DQMP Department of Quantum Matter Physics

Low dimensional quantum gases

T. Giamarchi

http://dqmp.unige.ch/giamarchi/

UNIVERSITÉ DE GENÈVE

FNSNF
FONDS NATIONAL SUISSE
SCHWEIZERISCHER NATIONALFONDS
FONDO NAZIONALE SVIZZERO
SWISS NATIONAL SCIENCE FOUNDATION

MaNEP SWITZERLAND

Plan of the lectures

- Equilibrium, basic notions of Tomonaga-Luttinger liquid
- Quantum transport
- Beyond 1D; Coupled 1D chains, Dimensional crossover

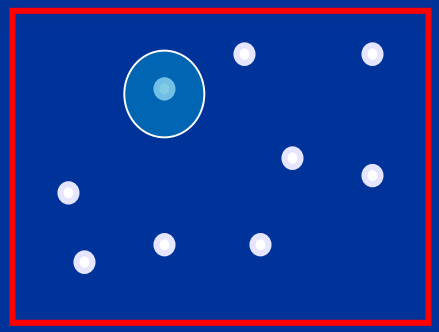
Why one dimension ?

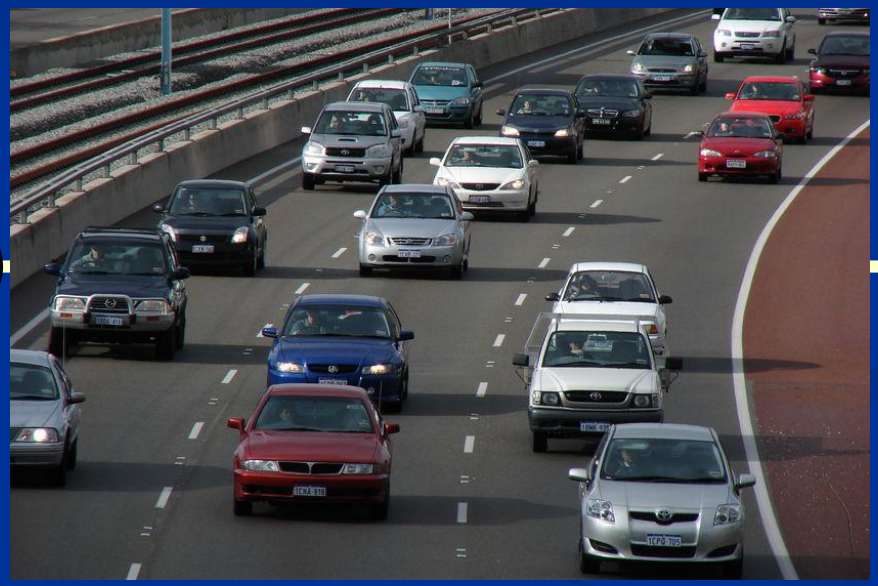
Three urban legends about 1D

- It is a toy model to understand higher dimensional systems.
- It does not exist in nature ! This is only for theorists !
- Everything is understood there anyway !

One dimension is specially interesting

- No individual excitation can exist (only collective ones)



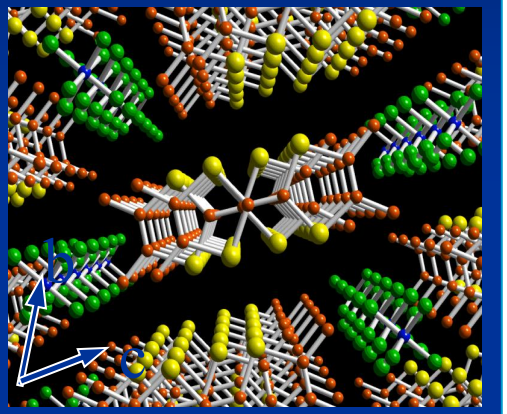


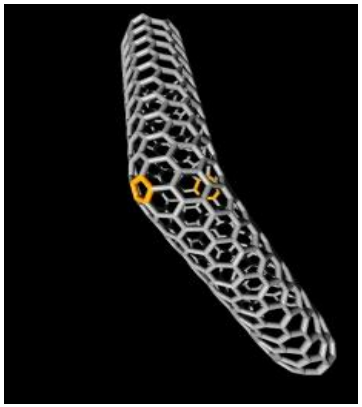
- Strong quantum fluctuations

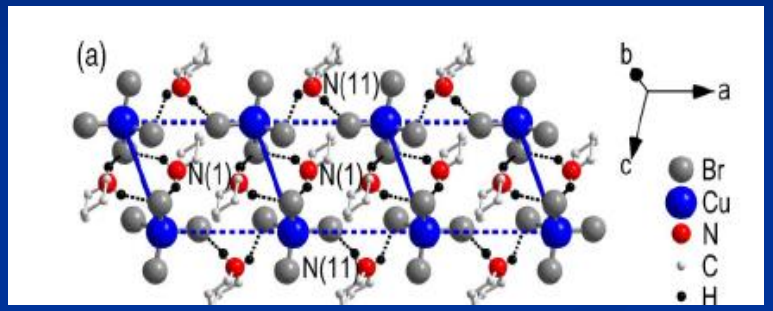
$$\psi = |\psi| e^{i\theta}$$

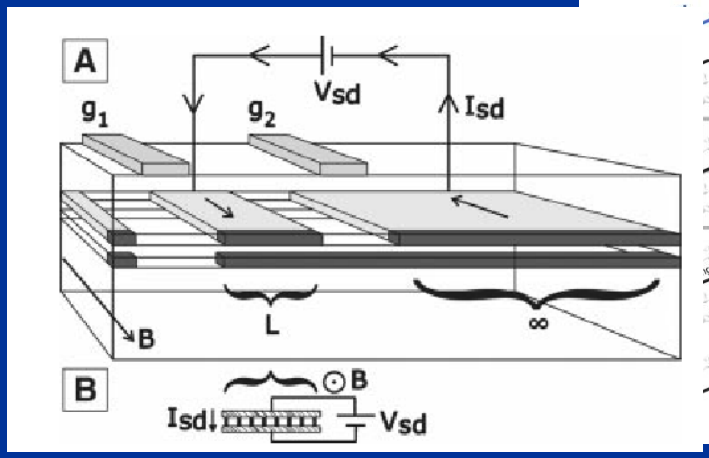
Difficult to order

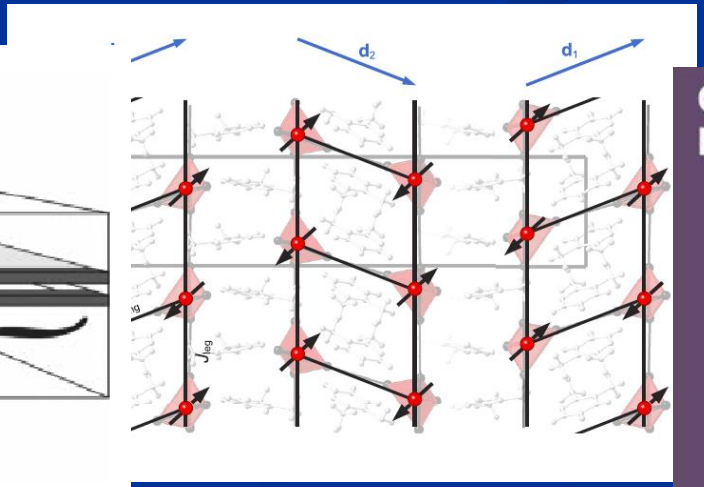
Many CM or cold atoms Systems

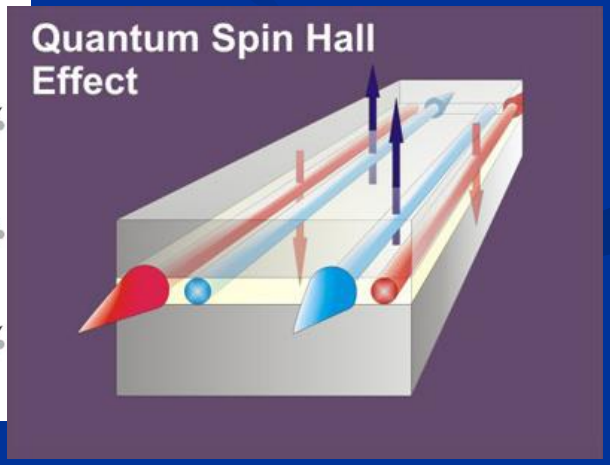












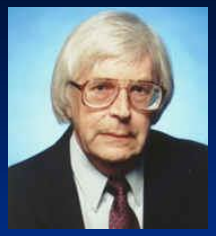
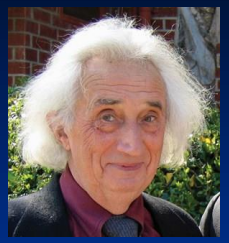
Drastic evolution of the 1d world

- New methods (DMRG, correlations from BA, etc.)
- New systems (cold atoms, magnetic insulators, etc.)
- New questions (strong SOC, out of equilibrium, etc)

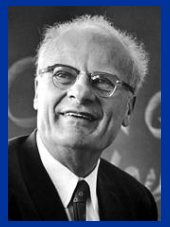
How to treat ?

- ``Standard'' many body theory

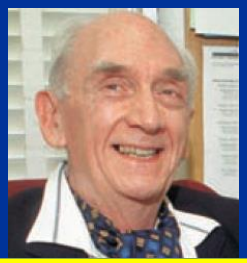
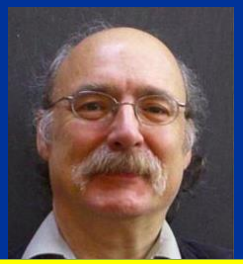



- Exact Solutions (Bethe ansatz)



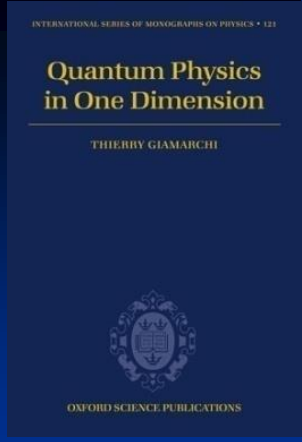

- Field theories (bosonization, CFT)

- Numerics (DMRG, MC, etc.)




References



TG, Quantum physics in one dimension, Oxford (2004)

