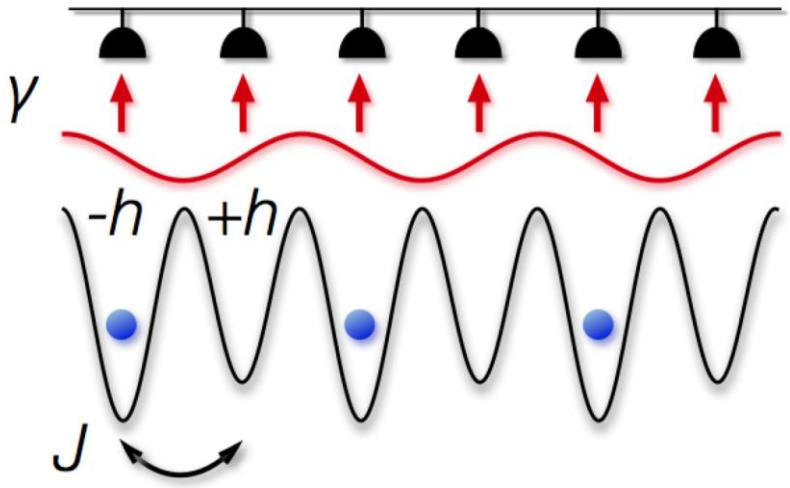
Propagation of correlation: Lindblad dynamics

At *t*=0, we switch on dissipation *γ* and on-site potential *h*, and consider the equal-time correlation function:

$$C(l,t) = \mathrm{Tr}[\hat{\rho}(t)\hat{c}_l^\dagger \hat{c}_0]$$

solution of Lindblad eq.

$$\hat{\rho}(t) = \sum_n P_n(t)\hat{\rho}^{(n)}(t)$$



Y. Ashida and MU, PRL **120**, 185301 (2018)

Propagation of correlation: Lindblad dynamics

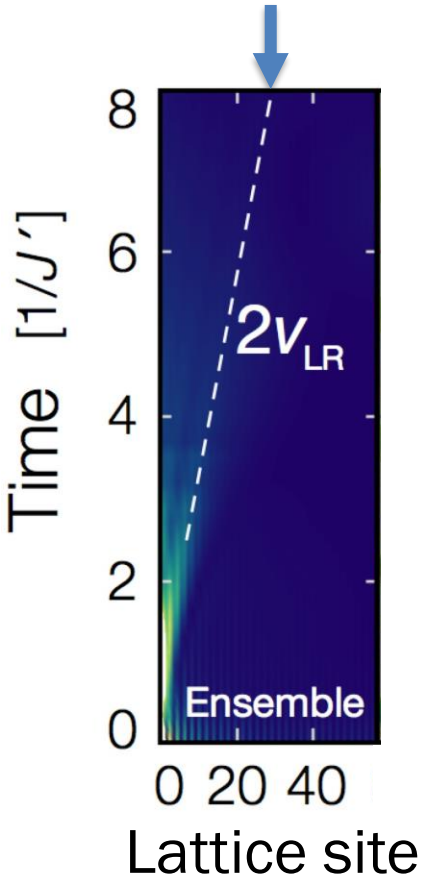
At *t*=0, we switch on dissipation *γ* and on-site potential *h*, and consider the equal-time correlation function:

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solution of Lindblad eq.

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Lieb-Robinson (LR) bound



At the ensemble (i.e., Lindblad) level, the propagation of correlation is limited by the Lieb-Robinson bound.

cf. D. Poulin, PRL **104**, 190401 (2010)

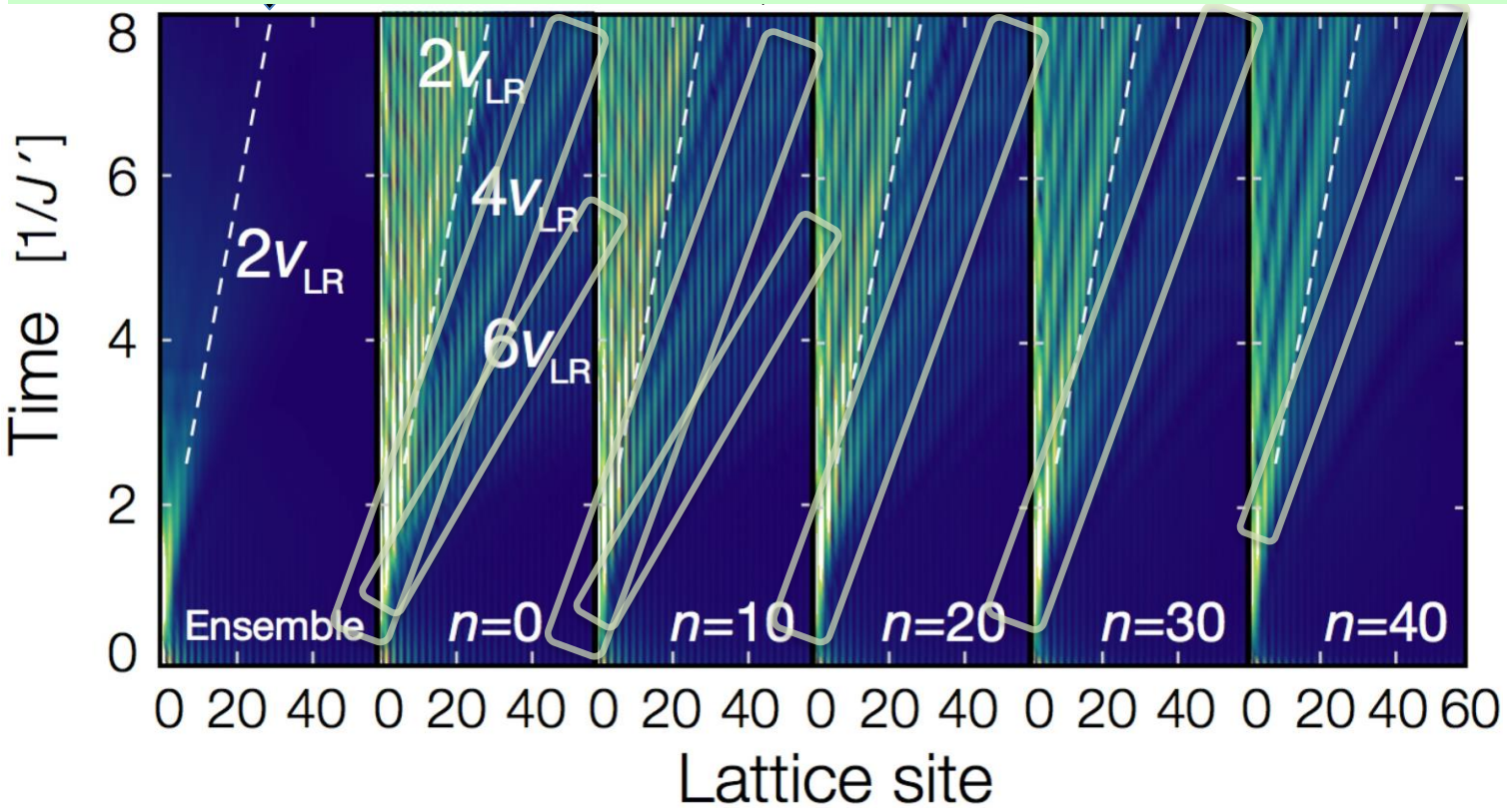
Y. Ashida and MU, PRL **120**, 185301 (2018)

Propagation of correlation: full-counting dynamics

Same quantity under the full-counting dynamics for different numbers of quantum jumps *n*=0, 10, 20, 30, 40:

$$C^{(n)}(l,t) = \mathrm{Tr}[\hat{\rho}^{(n)}(t)\hat{c}_l^\dagger \hat{c}_0] \qquad n\text{: number of jumps}$$

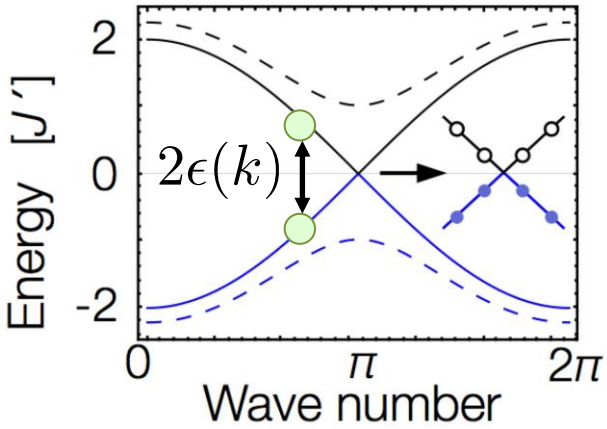
Faster propagation at integer multiples of LR bound!



Y. Ashida and MU, PRL **120**, 185301 (2018)

Physical origin of supersonic propagation

PT-symmetric non-Hermitian dynamics



The spectrum can be real for small dissipation γ, but eigenstates become non-orthogonal, mixing different eigenstates especially near the exceptional point.

→ norm oscillations at frequency 2ε(*k*).

Such higher harmonics enter the dispersion relation, leading to supersonic propagation.

$$v_{\mathrm{LR}} = \max_{k} \frac{\partial \epsilon(k)}{\partial (k/2)}$$

group velocity of quasiparticles

higher harmonics

2ε(k), 4ε(k), …

⟶

supersonic propagation

$4v_{\mathrm{LR}}$, $6\,v_{\mathrm{LR}}$,…

Y. Ashida and MU, PRL **120**, 185301 (2018)

Complex eigenspectrum:

intermediate-state engineering & magnetism reversal

M. Nakagawa*, et al*., Phys. Rev. Lett. **124**, 147203 (2020)

Nontrivial many-body physics arises from second- and higher-order perturbation theory:

$$E_n^{(2)} = \sum_m{}' \frac{|V_{mn}|^2}{E_n^{(0)} - E_m^{(0)}}$$

Here the intermediate states play a crucial role.

What new physics may arise if they are dissipative?

dissipative engineering of intermediate states

Quantum magnetism from spin-exchange interaction

Fermi-Hubbard model

$$H = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{h.c.}) + U \sum_{j} n_{j\uparrow} n_{j\downarrow}$$

Hopping Interaction

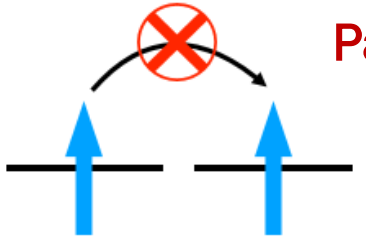
Suppose that on-site interaction *U* >> hopping amplitude *t*.

Quantum magnetism from spin-exchange interaction

◇ antiparallel spin configuration ⟶ The system can lower energy via virtual hopping to the neighboring site.