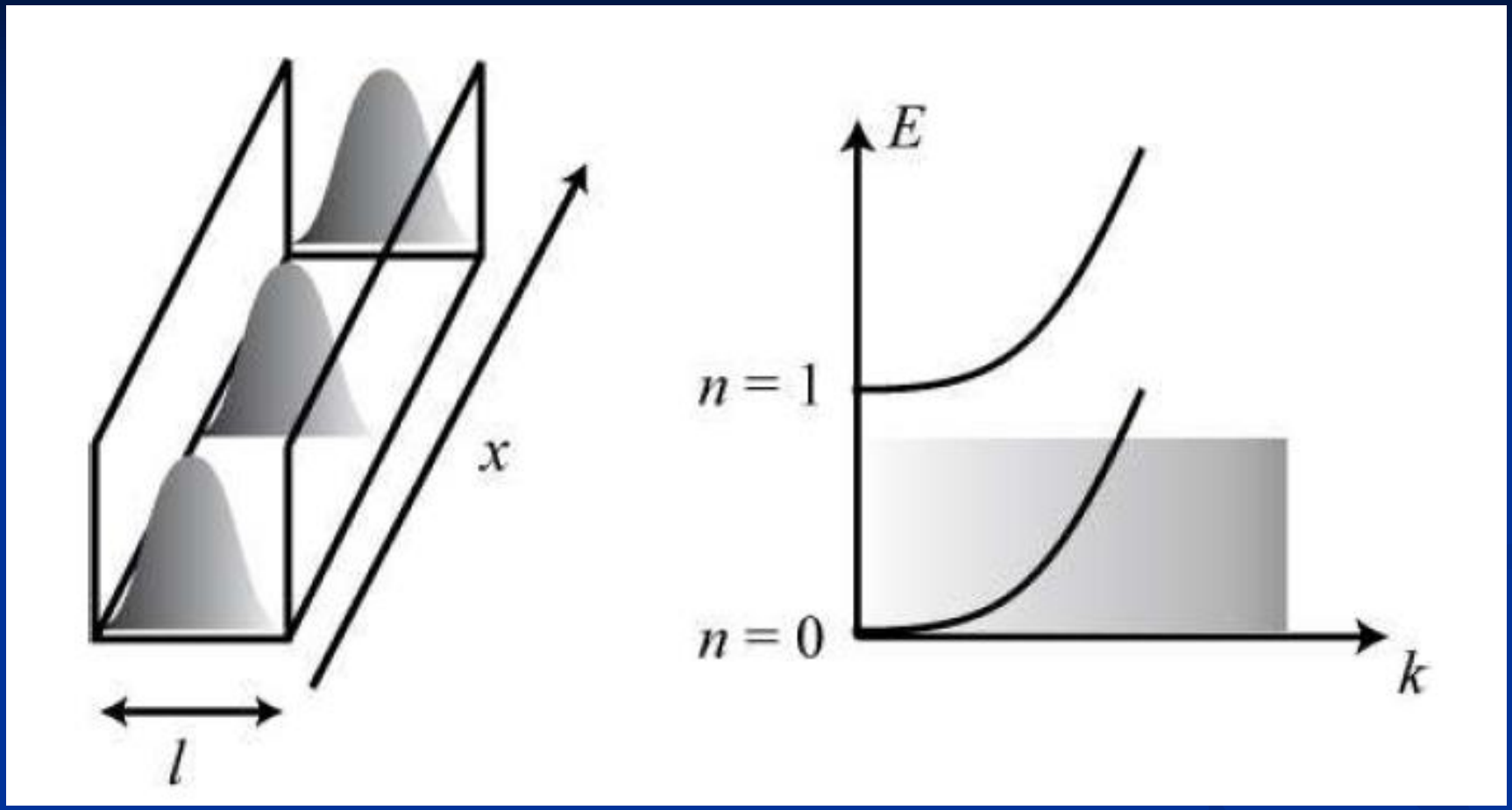
TG in ``Understanding Q. Phase Transitions", Ed. L. Carr, CRC Press (2010)

M. Cazalilla et al., Rev. Mod. Phys.83 1405 (2011)

TG, Int J. Mod. Phys. B 26 1244004 (2012)

TG, C. R. Acad. Sci. 17 322 (2016)

And now we start….

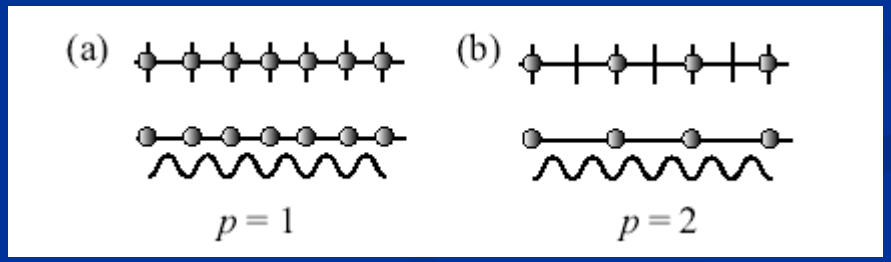

x
l
E
n = 1
n = 0
k

Typical problem (e.g. Bosons)

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu\int dx\, \rho(x)$$

- Lattice:



$$H = -J\sum_{\langle i,j\rangle} b_i^\dagger b_j + U\sum_i n_i(n_i-1) - \mu\sum_i n_i$$

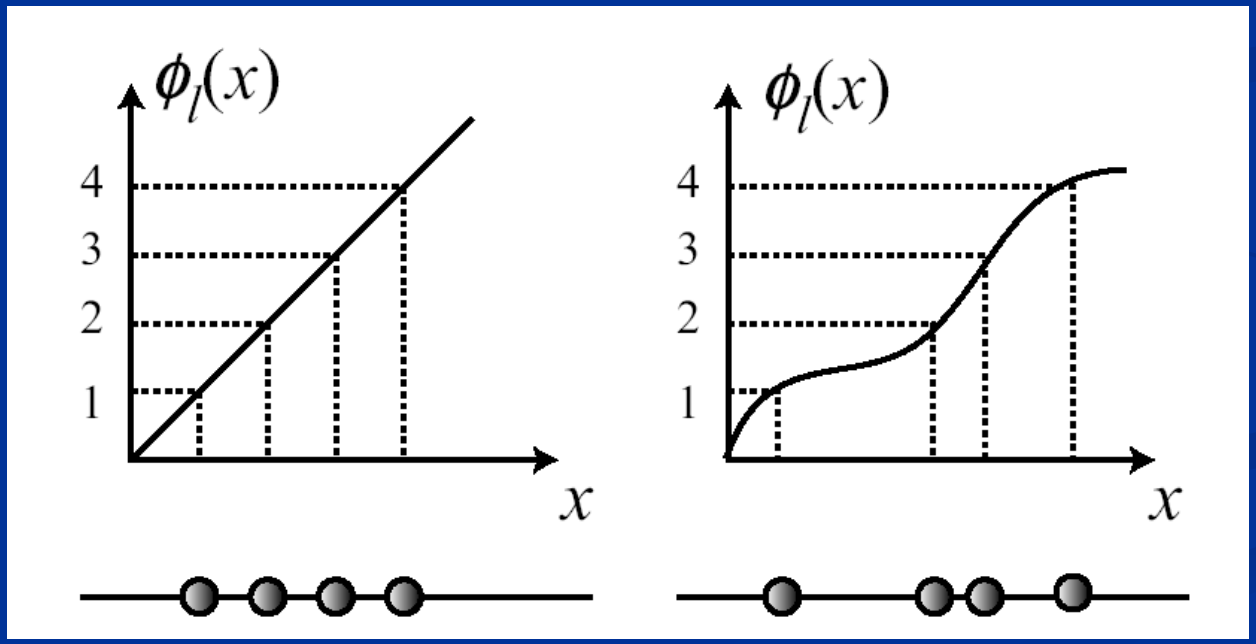
Luttinger liquid physics

Labelling the particles

$$\rho(x) = \sum_i \delta(x - x_i)$$

$$= \sum_n |\nabla \phi_l(x)| \delta(\phi_l(x) - 2\pi n)$$

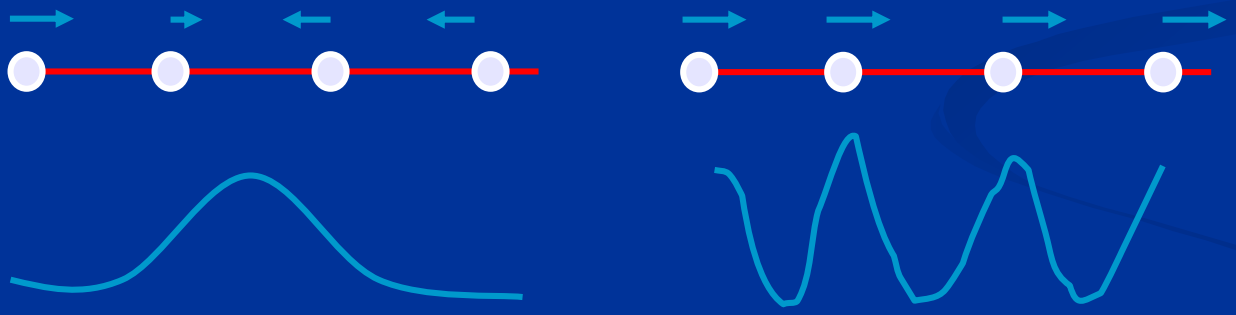
1D: unique way of labelling



$$\phi_l(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$\phi(x)$ varies slowly



CDW

$q \sim 0$ $\quad$ $q \sim 2\pi\rho_0$

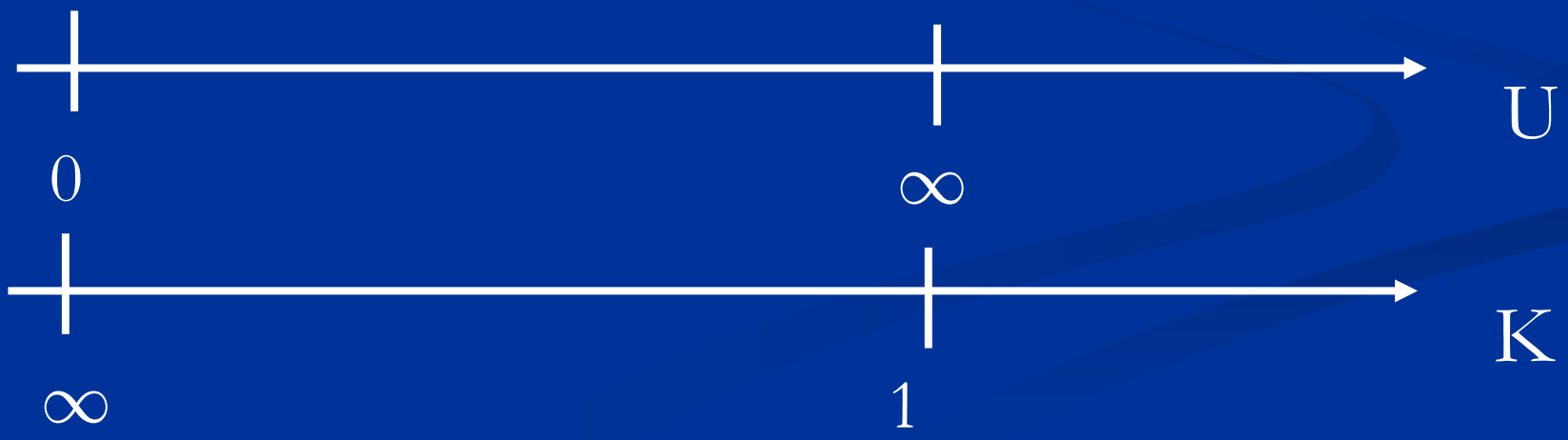
$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ: superfluid phase

$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum fluctuations

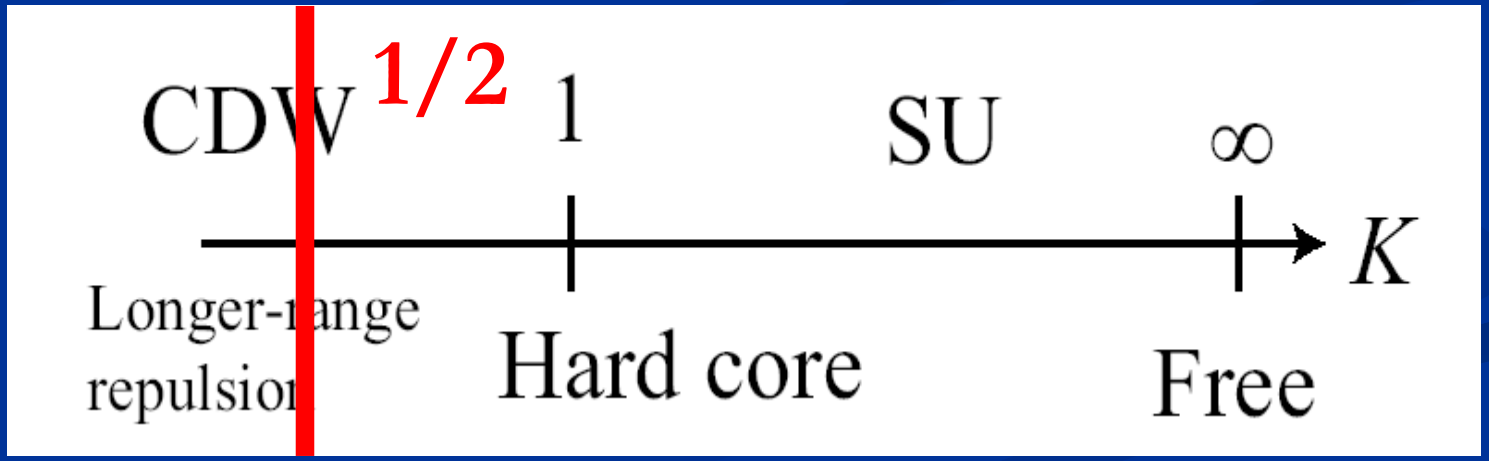
$$H = \frac{\hbar}{2\pi}\int dx[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$



Correlations

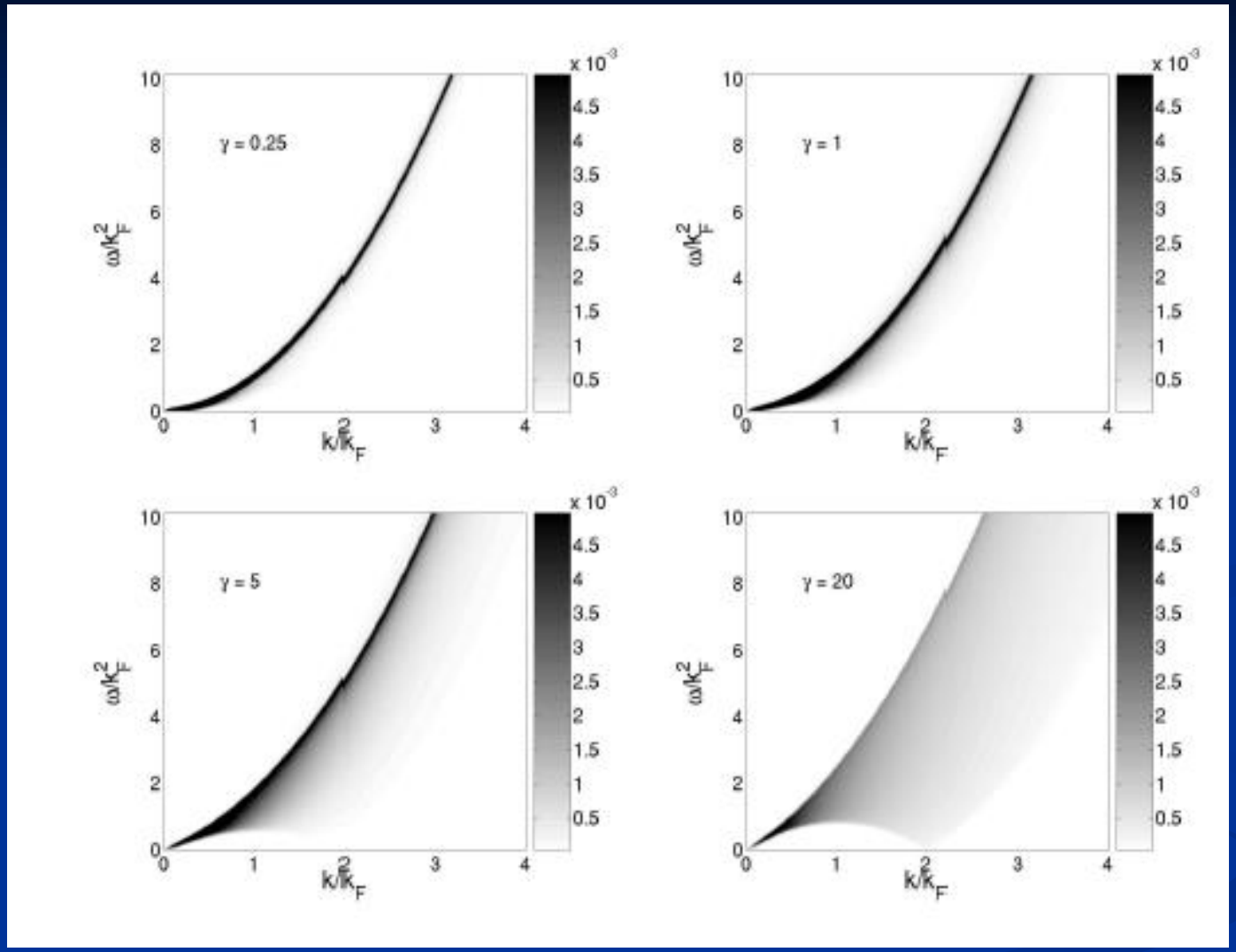
$$\langle \psi(r)\psi^\dagger(0)\rangle = A_1 \left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \cdots$$

$$\langle \rho(r)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K} + \cdots$$









S(q,ω) J.S. Caux et al PRA 74 031605 (2006)

Finite temperature

Conformal theory



β



Other 1D systems

Spins

Use boson or fermions mapping

$$S^{+} = (-1)^{i} e^{i\theta} + e^{i\theta} \cos(2\phi)$$

$$S^{z} = \frac{-1}{\pi} \nabla \phi + (-1)^{i} \cos(2\phi)$$

Powerlaw correlation functions

$$\langle S^z(x,0) S^z(0,0) \rangle = C_1 \frac{1}{x^2} + C_2 (-1)^x \left(\frac{1}{x}\right)^{2K}$$

$$\langle S^+(x,0) S^-(0,0) \rangle = C_3 \left(\frac{1}{x}\right)^{2K+\frac{1}{2K}} + C_4 (-1)^x \left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

Non universal exponents K(h,J)

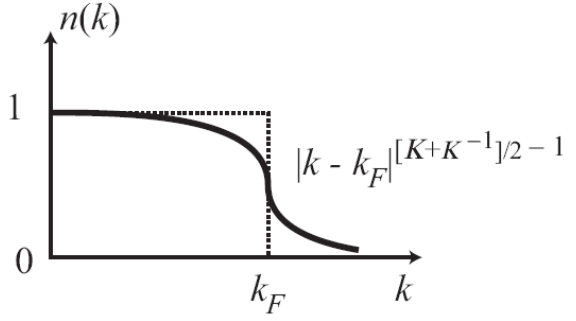
Fermions

$$\psi_F^\dagger(x) = \psi_B^\dagger(x) e^{i\frac{1}{2}\phi_l(x)}$$

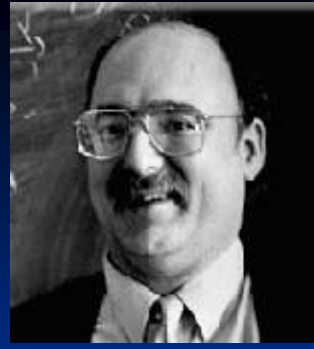
$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

Right ($+k_F$) and left ($-k_F$) particles

$$\langle \rho(x,\tau)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(x^2+y_\alpha^2)^2} + \rho_0^2 A_2 \cos(2\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{2K} + \rho_0^2 A_4 \cos(4\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{8K}$$



Luttinger liquid concept



- How much is perturbative ?

- Nothing (Haldane):
  provided the correct u,K are used

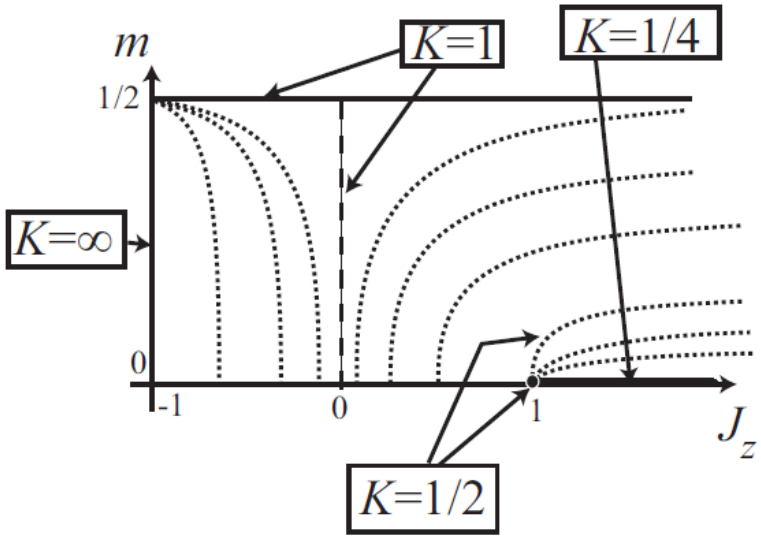
- Low energy properties: Luttinger liquid
(fermions, bosons, spins…)

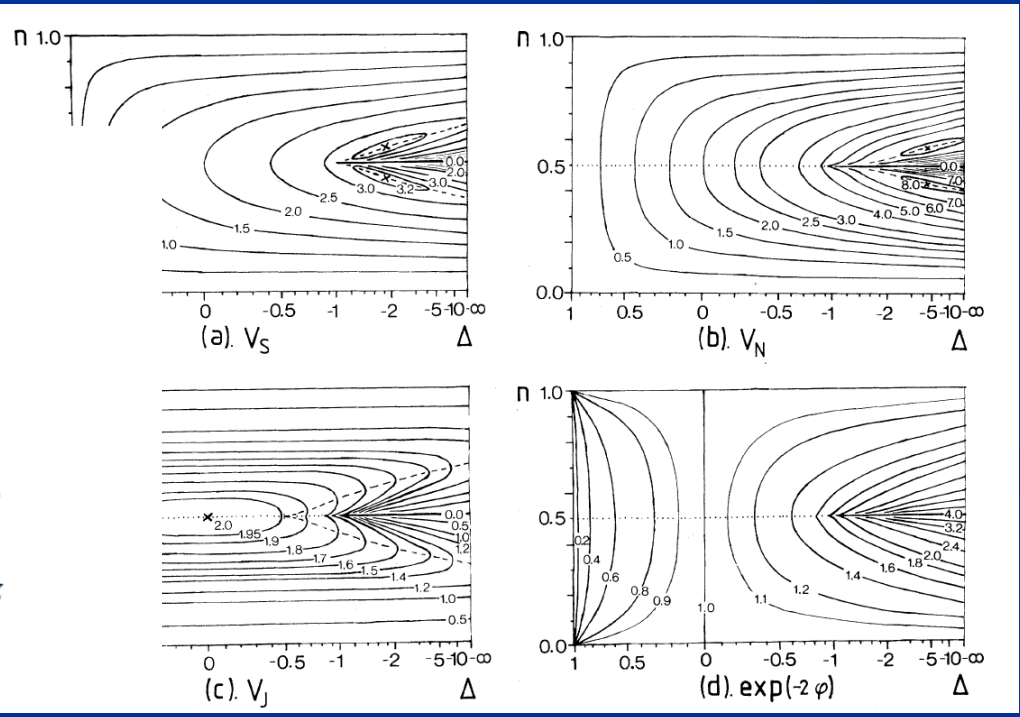
Luttinger parameters

General Relation of Correlation Exponents and Spectral Properties of One-Dimensional Fermi Systems: Application to the Anisotropic $S = \frac{1}{2}$ Heisenberg Chain