◇ parallel spin configuration ⟶ cannot lower energy (Pauli principle)



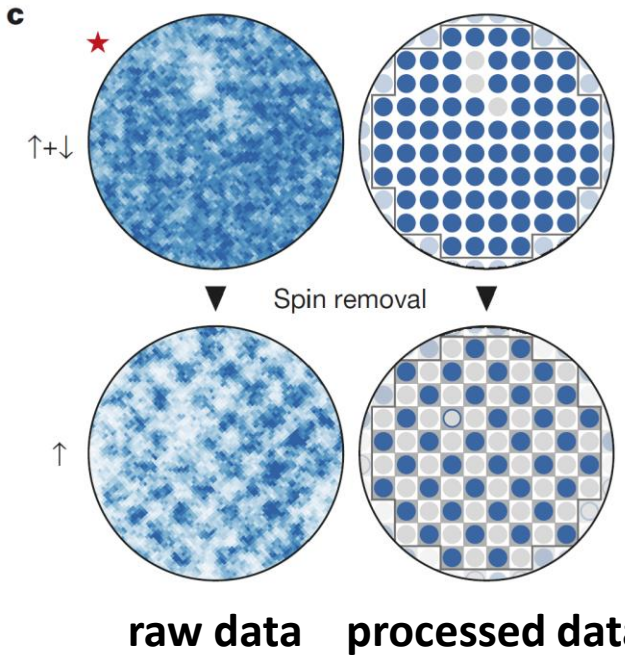
Pauli principle

antiferromagnetic Heisenberg model

$$H = J \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \quad (J > 0)$$

At half filling, an antiferromagnetic long-range order was observed in ^{6}Li using quantum-gas microscopy.

A. Mazurenko *et al*., Nature **545**, 462 (2017)



both spin components present

one spin components removed

This is what is expected from Hermitian systems.

What happens in the presence of inelastic collisions?

■ **Quantum master equation for the Fermi-Hubbard model**

$$\frac{d\rho(t)}{dt} = -i[H,\rho] + \sum_j (L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\})$$

$$= -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger) + \sum_j L_j \rho L_j^\dagger$$

quantum jumps
decrease particle number

■ **Jump operators describe two-body loss:** $L_j = \sqrt{2\gamma} c_{j\uparrow} c_{j\downarrow}$

What happens in the presence of inelastic collisions?

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$$\frac{d\rho(t)}{dt} = -i[H,\rho] + \sum_j (L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\})$$

$$= -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger) + \sum_j L_j \rho L_j^\dagger$$

quantum jumps
decrease particle number

■ **Jump operators describe two-body loss:** $L_j = \sqrt{2\gamma} c_{j\uparrow} c_{j\downarrow}$

$$H_{\text{eff}} = H - \frac{i}{2}\sum_j L_j^\dagger L_j$$

$$= -t \sum_{\langle i,j\rangle,\sigma} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + (U - i\gamma)\sum_j n_{j\uparrow} n_{j\downarrow}$$

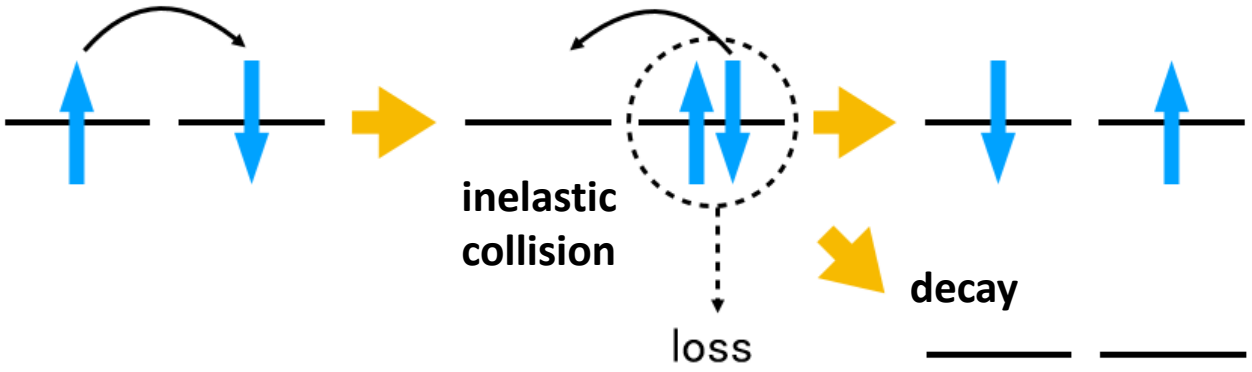
Non-Hermitian Hamiltonian reduces to the FH model with a complex interaction.

Because of this complex nature of the interaction, intermediate-state physics changes dramatically.

Dissipative spin-exchange interaction

M. Nakagawa, et al., PRL **124**, 147203 (2020)

◇ antiparallel spin configuration ⟶ The doubly occupied intermediate state decays due to inelastic collisions.

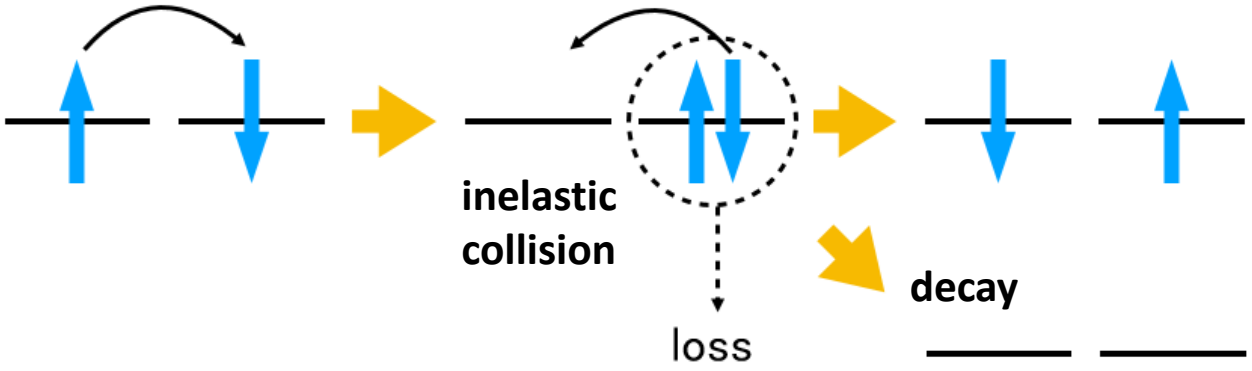


The antiferromagnetic interaction is suppressed by inelastic collisions.

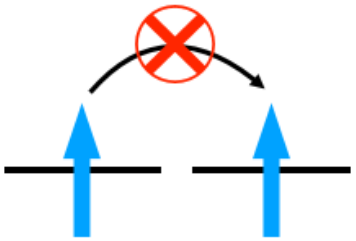
Dissipative spin-exchange interaction

M. Nakagawa, et al., PRL **124**, 147203 (2020)

◇ antiparallel spin configuration ⟶ The doubly occupied intermediate state decays due to inelastic collisions.



◇ parallel spin configuration ⟶ The system is free from loss and stable.



Higher-energy ferromagnetic states are stabilized by **dissipation.**

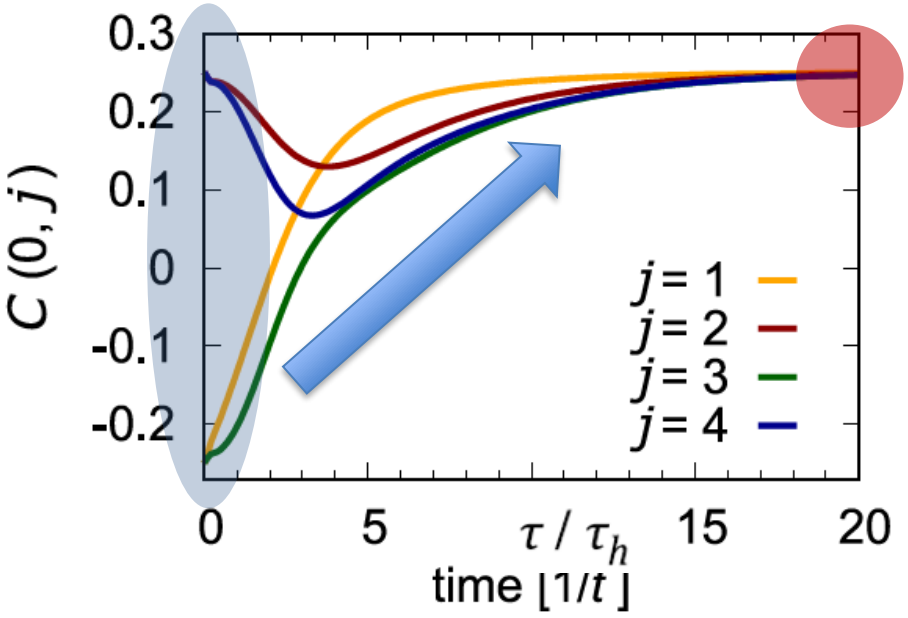
Dynamics of spin correlation of dissipative Hubbard model

M. Nakagawa, *et al.,* PRL **124**, 147203 (2020)

■ **8 particles, 8 sites, $U/t = \gamma/t = 10$**

initial state = AFM Néel state |↑↓↑↓…>

$$C(i,j) = \langle\psi(\tau)|\, \boldsymbol{S}_i \cdot \boldsymbol{S}_j \,|\psi(\tau)\rangle \,/\, \langle\psi(\tau)|\psi(\tau)\rangle$$



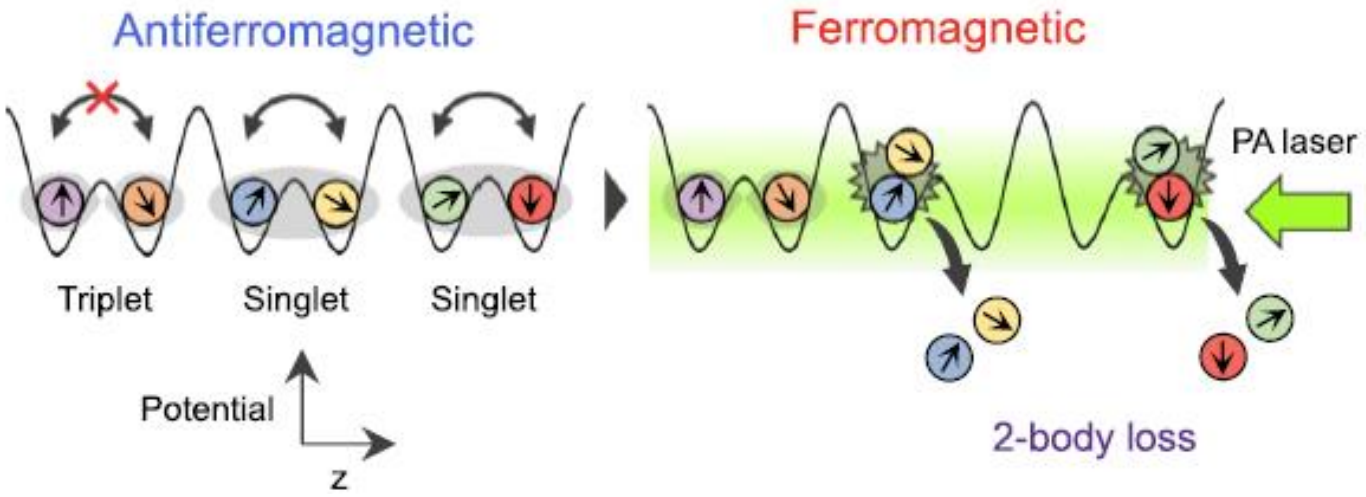
C (0, *j*) = 0.25 for all *j*
perfect ferromagnetism
corresponding to the
highest-energy state of
AFM Heisenberg model

Magnetism is inverted by dissipation!