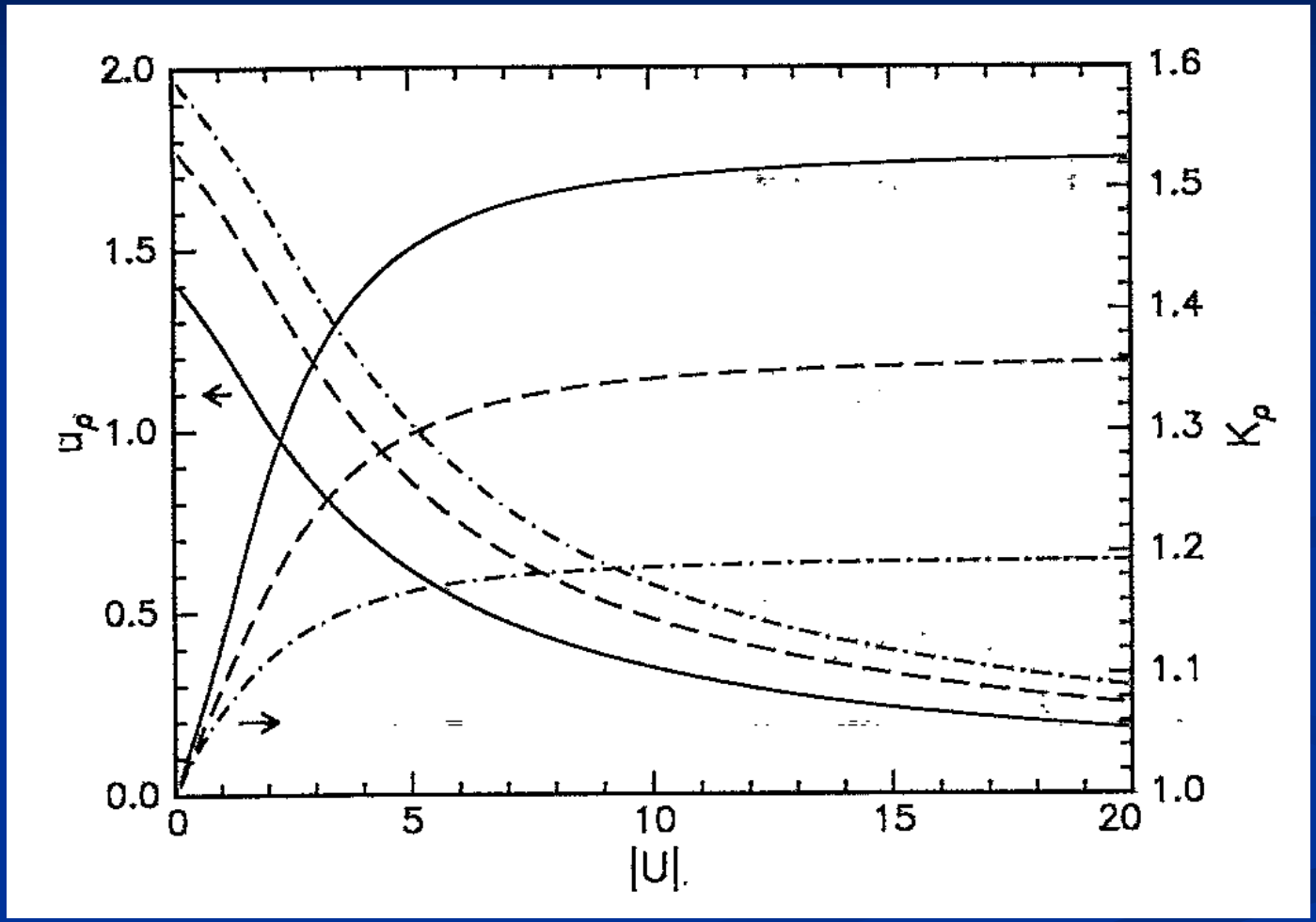
F. D. M. Haldane

Haldane, F. D. M. (1980). *Phys. Rev. Lett.*, **45**, 1358.





Attractive Hubbard model



TG + B. S. Shastry PRB 51 10915 (1995)

``Quantitative'' theory possible

- Trick: use thermodynamics and BA or numerics
- Compressibility: u/K
- Response to a twist in boundary: u K
- Specific heat : T/u
- Etc.

Tonks limit



U = ∞ : spinless fermions

Not for n(k) : $\psi_F \neq \psi_B$

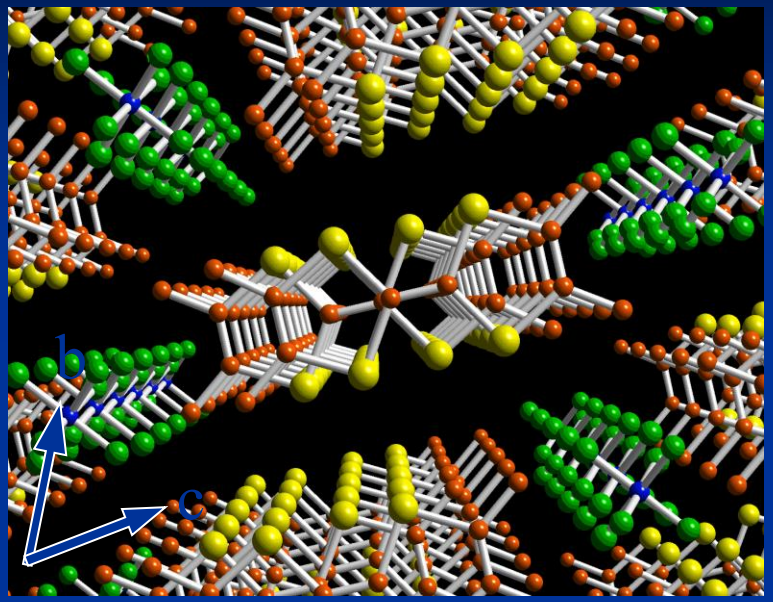
Free fermions: $\langle \rho(x)\rho(0)\rangle \propto \cos(2k_F x)\left(\frac{1}{x}\right)^2$

K=1

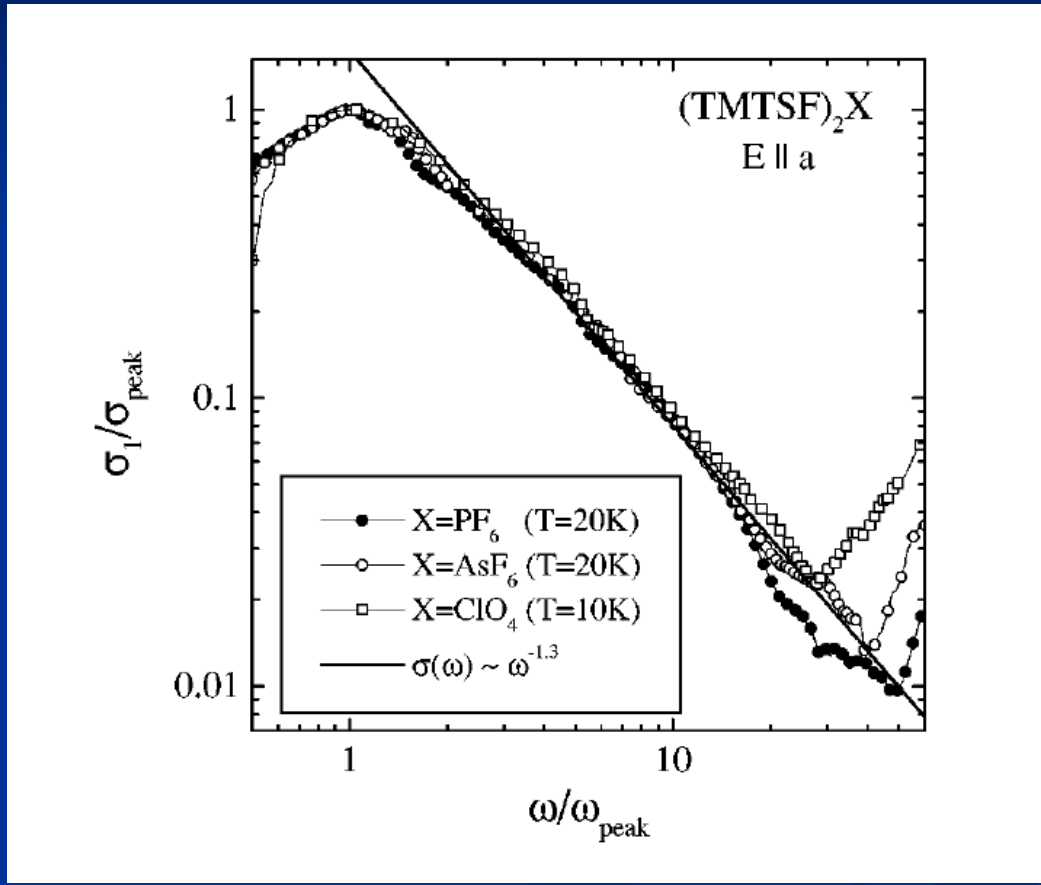
Note: $\langle \psi_B(x)\psi_B(0)^\dagger\rangle \propto \left(\frac{1}{x}\right)^{1/2}$

Tests of Luttinger liquids

Organic conductors



$$\sigma(\omega) \sim \omega^{-\nu}$$



TG PRB (91) :
Physica B 230 (1996)

A. Schwartz et al. PRB 58 1261 (1998)

First observation of LL/powerlaw !!

Cold gases

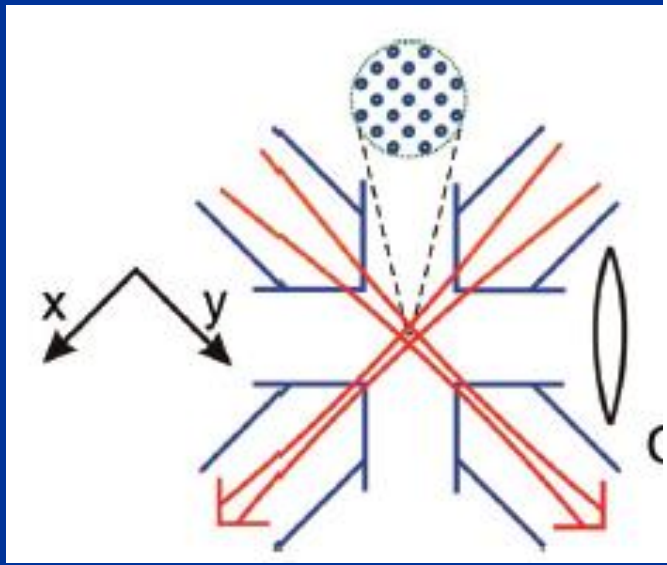
Bosons (continuum)

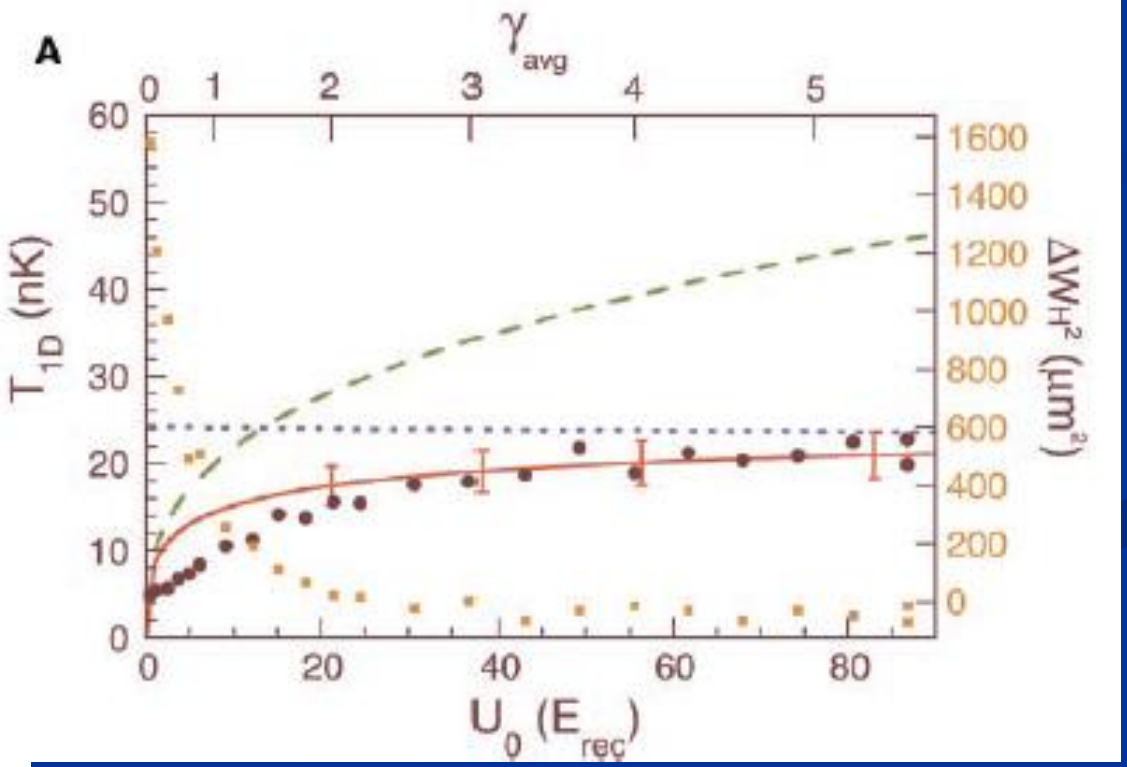


Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

SCIENCE VOL 305 20 AUGUST 2004 1125

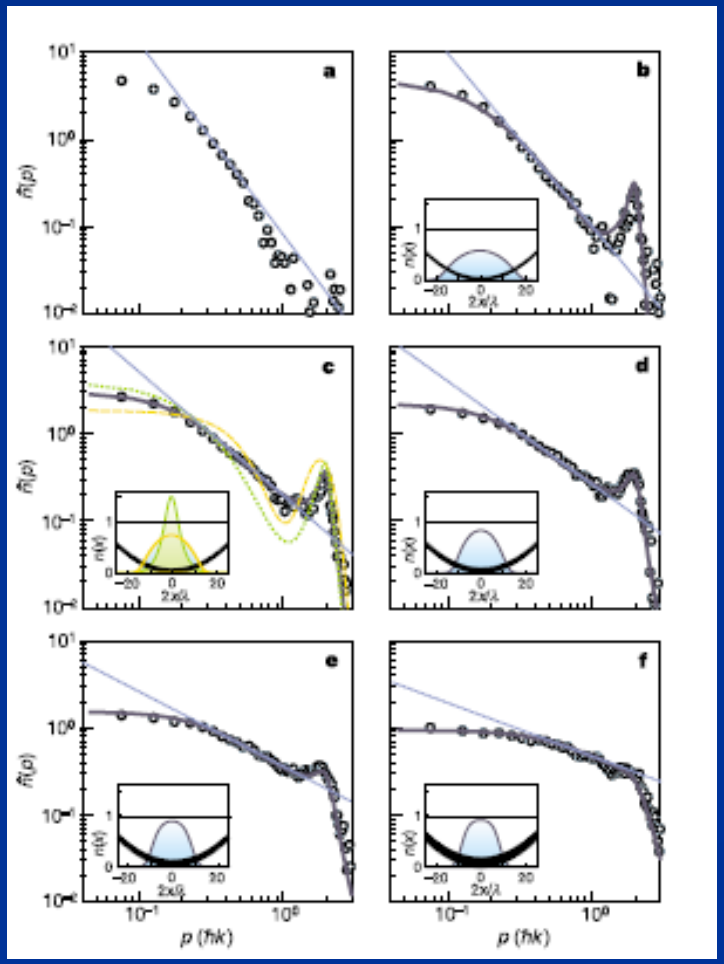


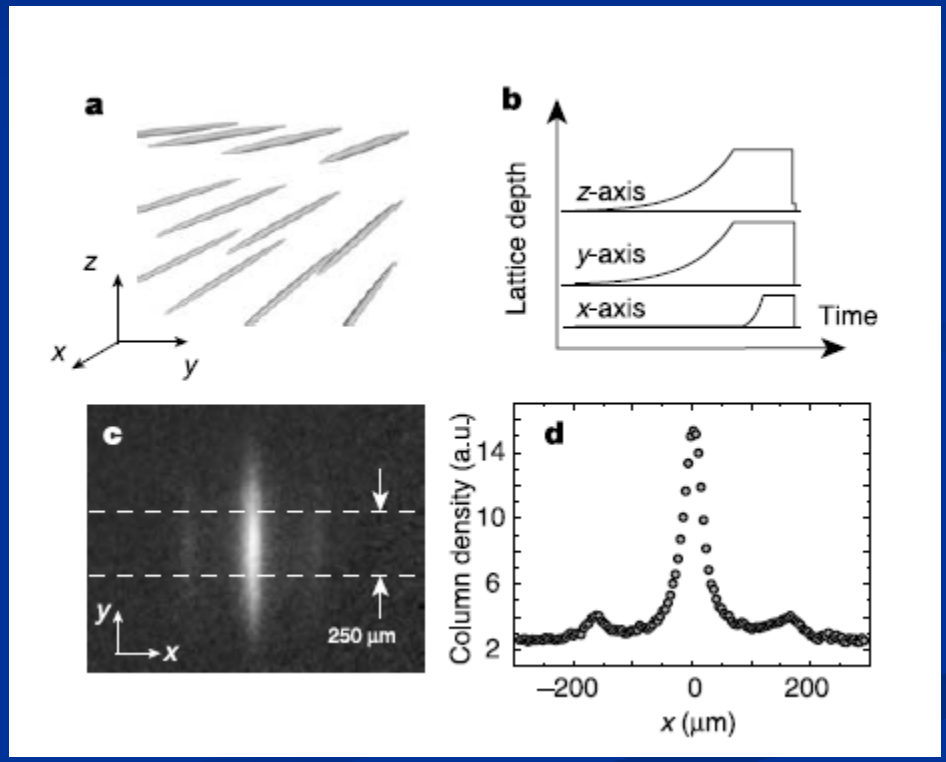


Optical lattices (dilute)



B. Paredes et al., Nature 429 277 (2004)

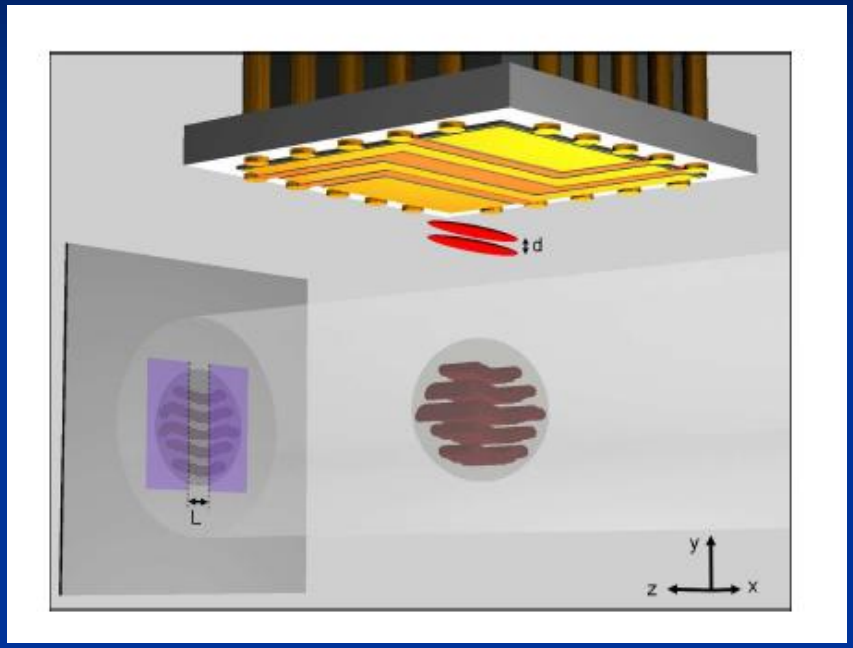


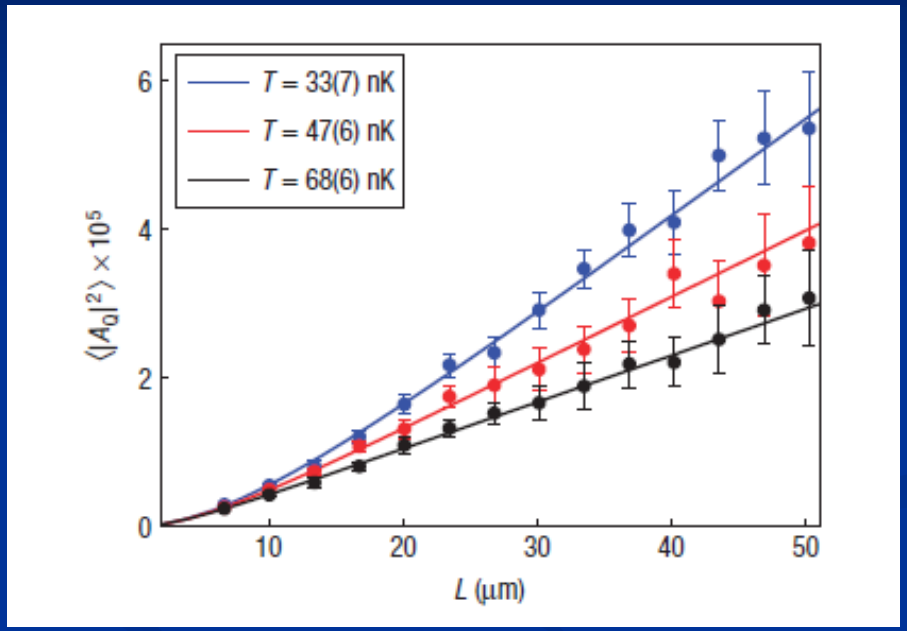


$$n(k) = \int dx e^{ikx} \langle \psi^\dagger(x)\psi(0) \rangle$$

Atom chips







$$\int_0^L dr\langle\psi(r)\psi^\dagger(0)\rangle$$

K large (42)

S. Hofferberth et al. Nat. Phys 4 489 (2008)









Charge velocity

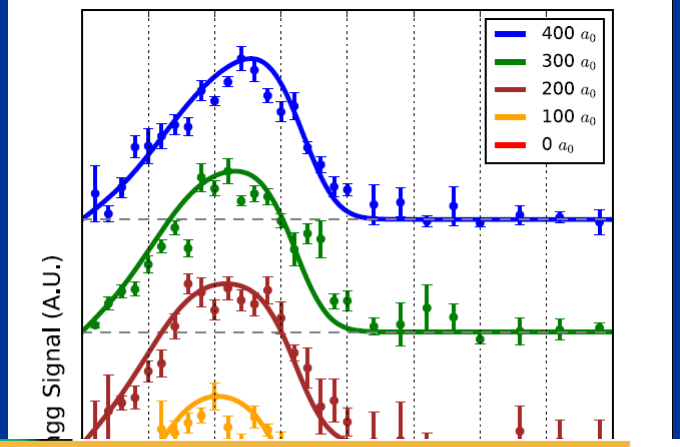


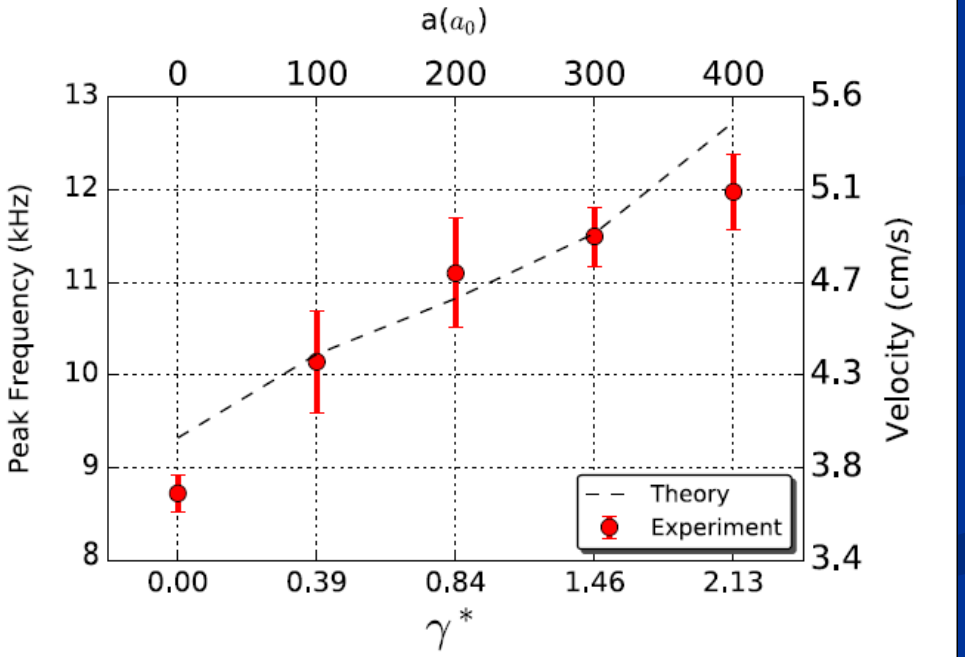
PHYSICAL REVIEW LETTERS **121,** 103001 (2018)