Observation of the Sign Reversal of Magnetic Correlation from AFM to FM in a Dissipative Fermi Gas

K. Honda, *et al*., PRL **130**, 063001 (2023)

^{173}Yb (m_F=-5/2, -3/2,..., 5/2) at T/T_F~0.15

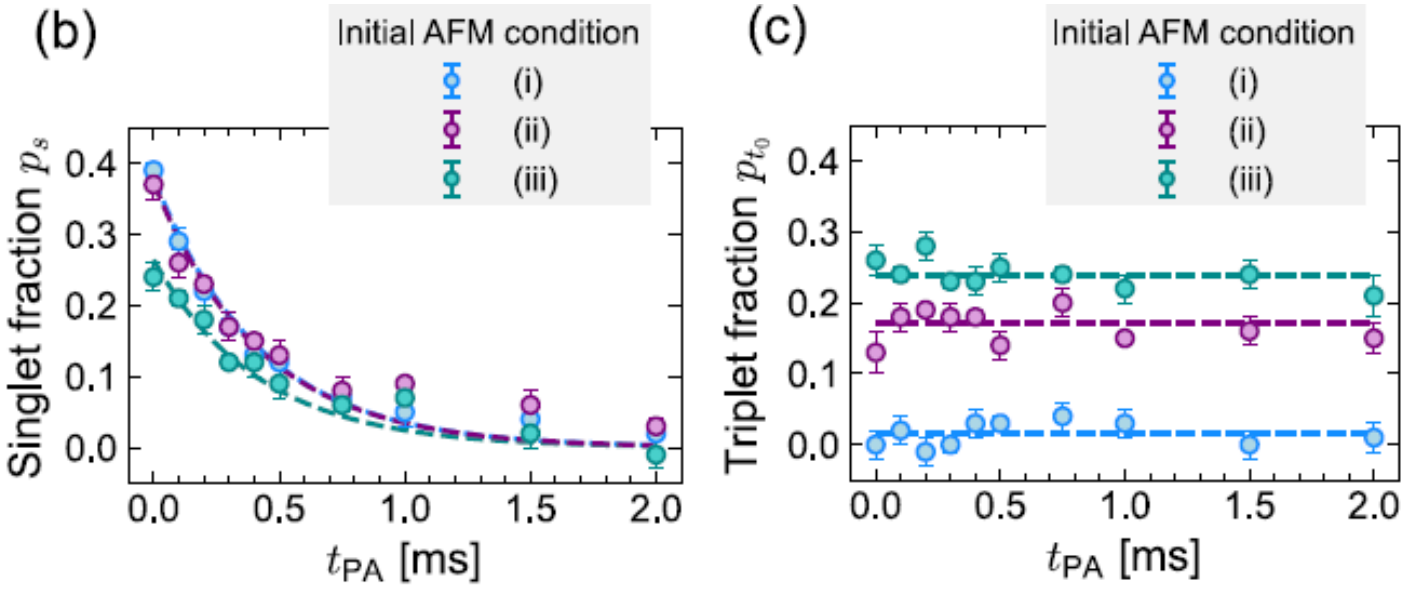


Spin-singlet pair: spatially symmetric wf → same site → loss

Spin-triplet pair: cannot occupy the same site → stable

Observation of the Sign Reversal of Magnetic Correlation from AFM to FM in a Dissipative Fermi Gas

K. Honda, *et al*., PRL **130**, 063001 (2023)



The singlet fraction decays exponentially, while the triplet fraction stays constant, indicating the ferromagnetic transition.

Non-orthogonal eigenstates:

Continuous phase transition w/o gap closing

N. Matsumoto, *et al*., Phys. Rev. Lett. **125**, 260601 (2020)

In Hermitian systems, quantum phase transitions are accompanied by either

gap closing	**or**	**level crossing**
continuous QPT		**1st order QPT**

The gap closing causes divergence in the fidelity susceptibility which is a signature of continuous QPT.

In non-Hermitian systems, non-orthogonality of eigenstates could lead to divergence of the fidelity susceptibility without gap closing.

Fidelity susceptibility

$H(\lambda) = H_0 + \lambda V$ V causes a critical phenomenon.

$|\psi_n^R(\lambda)\rangle$ $(\langle\psi_n^L(\lambda)|)$ right (left) eigenstate with eigenenergy $E_n(\lambda)$

small variation of the coupling constant

$$F(\lambda, \delta\lambda) := \left|\langle\psi_0^R(\lambda)|\psi_0^R(\lambda+\delta\lambda)\rangle\right|$$

$$= 1 - \frac{1}{2}\delta\lambda^2 \sum_{m,n\neq 0} \frac{\langle\psi_0^R|V^\dagger|\psi_m^L\rangle}{(E_0 - E_m)^*} \frac{\langle\psi_n^L|V|\psi_0^R\rangle}{E_0 - E_n} \left(\langle\psi_m^R|\psi_n^R\rangle - \langle\psi_m^R|\psi_0^R\rangle\langle\psi_0^R|\psi_n^R\rangle\right)$$

fidelity susceptibility $\chi_F(\lambda)$

For Hermitian systems, this double sum reduces to a single sum due to the orthogonality of eigenstates:

$$\chi_F(\lambda) = \sum_n{}' \frac{|\langle\psi_n(\lambda)|V|\psi_0(\lambda)\rangle|^2}{|E_0(\lambda) - E_n(\lambda)|^2}$$

divergence ↔ gap closes

single sum

The denominator should vanish.

Nonorthogonality-induced divergence

However, for non-Hermitian systems, eigenstates are not orthogonal to each other.

small variation of the coupling constant

$$F(\lambda,\delta\lambda) := \left|\left\langle\psi_0^R(\lambda)\middle|\psi_0^R(\lambda+\delta\lambda)\right\rangle\right|$$

$$= 1 - \frac{1}{2}\delta\lambda^2 \sum_{m,n\neq 0} \frac{\left\langle\psi_0^R\middle|V^\dagger\middle|\psi_m^L\right\rangle}{(E_0-E_m)^*} \frac{\left\langle\psi_n^L\middle|V\middle|\psi_0^R\right\rangle}{E_0-E_n} \left(\left\langle\psi_m^R\middle|\psi_n^R\right\rangle - \left\langle\psi_m^R\middle|\psi_0^R\right\rangle\left\langle\psi_0^R\middle|\psi_n^R\right\rangle\right)$$

fidelity susceptibility $\chi_F(\lambda)$

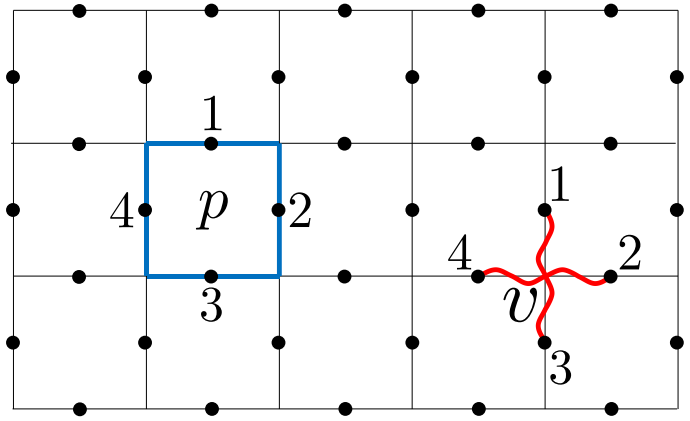
double sum due to nonorthogonality

This double sum can cause divergence, which leads to a quantum phase transition without gap closing!

Toric-Code Model

A. Yu. Kitaev, Ann. Phys. **303**, 2 (2003)

A ½ spin is placed on every edge of a square lattice.



$$H(\beta) = -\sum_{v\in\{\text{vertex}\}} \sigma^{\beta}_{v,1}\sigma^{\beta}_{v,2}\sigma^{\beta}_{v,3}\sigma^{\beta}_{v,4} - \sum_{p\in\{\text{plaquette}\}} \sigma^{z}_{p,1}\sigma^{z}_{p,2}\sigma^{z}_{p,3}\sigma^{z}_{p,4}$$

with $\beta = x$

Non-Hermitian Toric-Code Model

N. Matsumoto *et al.*, PRL **125**, 260601 (2020)

$$H(\beta) = -\sum_{v\in\{\text{vertex}\}} \sigma^{\beta}_{v,1}\sigma^{\beta}_{v,2}\sigma^{\beta}_{v,3}\sigma^{\beta}_{v,4} - \sum_{p\in\{\text{plaquette}\}} \sigma^{z}_{p,1}\sigma^{z}_{p,2}\sigma^{z}_{p,3}\sigma^{z}_{p,4}$$

Pauli β

$$\sigma_i^{\beta} := \cosh(\beta)\sigma_i^x + i\sinh(\beta)\sigma_i^y = \begin{pmatrix} 0 & \mathrm{e}^{\beta} \\ \mathrm{e}^{-\beta} & 0 \end{pmatrix}$$

asymmetric spin flip

$(\beta > 0)$

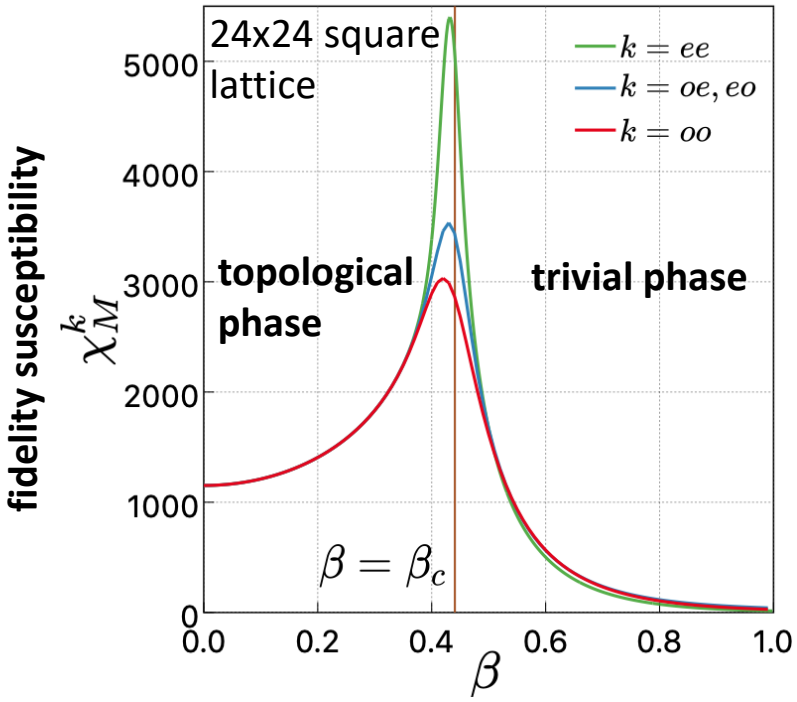
β parametrizes the strength of non-Hermiticity.