Editors' Suggestion

Measurement of the Dynamical Structure Factor of a 1D Interacting Fermi Gas

T. L. Yang,[1] P. Grišins,[2] Y. T. Chang,[1] Z. H. Zhao,[1] C. Y. Shih,[1] T. Giamarchi,[2] and R. G. Hulet[1]





ULTRACOLD ATOMS CONFIRM 55-YEAR-OLD PHYSICS THEORY

https://www.futurity.org/one-dimensional-electrons-physics-1858622/

Plan of the lectures

- Equilibrium, basic notions of Tomonaga-Luttinger liquid
- Equilibrium, Beyond 1D; Coupled 1D chains, Dimensional crossover
- Out of equilibrium, Quantum transport

Other important 1D properties
Fractionalization of excitations

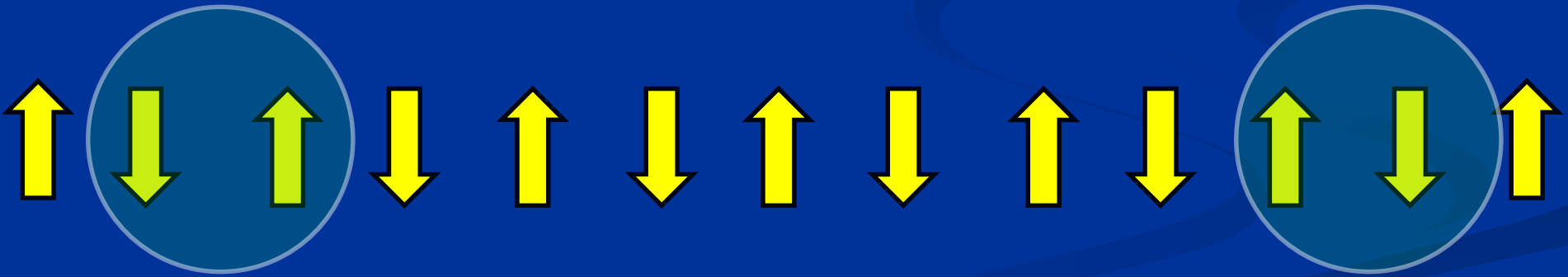
Fractionalization of excitations

$$H = J\sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$



$\Delta S = -1$ $E = \grave{o}(q)$

Magnon



$\Delta S = -1/2$

Spinons

$\Delta S = -1/2$

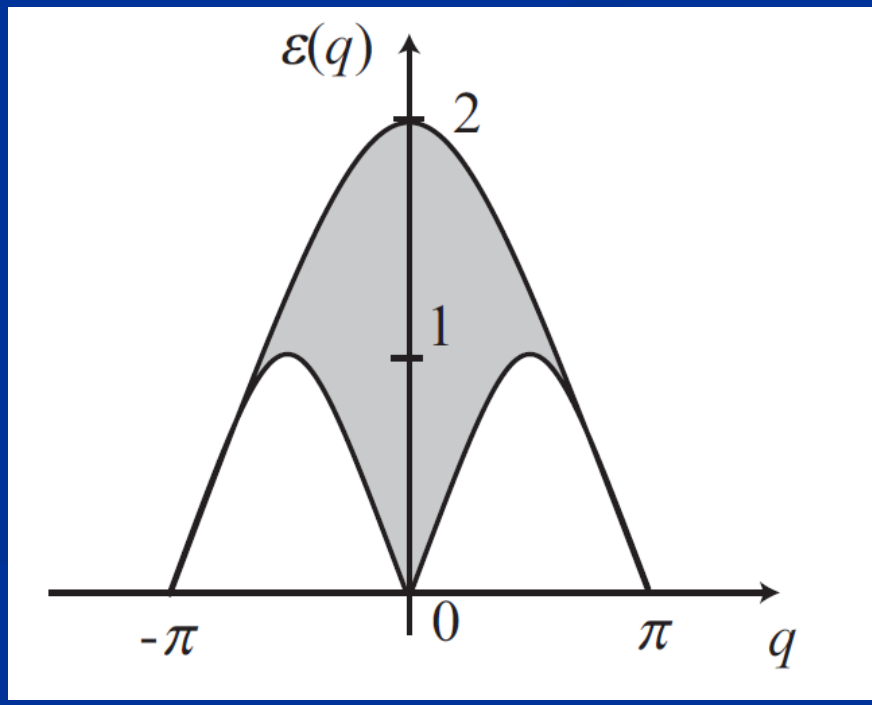
Magnons and spinons: 1 = ½ + ½



- Hidden (topological) order parameters
- Continuum of excitations

$$E(k) = \cos(k_1) + \cos(k_2)$$

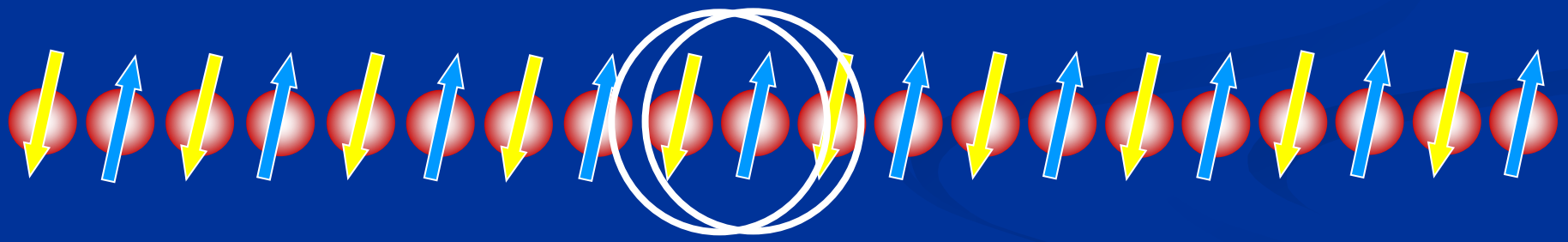
$$k = k_1 + k_2$$



Particles with “spins”: spin-charge separation

Spin

Charge



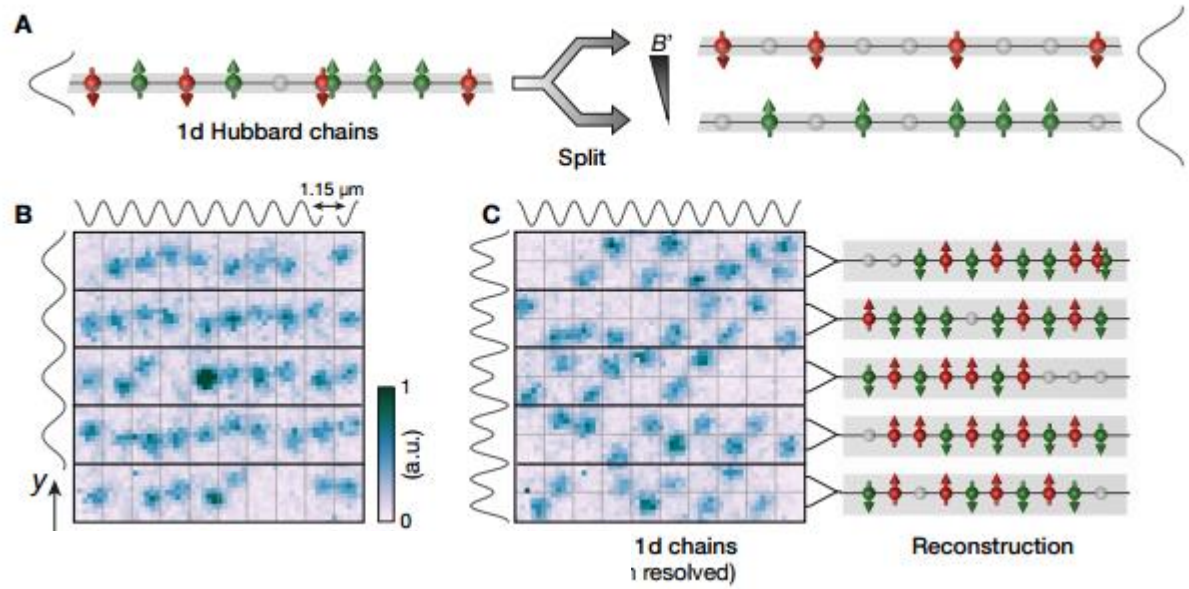
Spinon

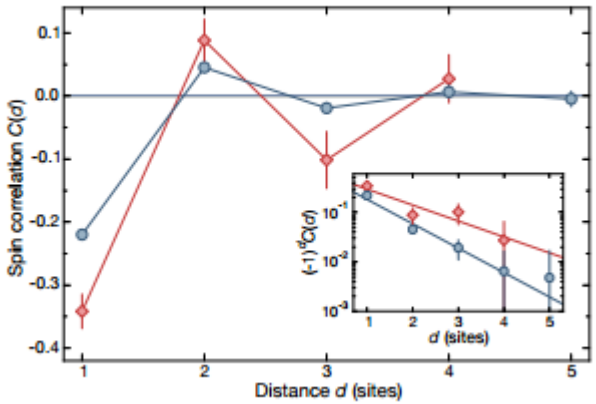
Holon

Spin and Charge Resolved Quantum Gas Microscopy of Antiferromagnetic Order in Hubbard Chains

Martin Boll[1*], Timon A. Hilker[1*], Guillaume Salomon[1*], Ahmed Omran[1], Immanuel Bloch[1,2], and Christian Gross[1†]

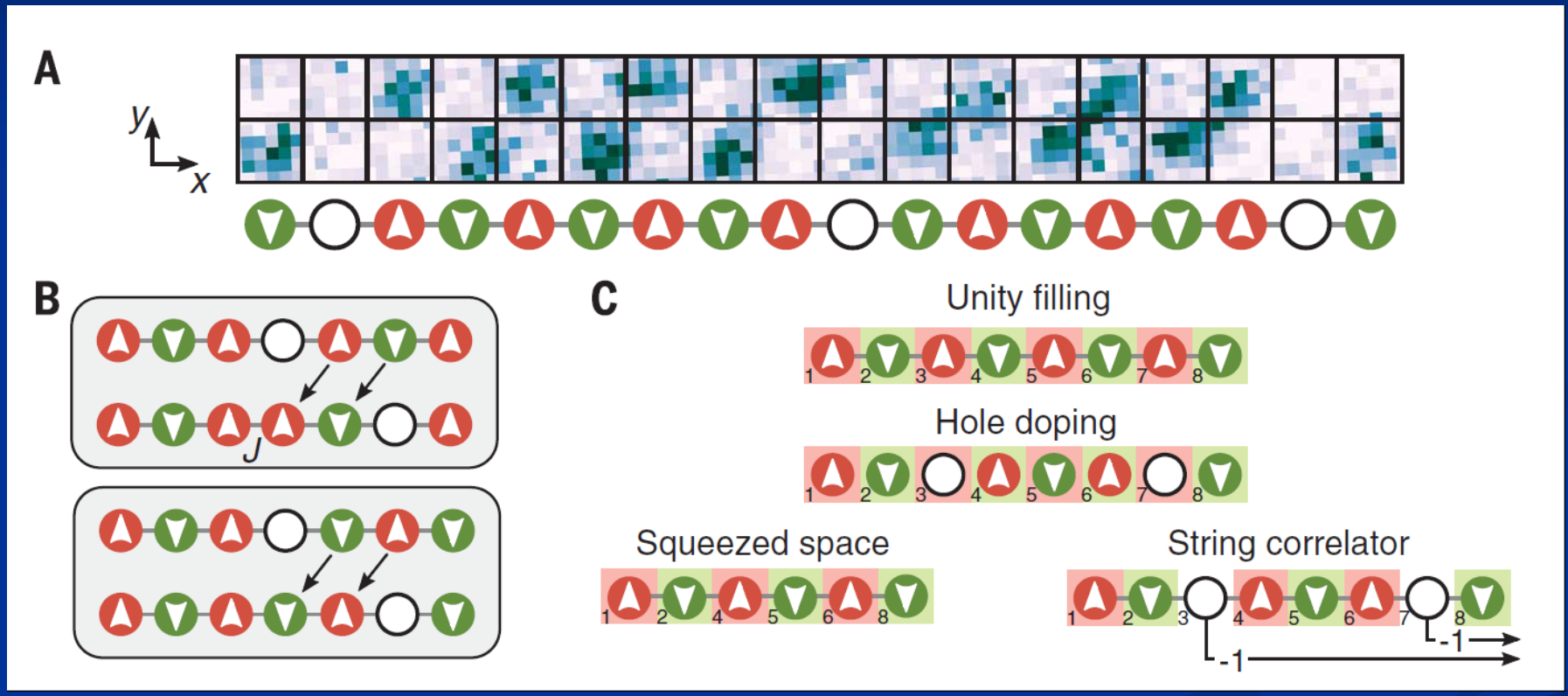
arXiv:1605.05661v2





The inset shows the decay of the rectified spin correlations $(-1)^d C(d)$ in a logarithmic plot together with an exponential fit $C(d) \propto \exp(-d/\xi)$, which reveals an average correlation length of $\xi = 0.9(1)$ sites and $\xi = 1.4(4)$ sites for the lowest entropy tube. All error bars represent one s.e.m.

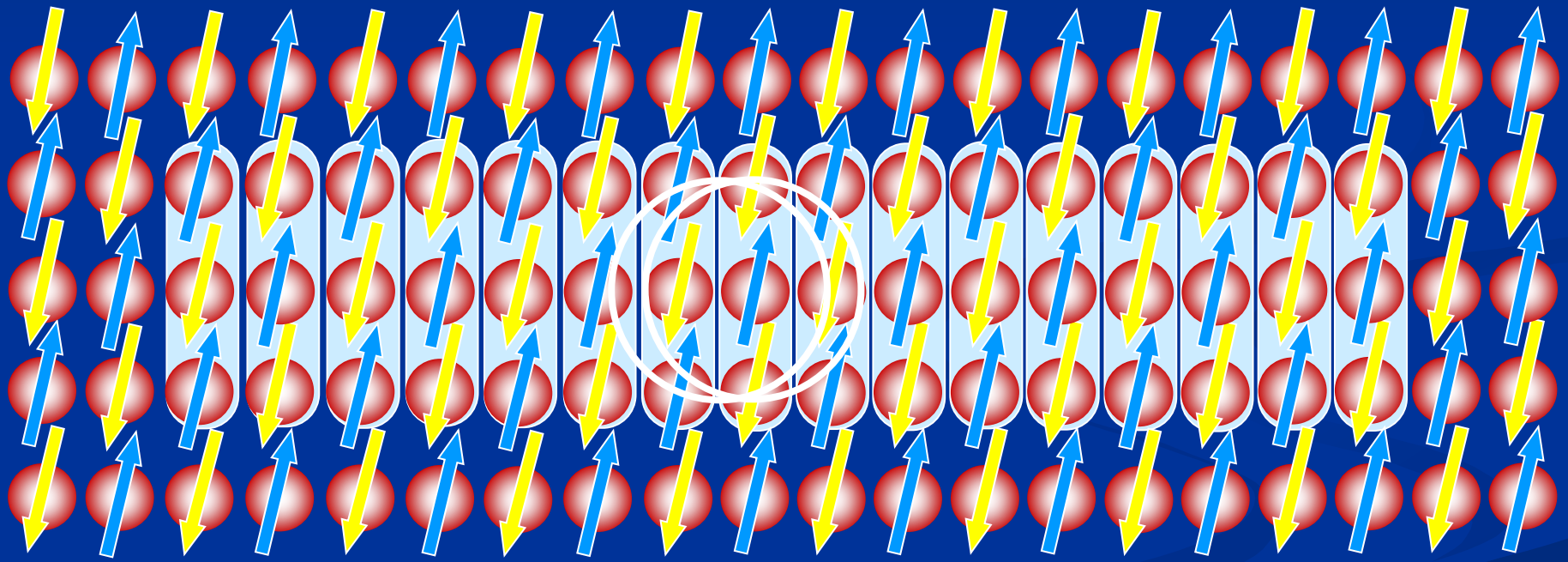
Doped Hubbard model



Hilker *et al.*, *Science* **357**, 484–487 (2017)

Spin-Charge Separation higher D ?

Spin Charge

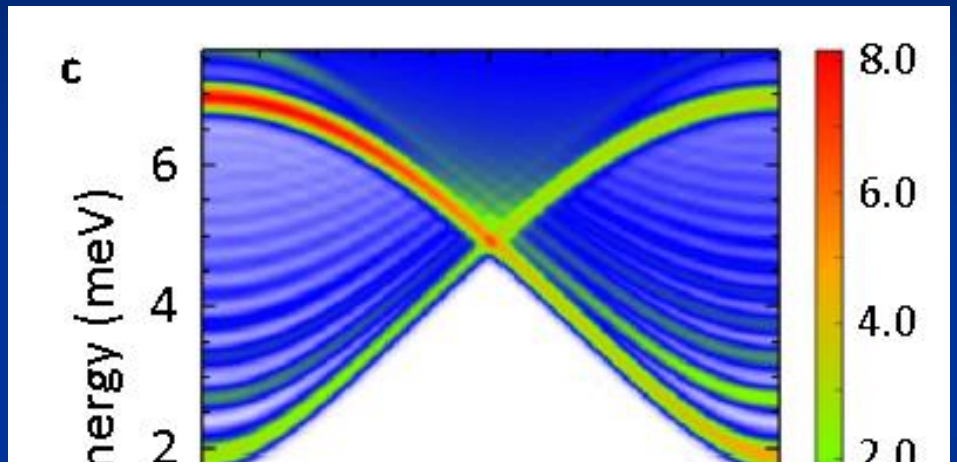


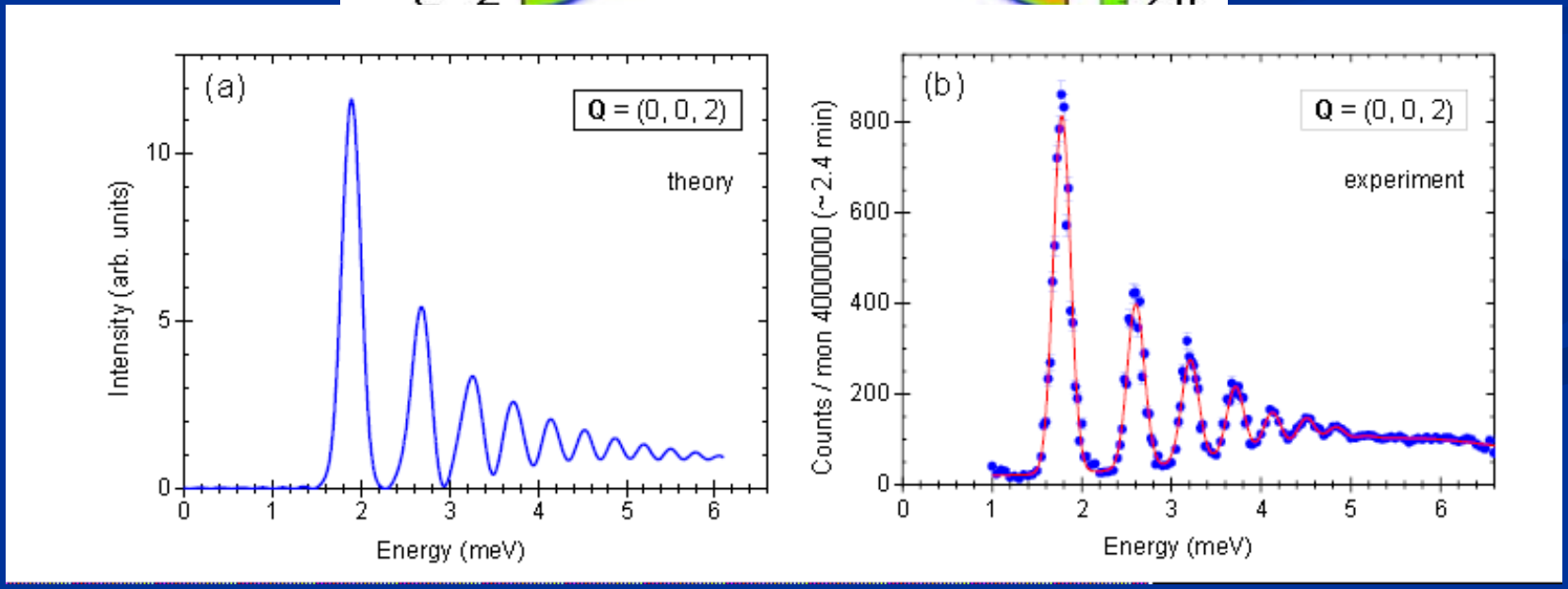
Energy increases with spin-charge separation

Confinement of spin-charge: « quasiparticle »

Confinement of spinons BaCoVO

Q. Faure, S. Takayoshi, et al. Nat. Phys 14, 716 (2018)

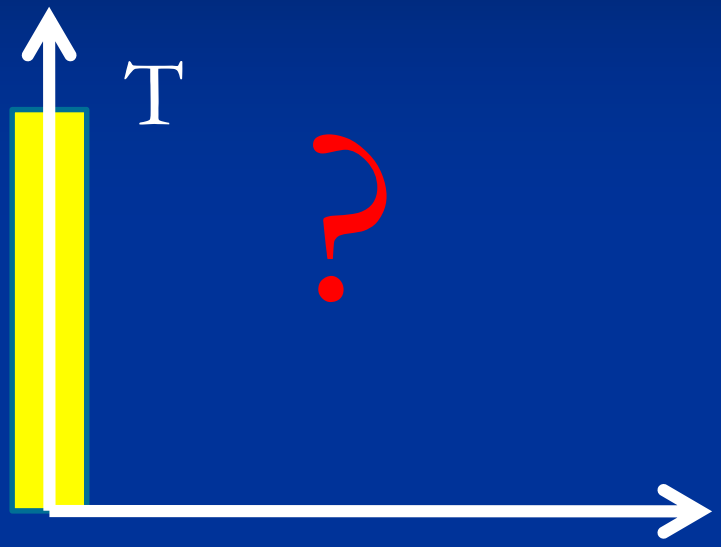


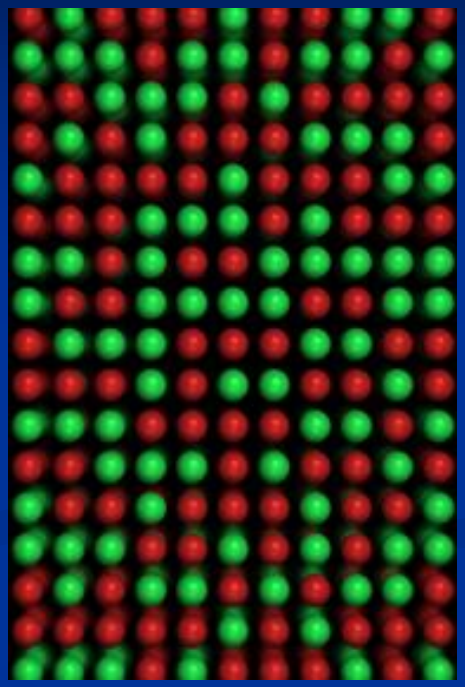


TLL as a stepping stone to go beyond

Effect of lattices: Mott transition

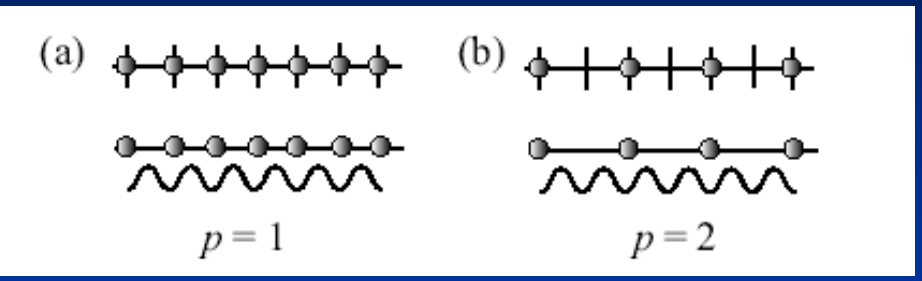
Mott transition





- Mott insulator (n=1)
- T < T_N : antiferromagnetic phase

Periodic lattice



$$H = \int dx V_0 \cos(Qx)\rho(x)$$

$$H = \int dx V_0 \cos(Qx)\rho_0 e^{i(2\pi\rho_0 x - 2\phi(x))}$$

- Incommensurate: $Q \neq 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x) + \delta x)$$