β=0→Hermitian toric-code model.

We can show that in this model there is no gap closing.

Non-Hermitian Toric-Code Model

N. Matsumoto *et al.*, PRL **125**, 260601 (2020)



Yet, the fidelity susceptibility diverges at the transition point between topological and trivial phases.

Summary: fundamentals

- **nonunitary dynamics / *PT*-symmetry**

  → **surpass the Lieb-Robinson bound** PRL 120, 185301 (2018)

- **complex eigenspectrum**

  → **intermediate-state eng., esp. magnetism reversal**

  PRL 124, 147203 (2020)

- **nonorthogonality of eigenstates**

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  PRL 125, 260601 (2020)

End of Day 1

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Yesterday: fundamentals

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  PRL 125, 260601 (2020)

Today: applications

- **Statistical physics:**

  Yang-Lee zeros & nonunitary criticality

  PR Res. 4, 033250 (2022)
  PRL 132, 176601 (2024)

- **Symmetry and topology in physics**

  Nat. Commun. 10, 297 (2019)
  PRX 8, 031079 (2018)
  PRX 9, 041015 (2020)

Applications to statistical physics:

Yang-Lee zeros

What's Yang-Lee Zeros?

C. N. Yang & T. D. Lee, Phys. Rev. **87**, 404 (1952)
T. D. Lee & C. N. Yang, Phys. Rev. **87**, 410 (1952)

Yang-Lee zeros are the zero points of the partition function.

The partition function vanishes only when some physical parameters, such as an external magnetic field and temperature, are made complex.

This is definitely unphysical. But if we know the distribution of zeros, analytic properties of the partition function can be found.

Yang and Lee noted this fact to investigate fundamental properties of phase transitions, such as critical exponents.

Yang-Lee Zeros and Edge Singularity

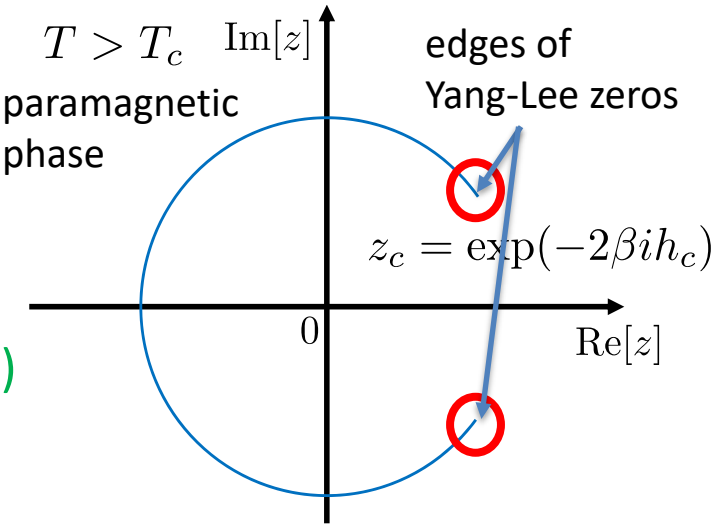
Yang and Lee proved that zeros of the grand partition function are distributed on a unit circle of a complex fugacity $z=\exp(-2\beta H)$.

C. N. Yang & T. D. Lee, Phys. Rev. **87**, 404 (1952)
T. D. Lee & C. N. Yang, Phys. Rev. **87**, 410 (1952)

The edges of the distribution are shown to exhibit nonunitary critical phenomena known as the Yang-Lee edge singularity.

P. Kortman & R. Griffiths, PRL **27**, 1439 (1971)
M. Fisher, PRL **40**, 1610 (1978)

Since the Yang-Lee edges appear for imaginary magnetic fields ($H=\pm ih$), physical implementation of Yang-Lee zeros is highly nontrivial.



density of the distribution of Yang-Lee zeros near the edges

$$g(h) \propto (h - h_c)^{\sigma}$$

(critical exponent $\sigma < 1$)

Mapping and Embedding

Lee and Yang considered the classical one-dimensional Ising model under an imaginary magnetic field:

$$H = -J \sum_j \sigma_j \sigma_{j+1} - ih \sum_j \sigma_j$$

This classical system can be mapped via the quantum-classical correspondence to a non-Hermitian quantum system described by a *PT*-symmetric Hamiltonian:

$$H_{\mathrm{PT}} = R(\cos\phi)\sigma^x + iR(\sin\phi)\sigma^z$$

$$[H_{\mathrm{PT}}, \mathcal{PT}] = 0 \text{ with } \mathcal{P} = \sigma^x \text{ and } \mathcal{T} = \mathcal{K}$$

N. Matsumoto *et al.*, Phys. Rev. Res. **4**, 033250 (2022)

Embedding the non-Hermitian system in an extended Hermitian system

U. Günther & B. F. Samsonov, Phys. Rev. Lett. **101**, 230404 (2008)
K. Kawabata *et al*., Phys. Rev. Lett. **119,** 190401 (2017)

The dynamics governed by H_{PT} is realized by embedding it in an extended Hermitian system of the system and ancilla:

$$H_{\mathrm{tot}} = r\sin\theta\, I_{\mathrm{A}} \otimes \sigma^x + r\cos\theta\, \sigma^y_{\mathrm{A}} \otimes \sigma^z$$

$$|\psi\rangle^{\mathrm{PT}}_{\mathrm{tot}} = |\uparrow\rangle_{\mathrm{A}} \otimes |\psi\rangle + |\downarrow\rangle_{\mathrm{A}} \otimes (\eta|\psi\rangle) \quad \textbf{(initial state)}$$

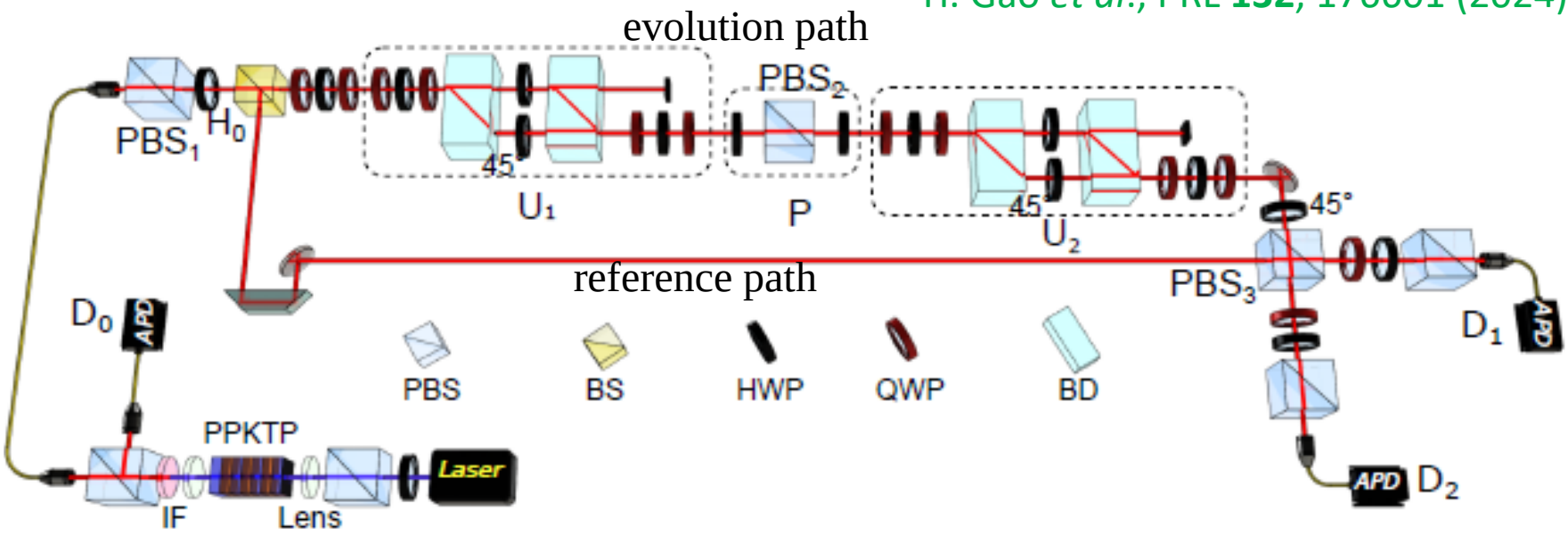
$$e^{-itH_{\mathrm{tot}}}|\psi\rangle^{\mathrm{PT}}_{\mathrm{tot}} = |\uparrow\rangle_{\mathrm{A}} \otimes \boxed{e^{-itH_{\mathrm{PT}}}|\psi\rangle} + |\downarrow\rangle_{\mathrm{A}} \otimes \eta e^{-itH_{\mathrm{PT}}}|\psi\rangle$$

By measuring the ancilla qubit after time evolution and postselecting the ancilla state $|\uparrow\rangle_{\mathrm{A}}$, we obtain the desired time evolution of the system governed by H_{PT}.

N. Matsumoto *et al*., Phys. Rev. Res. **4**, 033250 (2022)

Direct Observation of Yang-Lee Physics using Heralded Photons

H. Gao *et al.*, PRL **132**, 176601 (2024)



The Yang-Lee physics of the mapped *PT*-symmetric system has been experimentally implemented using heralded photons.

The horizontal and vertical polarization states of a heralded single photon are used for a qubit system ($|0\rangle = |H\rangle$, $|1\rangle = |V\rangle$) and optical loss is fine-tuned to dynamically simulate the *PT*-symmetric system.

Advantages of Photonic Experiments

H. Gao *et al*., PRL **132**, 176601 (2024)

Typical measurements yield a maximum of 240,000 photon counts per second, ensuring sufficient statistical accuracy for testing nonunitary critical phenomena.

Not just physical quantities such as magnetization *m*, but also the partition function *Z* can be measured directly.

$$m = \langle \sigma_z \rangle = \frac{\langle H| e^{-\beta H_{\mathcal{PT}}} |H\rangle - \langle V| e^{-\beta H_{\mathcal{PT}}} |V\rangle}{\langle H| e^{-\beta H_{\mathcal{PT}}} |H\rangle + \langle V| e^{-\beta H_{\mathcal{PT}}} |V\rangle}$$

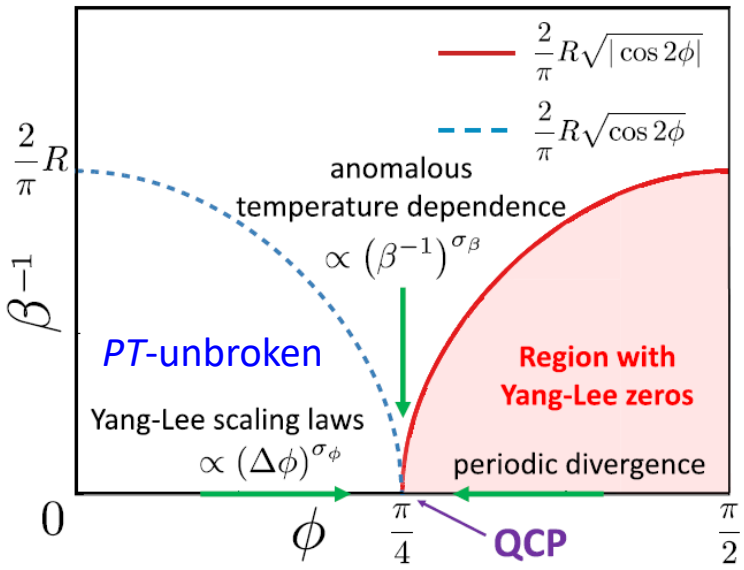
$$Z = \langle H| e^{-\beta H_{\mathcal{PT}}} |H\rangle + \langle V| e^{-\beta H_{\mathcal{PT}}} |V\rangle \quad (t = -\mathrm{i}\,\beta)$$

PT-unbroken phase: classical Yang-Lee scaling

H. Gao *et al.*, PRL **132**, 176601 (2024)

In the *PT*-unbroken phase, experiments on m and χ show scaling laws consistent with the classical Yang-Lee scaling laws:

$$m \to \frac{-i\sin\phi}{\sqrt{\cos 2\phi}} \propto \Delta\phi^{-\frac{1}{2}}, \qquad \chi \to \frac{-i\cos^3\phi}{\cos^{\frac{3}{2}} 2\phi} \propto \Delta\phi^{-\frac{3}{2}}$$

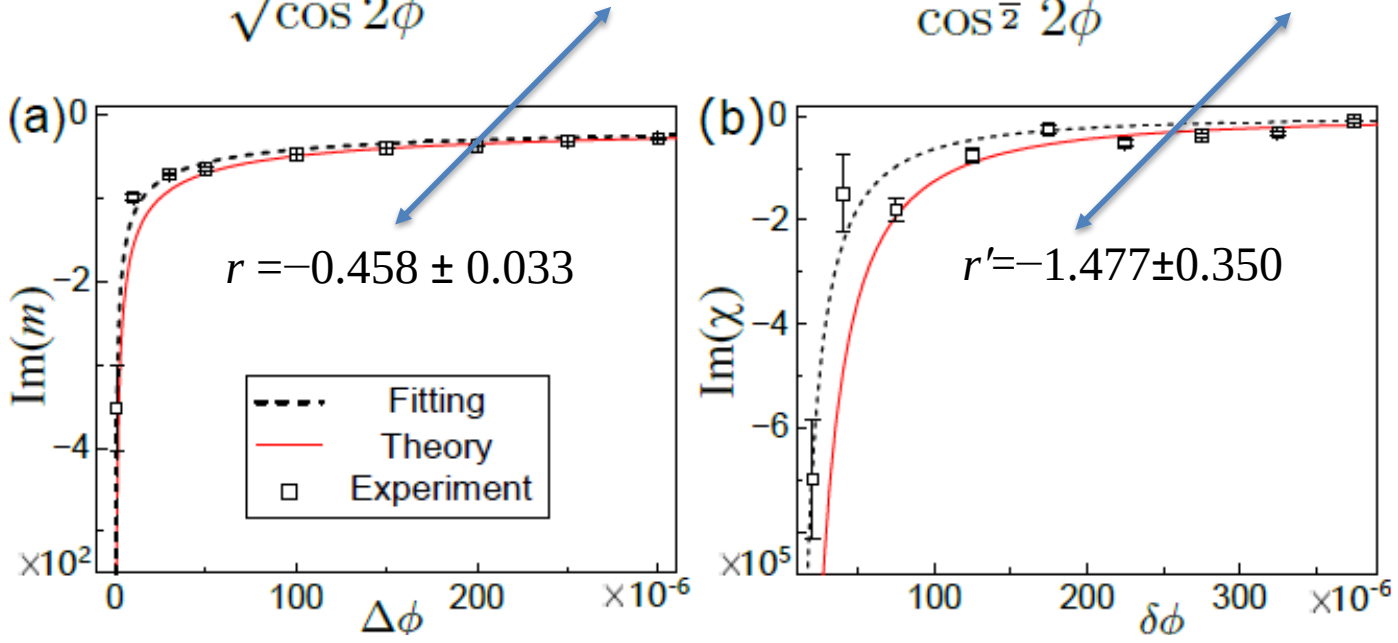


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The measured results are consistent with theoretical predictions.

At the *PT* transition point: anomalous scaling

H. Gao *et al*., PRL **132**, 176601 (2024)

At the *PT* transition point, the measured m and χ show anomalous scaling with negative critical exponents (-1 and -3):

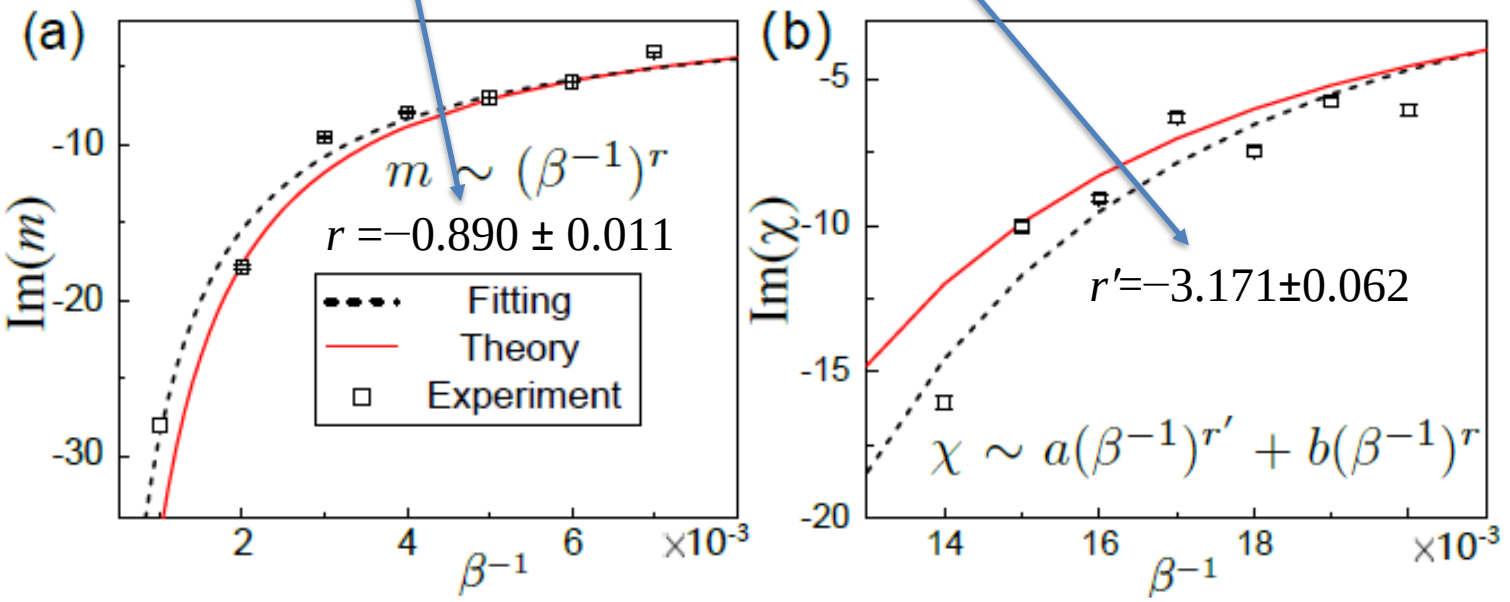
$$m \to -\frac{i}{\sqrt{2}}\beta R, \quad \chi \to -\frac{i}{3\sqrt{2}}(\beta^3 R^3 + \frac{3}{2}\beta R) \quad t = -i\beta$$

At the *PT* transition point: anomalous scaling

H. Gao *et al.*, PRL **132**, 176601 (2024)

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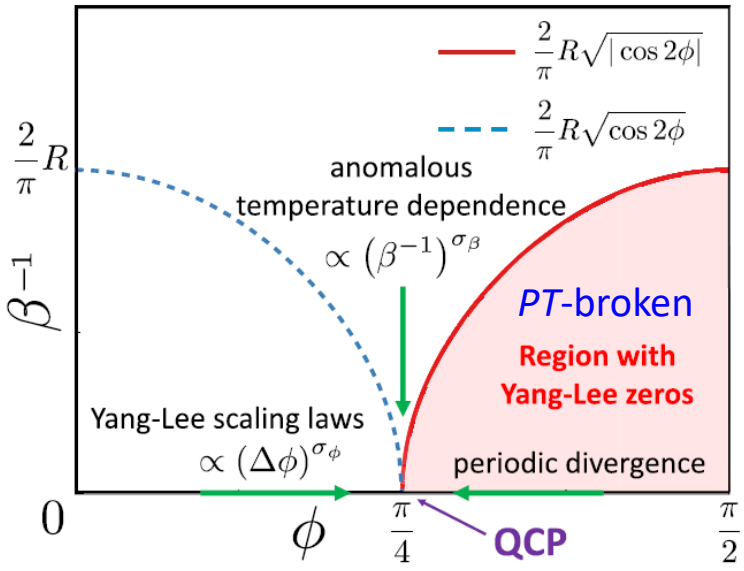


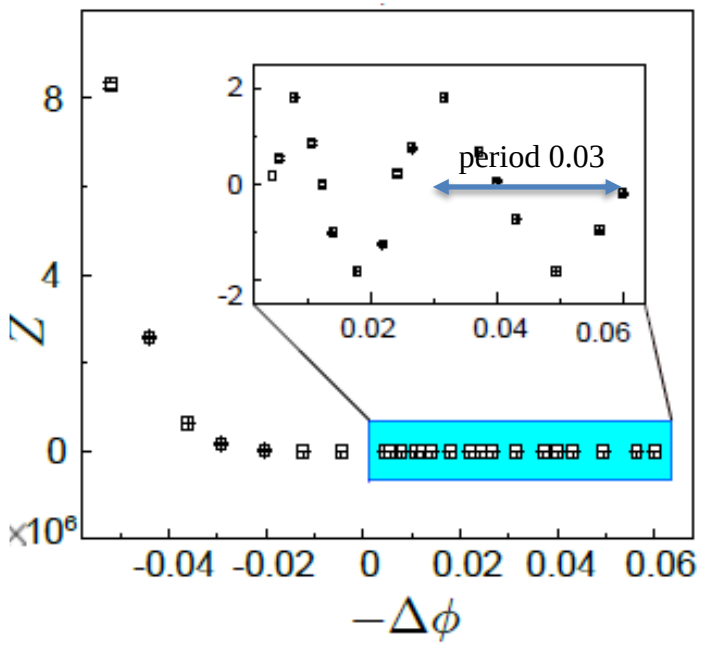
The measured results are consistent with theoretical predictions.

Direct observation of Yang-Lee zeros

H. Gao *et al.*, PRL **132**, 176601 (2024)

In the *PT*-broken phase, the measured partition function *Z* show periodic oscillations with period ~ 0.03 consistent with the theoretical prediction of π/100.

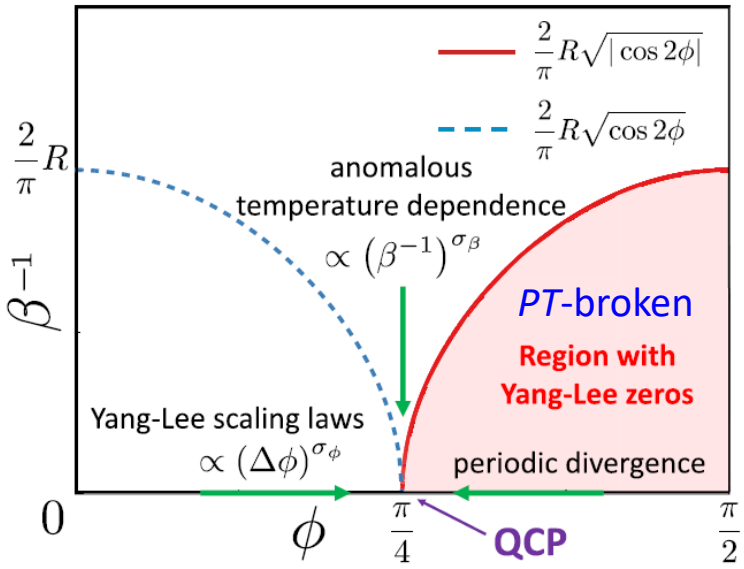


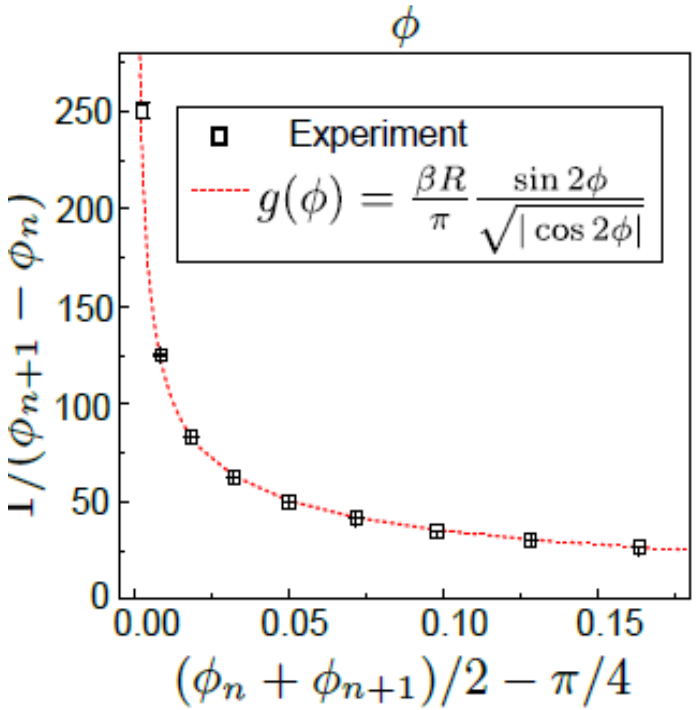


Observation of Yang-Lee edge singularity

H. Gao *et al.*, PRL **132**, 176601 (2024)

The observed density of Yang-Lee zeros $1/(\phi_{n+1}-\phi_n)$ for n=1,2, …,9 show divergent behavior near the critical point π/4, which agrees with our prediction of the Yang-Lee edge singularity.





Yang-Lee Zeros in BCS Superconductivity

H. Li, *et al*., Phys. Rev. Lett. **131**, 216001 (2023)

BCS Superfluidity with Complex Coupling

K. Yamamoto, *et al*., Phys. Rev. Lett. **127**, 123601 (2019)

We consider a BCS model with a complex coupling constant U:

$$H = \sum_{k\sigma} \xi_k c^\dagger_{k\sigma} c_{k\sigma} - \frac{U}{N} \sum_{k,k'}{}' c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow} \quad U = U_R + iU_I$$

The BEC wavefunction has the left and right components:

$$|\mathrm{BCS}\rangle_R = \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0\rangle \qquad u_k^2 + v_k \bar{v}_k = 1$$

$$|\mathrm{BCS}\rangle_L = \prod_k (u^*_k + \bar{v}^*_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0\rangle$$

The Bogoliubov energy spectrum and the energy gap:

$$E_k = \sqrt{\xi_k^2 + \Delta_k^2} \qquad \Delta_k = \Delta_0 \theta(\omega_D - |\xi_k|)$$

$$\Delta_0 = \frac{\omega_D}{\sinh\left(\frac{1}{\rho_0 U}\right)}$$

Δ_0 is complex in general, so is E_k.

The Semicircle Theorem

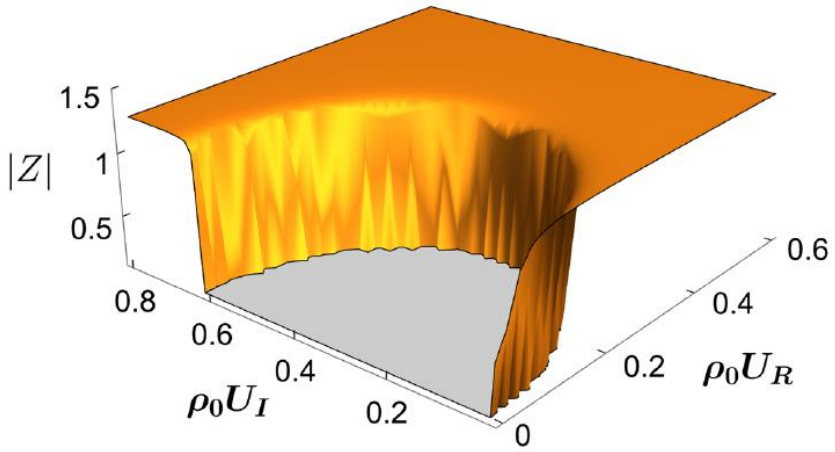
H. Li, *et al.*, Phys. Rev. Lett. **131**, 216001 (2023)

The zeros of the partition function are given by

$$Z = \prod_{k,\sigma} Z_k = \prod_{k,\sigma} (1 + e^{-\beta E_k}) = 0 \quad \Rightarrow \quad \begin{aligned} &\mathrm{Re}(E_k) = 0 \text{ and} \\ &\mathrm{Im}(\beta E_k) = (2n+1)\pi \end{aligned}$$

$$(\rho_0 \pi U_R)^2 + (\rho_0 \pi U_I - 1)^2 = 1, \quad U_R > 0 \qquad U = U_R + iU_I$$

Yang-Lee zeros are distributed on a semicircle on the complex plane of the coupling constant in sharp contrast to the full circle theorem of the Ising model.



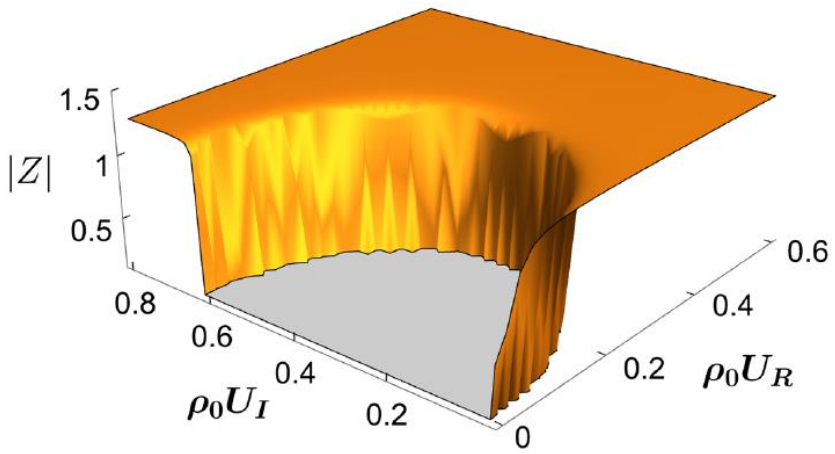
Why semicircle rather than full circle?

H. Li, *et al.*, Phys. Rev. Lett. **131**, 216001 (2023)

This is due to the attractive interaction ($U_R > 0$) needed for pairing instability.

$$(\rho_0 \pi U_R)^2 + (\rho_0 \pi U_I - 1)^2 = 1, \quad U_R > 0 \qquad U = U_R + iU_I$$

Thus the semicircle arises from the Fermi surface instability!



Physical Meaning of Yang-Lee zeros

H. Li, *et al.*, Phys. Rev. Lett. **131**, 216001 (2023)

The number χ of Yang-Lee zeros is directly connected with Δ_0.

Yang-Lee zeros are specified by the conditions:

$$\mathrm{Re}(E_{\boldsymbol{k}}) = 0 \quad \text{and} \quad \mathrm{Im}(\beta E_{\boldsymbol{k}}) = (2n+1)\pi$$

On the semicircle, the gap becomes pure imaginary, so the number of Yang-Lee zeros becomes equal to the number of n's that satisfy $\mathrm{Im}(\beta E_{\mathrm{k}})=(2n+1)\pi$ for $\mathrm{Im}(\beta E_{\mathrm{k}}) \in [0, \beta\, \mathrm{Im}(\Delta_0)]$. Thus

$$\chi \simeq \frac{\beta \mathrm{Im}(\Delta_0)}{\pi} = \frac{\beta\omega_D}{\pi \cosh\left(\frac{U_R}{\rho_0 |U|^2}\right)}$$

Remarkably, the number of Yang-Lee zeros gives the SC gap. Underlying physics deserves deeper understanding.

Symmetry and Topology in Open Systems

Topological phases in closed systems

In closed (i.e., Hermitian) systems, the number of topologically distinct phases is known to be 10.

A. Altland and M. R. Zirnbauer, PRB **55**, 1142 (1997)

Symmetry Class	Time reversal symmetry	Particle hole symmetry	Chiral symmetry
A	No	No	No
AIII	No	No	Yes
AI	Yes, $T^2 = 1$	No	No
BDI	Yes, $T^2 = 1$	Yes, $C^2 = 1$	Yes
D	No	Yes, $C^2 = 1$	No
DIII	Yes, $T^2 = -1$	Yes, $C^2 = 1$	Yes
AII	Yes, $T^2 = -1$	No	No
CII	Yes, $T^2 = -1$	Yes, $C^2 = -1$	Yes
C	No	Yes, $C^2 = -1$	No
CI	Yes, $T^2 = 1$	Yes, $C^2 = -1$	Yes

Topological phases in open systems

How many topologically dintinct phases are possible once we remove the Hermiticity condition?

A Key Insight

A key insight can be obtained if we note

$$\text{reciprocity } (\boldsymbol{O^T}) = \text{time reversal } (\boldsymbol{O^*})$$

Hermiticity

In Hermitian physics, reciprocity and time reversal are one and the same.

Chiral symmetry ($\Gamma H^\dagger \Gamma^{-1} = -H$) and sublattice symmetry ($\Gamma H \Gamma^{-1} = -H$) are also equivalent in Hermitian physics.

If we remove these constraints, we can achieve

Topological unification and ramification

Z. Gong, *et al*., PRX **8**, 031079 (2018)

K. Kawabata, *et al*., Nat. Commun. **10**, 297 (2019)

K. Kawabata, *et al*., PRX **9**, 041015 (2019)

Y. Ashida, *et al*., Adv. Phys. **69**, 249 (2020

Hermitian **non-Hermitian**

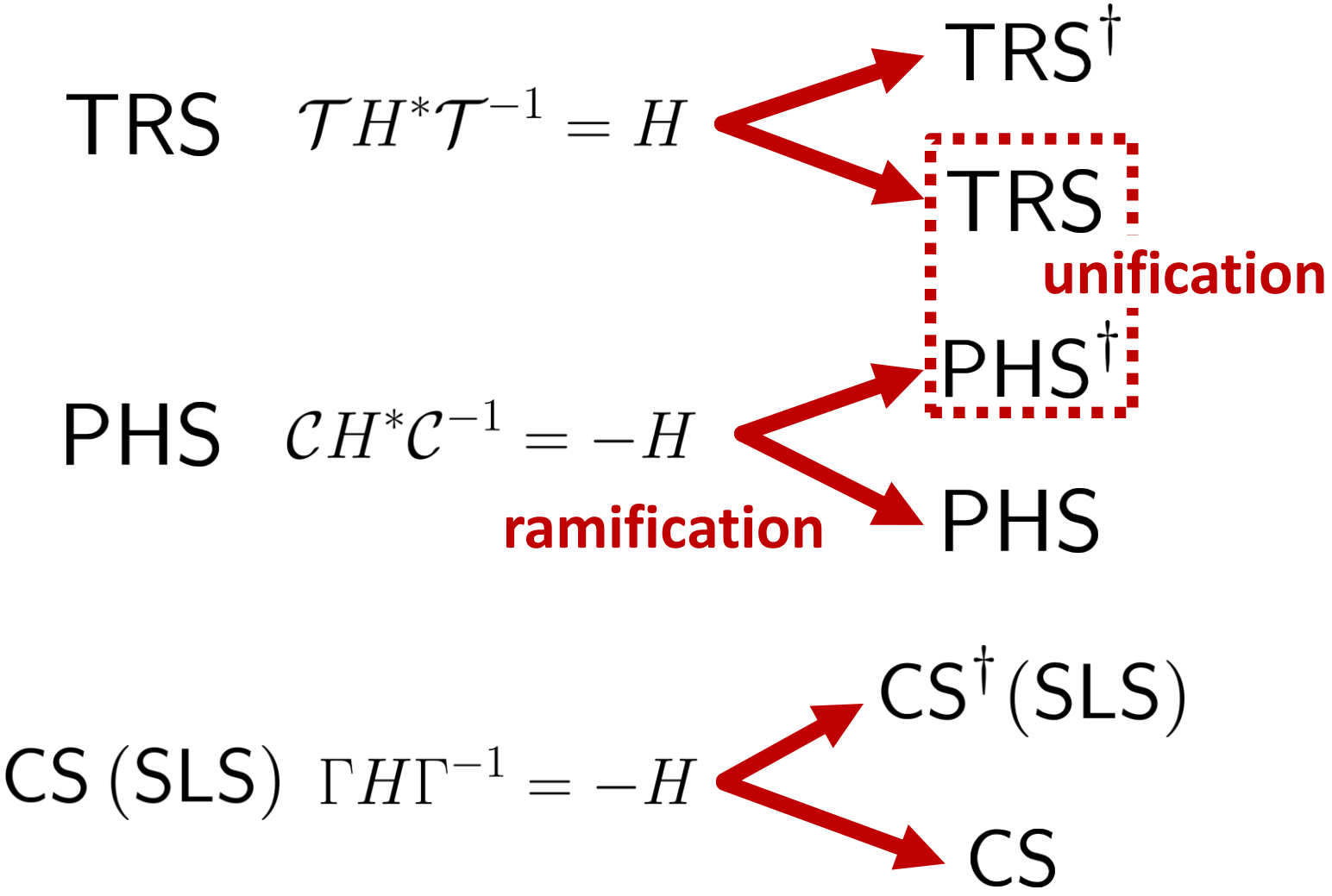
TRS $\mathcal{T}H^*\mathcal{T}^{-1} = H$ → TRS†, TRS

PHS $\mathcal{C}H^*\mathcal{C}^{-1} = -H$ → PHS†, PHS

ramification

CS (SLS) $\Gamma H\Gamma^{-1} = -H$ → CS† (SLS), CS

Hermitian **non-Hermitian**



Symmetry Ramification: TRS

Removal of the Hermitian constraint **ramifies** symmetry.

K. Kawabata, *et al*., PRX **9**, 041015 (2019)

Hermitian TRS

$$\mathcal{T} H^* \mathcal{T}^{-1} = H$$

unitary operator

Symmetry Ramification: TRS

Removal of the Hermitian constraint **ramifies** symmetry.

K. Kawabata, *et al*., PRX **9**, 041015 (2019)

Hermitian TRS

$$\mathcal{T} H^* \mathcal{T}^{-1} = H$$

$$\| \quad \text{c.c.} \quad \text{transposition}$$

$$H^* = H^T$$

$$\mathcal{T} H^T \mathcal{T}^{-1} = H$$

Symmetry Ramification: TRS

Removal of the Hermitian constraint **ramifies** symmetry.

K. Kawabata, *et al*., PRX **9**, 041015 (2019)

Hermitian TRS → **non-Hermitian TRS**

$$\mathcal{T}H^*\mathcal{T}^{-1} = H \quad \longrightarrow \quad \mathcal{T}H^*\mathcal{T}^{-1} = H$$

$$\| \; H^* = H^T \qquad \text{nonequivalent} \qquad \not\| \; H^* \neq H^T$$

$$\mathcal{T}H^T\mathcal{T}^{-1} = H \quad \longrightarrow \quad \mathcal{T}H^T\mathcal{T}^{-1} = H$$

Time reversal symmetry ramifies into two distinct classes.

Symmetry Ramification: TRS

Removal of the Hermitian constraint **ramifies** symmetry.

K. Kawabata, *et al*., PRX **9**, 041015 (2019)

Hermitian TRS

$$\mathcal{T}H^*\mathcal{T}^{-1} = H$$

$$H^* = H^T$$

$$\mathcal{T}H^T\mathcal{T}^{-1} = H$$

non-Hermitian TRS

time reversal

$$\mathcal{T}H^*\mathcal{T}^{-1} = H$$

$$H^* \neq H^T$$

$$\mathcal{T}H^T\mathcal{T}^{-1} = H$$

reciprocity

Symmetry Ramification: TRS

Removal of the Hermitian constraint **ramifies** symmetry.

K. Kawabata, *et al*., PRX **9**, 041015 (2019)

Hermitian TRS → **non-Hermitian TRS**

time reversal

$$\mathcal{T}H^*\mathcal{T}^{-1} = H \quad \longrightarrow \quad \mathcal{T}H^*\mathcal{T}^{-1} = H$$

$$H^* = H^T \qquad H^* \neq H^T$$

$$\mathcal{T}H^T\mathcal{T}^{-1} = H \quad \longrightarrow \quad \mathcal{T}H^T\mathcal{T}^{-1} = H$$

reciprocity

They lead to the same physics for Hermitian Hamiltonians but predict different physics for non-Hermitian Hamiltonians.

Examples -- symmetry-protected topological lasers
dissipative topological superconductors

Symmetry Ramification: CS

Removal of the Hermitian constraint **ramifies** symmetry.

K. Kawabata, *et al*., PRX **9**, 041015 (2019)

Hermitian CS=SLS

$$\Gamma H \Gamma^{-1} = -H$$

Symmetry Ramification: CS

Removal of the Hermitian constraint **ramifies** symmetry.

K. Kawabata, *et al*., PRX **9**, 041015 (2019)

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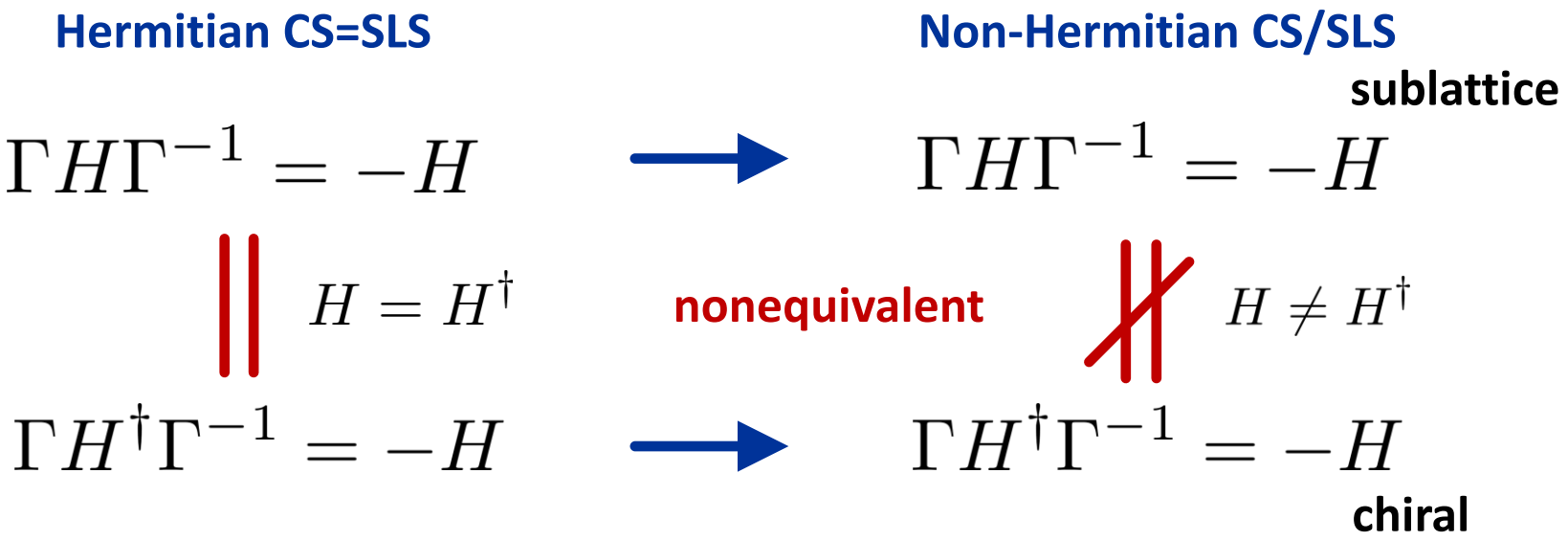
$$\| \quad H = H^\dagger$$

$$\Gamma H^\dagger \Gamma^{-1} = -H$$

Symmetry Ramification: CS

Removal of the Hermitian constraint **ramifies** symmetry.

K. Kawabata, *et al*., PRX **9**, 041015 (2019)



Hermitian **Non-Hermitian**

TRS $\mathcal{T}H^*\mathcal{T}^{-1} = H$

- TRS† $\mathcal{T}H^T\mathcal{T}^{-1} = H$
- TRS $\mathcal{T}H^*\mathcal{T}^{-1} = H$

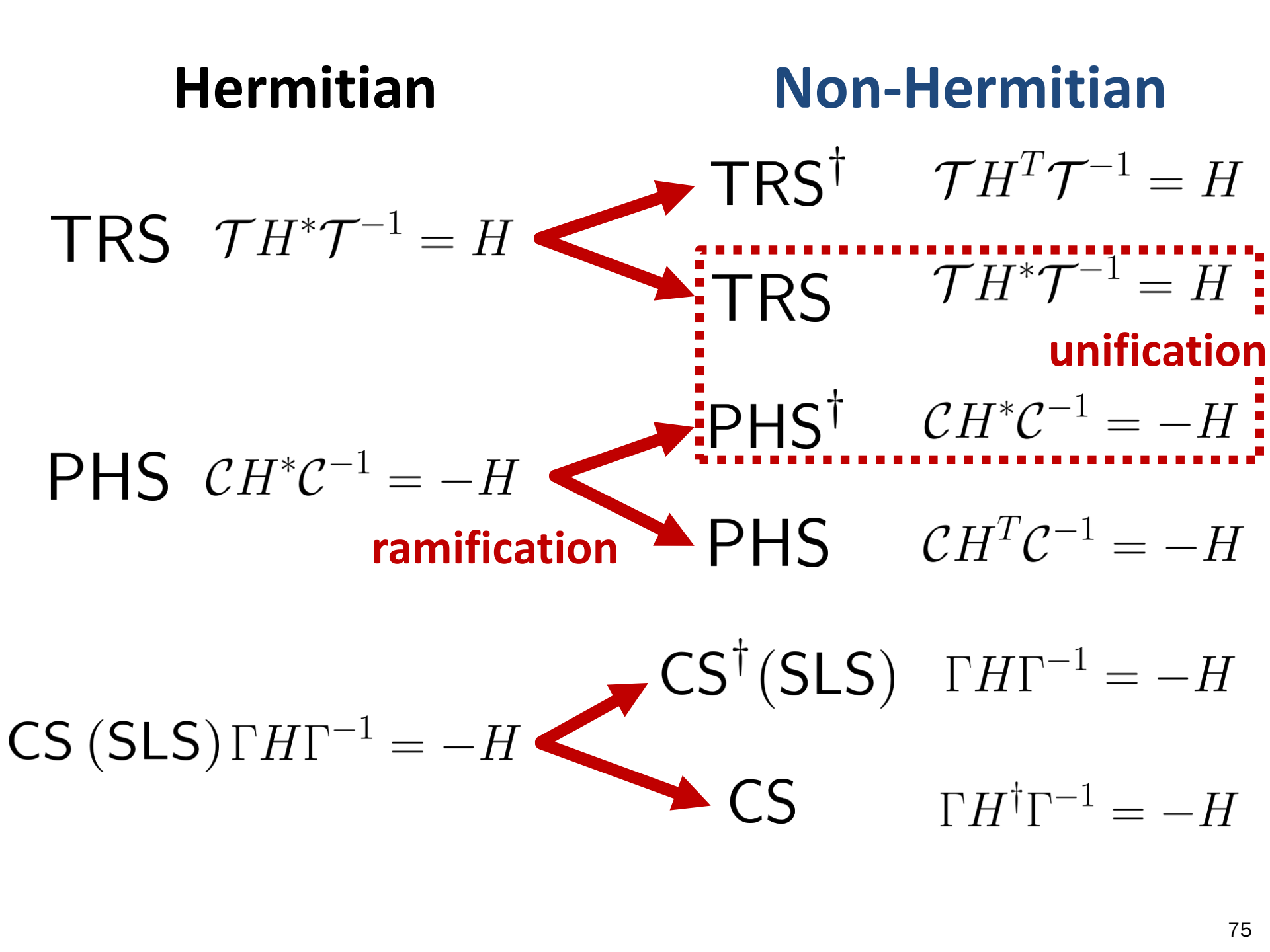
PHS $\mathcal{C}H^*\mathcal{C}^{-1} = -H$

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- PHS $\mathcal{C}H^T\mathcal{C}^{-1} = -H$

ramification

CS (SLS) $\Gamma H\Gamma^{-1} = -H$

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Hermitian
Non-Hermitian
TRS $\mathcal{T}H^*\mathcal{T}^{-1}=H$
PHS $\mathcal{C}H^*\mathcal{C}^{-1}=-H$
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ramification
TRS† $\mathcal{T}H^T\mathcal{T}^{-1}=H$
TRS $\mathcal{T}H^*\mathcal{T}^{-1}=H$
unification
PHS† $\mathcal{C}H^*\mathcal{C}^{-1}=-H$
PHS $\mathcal{C}H^T\mathcal{C}^{-1}=-H$
CS† (SLS) $\Gamma H\Gamma^{-1}=-H$
CS $\Gamma H^\dagger\Gamma^{-1}=-H$

Symmetry Unification

K. Kawabata *et al.*, Nat. Commun. **10**, 297 (2019)

Time-revsersal symmetry and the Hermitian conjugate of particle-hole symmetry are defined by

$$\left.\begin{array}{ll} \text{TRS} & \mathcal{T}H^*\mathcal{T}^{-1} = H \\ \text{PHS}^\dagger & \mathcal{C}H^*\mathcal{C}^{-1} = -H \end{array}\right\} \text{ antiunitary}$$

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K. Kawabata *et al.*, Nat. Commun. **10**, 297 (2019)

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$$\text{TRS} \quad \mathcal{T}H^*\mathcal{T}^{-1} = H$$
$$\text{PHS}^\dagger \quad \mathcal{C}H^*\mathcal{C}^{-1} = -H$$

These symmetry classes become equivalent to each other once the Hermitian constraint is removed.

$$\mathcal{T}H^*\mathcal{T}^{-1} = H \longleftrightarrow \mathcal{T}\left[\mathrm{i}H\right]^*\mathcal{T}^{-1} = \underline{\underline{-}}\left[\mathrm{i}H\right]$$

one-to-one mapping
(not change the eigenstates)

Symmetry Unification

K. Kawabata *et al.*, Nat. Commun. **10**, 297 (2019)

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These symmetry class… equivalent to each other once the Hermiti… removed.

$$= H \longleftrightarrow \mathcal{T}\left[\mathrm{i}H\right]^*\mathcal{T}^{-1} = \underline{=}\left[\mathrm{i}H\right]$$

one-to-one mapping
(not change the eigenstates)

The antiunitary symmetries that are distinct under the Hermitian constraint can thus be unified in non-Hermitian systems.

Symmetry Classification

In Hermitian systems, the celebrated 10-fold Altrand-Zirnbauer symmetry classes constitute the fundamental symmetry classification.

A. Altland and M. R. Zirnbauer, PRB **55**, 1142 (1997)

Symmetry Class	Time reversal symmetry	Particle hole symmetry	Chiral symmetry
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38 Non-Hermitian Symmetry Classes

K. Kawabata, *et al*., PRX **9**, 041015 (2019)

H. Zhou and J. Y. Lee, PRB**99**, 235112 (2019)

Y. Ashida, *et al*., Adv. Phys. **69**, 249 (2020)

- **10** non-Hermitian AZ symmetry classes

$$\mathcal{T}_+ H^* \mathcal{T}_+ = H, \;\; \mathcal{C}_- H^T \mathcal{C}_- = -H, \;\; \Gamma H^\dagger \Gamma = -H$$

- **10** non-Hermitian AZ† symmetry classes

$$\mathcal{C}_+ H^T \mathcal{C}_+ = H, \;\; \mathcal{T}_- H^* \mathcal{T}_- = -H, \;\; \Gamma H \Gamma = -H$$

(**Hermitian conjugate** of the AZ class)

- **22** non-Hermitian AZ symmetry classes with **sublattice symmetry**

$$\mathcal{S} H \mathcal{S} = -H$$ (**NOT** equivalent to chiral symmetry)

38 Non-Hermitian Symmetry Classes

K. Kawabata, *et al*., PRX **9**, 041015 (2019)

H. Zhou and J. Y. Lee, PRB**99**, 235112 (2019)

Y. Ashida, *et al*., Adv. Phys. **69**, 249 (2020)

10 + 10 + 22 – 4 = 38 symmetry classes

overcount due to symmetry unification

- **In total, we have 38 symmetry classes.**
- **This 38-fold symmetry classification is a non-Hermitian generalization of the Altrand-Zirnbauer symmetry classification.**

Summary: toolbox of beyond-Hermitian physics

- **Nonunitary dynamics/ *PT* symmetry**
  - **transport beyond LR bound** PRL **120**, 185301 (2018)
  - **quantum criticality** Nat. Commun. **8**, 15791 (2017)
- **Complex spectra** PRL **124**, 147203 (2020)
  - **intermediate-state engineering & magnetism reversal**
- **Non-orthogonal eigenstates** PRL **125**, 260601 (2020)
  - **continuous phase transitions without gap closing**
- **Applications to statistical physics: Yang-Lee zeros**
  - **embedding & postselection** Phys. Rev. Res. **4**, 033250 (2022)
  - **semicircle theorem in BCS superconductivity** PRL **131**, 216001 (2023)
- **Applications to general physics: topological classes 10 → 38** PRX **8**, 031079 (2018); ibid. **9**, 041015 (2019)

These new phenomena exemplify only a tip of the iceberg of very rich beyond-Hermitian physics!

For a comprehensive review, Adv. Phys. **69**, 249 (2020)

Thank you very much!