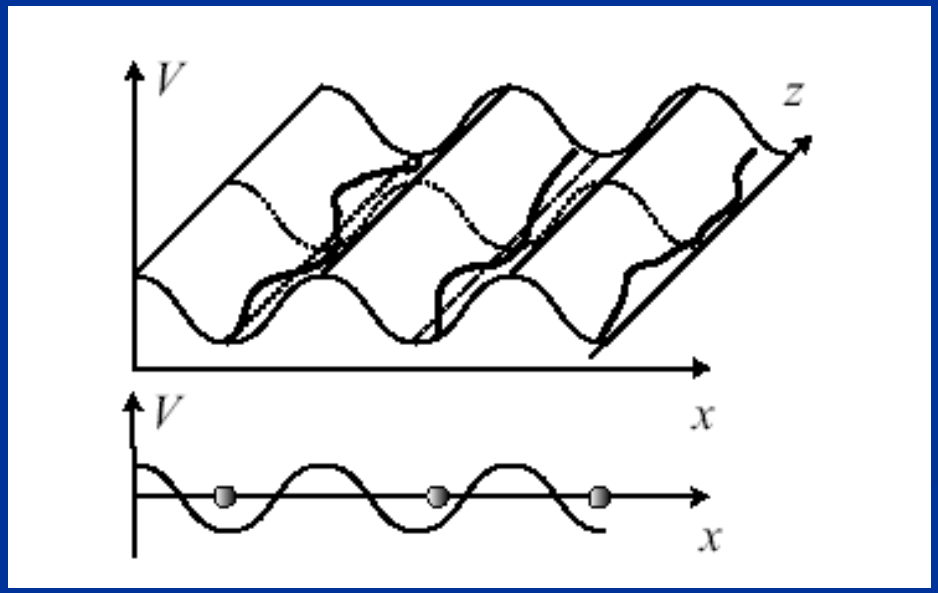
- Commensutate: $Q = 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x))$$

Competition

$$S_0 = \int \frac{dx d\tau}{2\pi K} [\tfrac{1}{u} (\partial_\tau \varphi(x,\tau))^2 + u(\partial_x \varphi(x,\tau))^2]$$

$$S_L = -V_0 \rho_0 \int dx d\tau \cos(2\varphi(x))$$



Beresinskii-Kosterlitz-Thouless transition at K=2

String order parameter

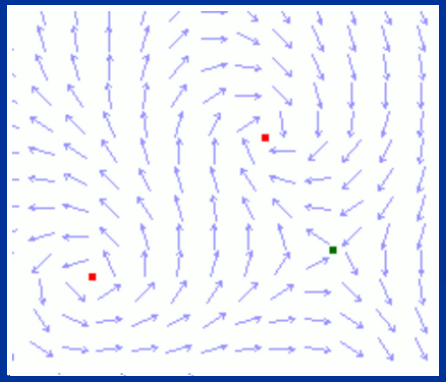


BKT transition



$$H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j)$$

- BKT: remarkable transision going outside the paradigm of Landau's phase transitions
- A transition wihout an order parameter
- Topological Vortex excitations

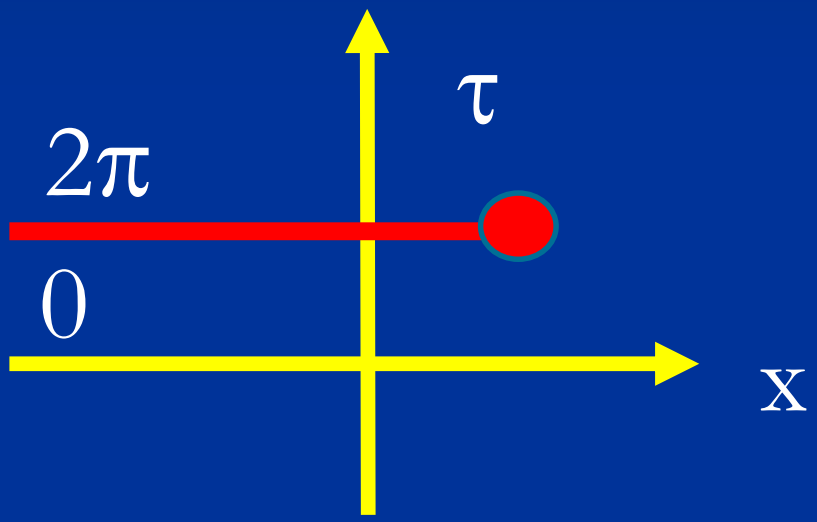




Vortex operator

$e^{iaP}\,|\,x\rangle = |x+a\rangle$

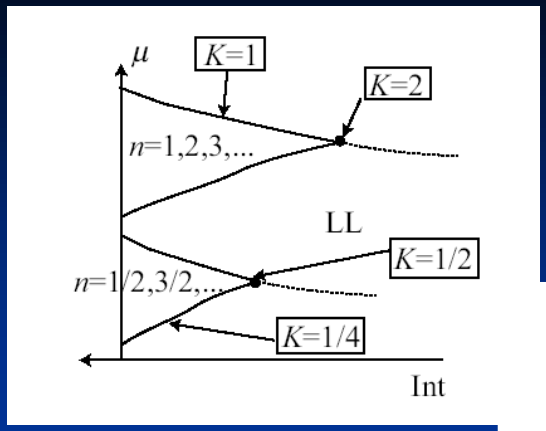
$\phi(x,\tau) = \pi\int_{-\infty}^{x} dx'\Pi_{\theta}(x',\tau)$

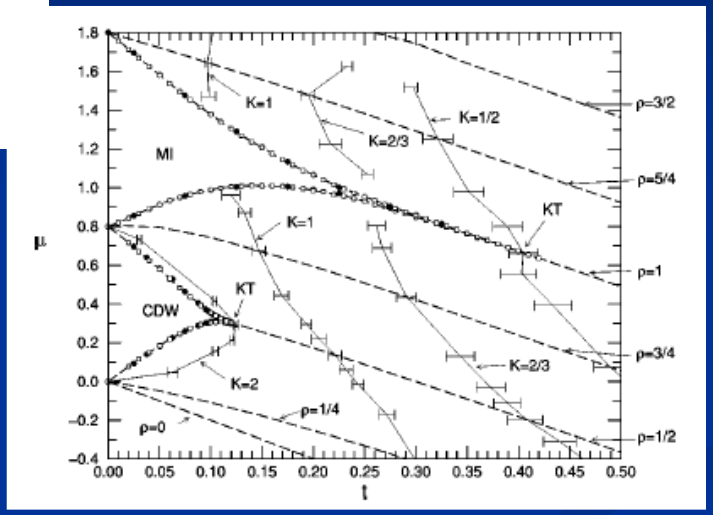


$\cos(2\phi(x_1,\tau_1))$

- Vortex operator for θ
- K : inverse temperature
- g : vortex fugacity

$$S = \frac{K}{2\pi}\int dx d\tau\left[\frac{1}{u}(\partial_\tau\theta) + u(\partial_x\theta)^2\right] - g\int dx\cos(2\phi)$$





T. Kuhner et al. PRB 61 12474 (2000)

Gap in the excitation spectrum

$$G(r) \propto e^{-r/\xi}$$

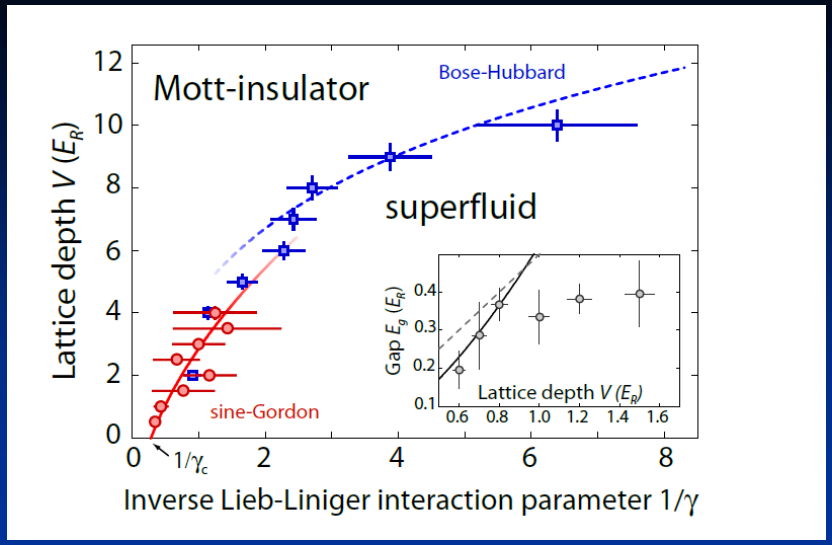
Mott insulator: ϕ is locked

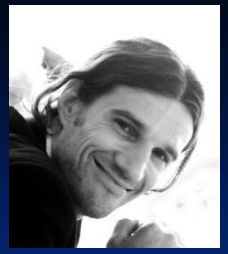
Density is fixed

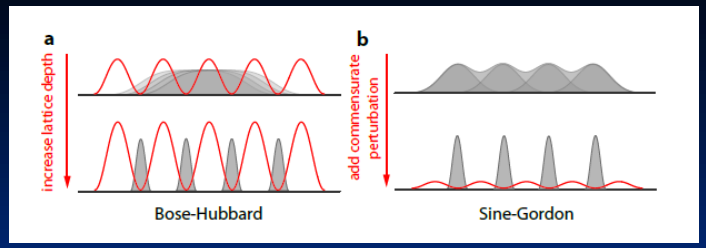
TG, Physica B 230 975(97): arXiv/0605472 (Salerno lectures);

Oxford (2004);

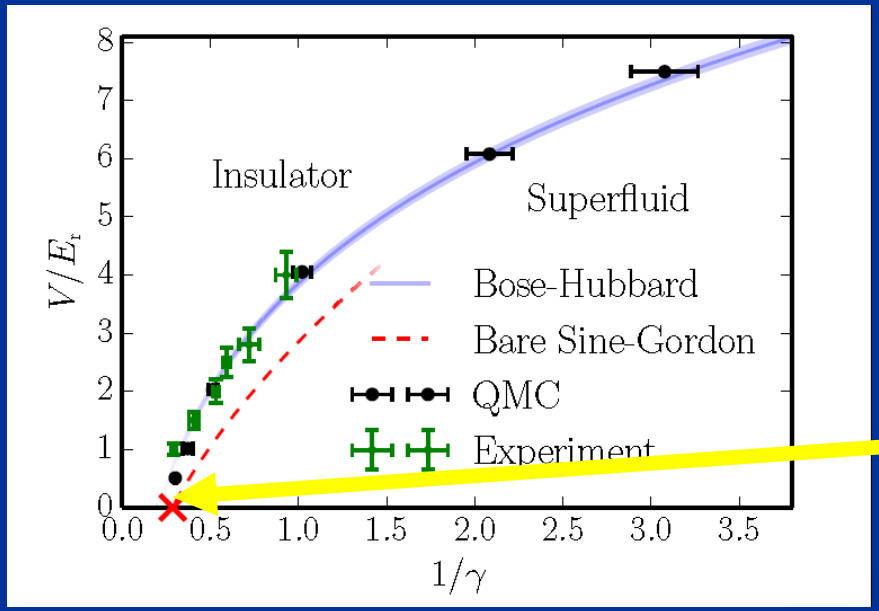
M. Cazalilla et al., Rev. Mod. Phys.83 1405 (2011)







E. Haller et al. Nature 466 597 (2010)







Renormalized Sine-Gordon

Shows:

$K^* = 2$

G. Boeris et al. PRA 93 93, 011601(R) (2016)

Non local (topological) order

$$\rho(x) \sim \nabla \phi(x)$$

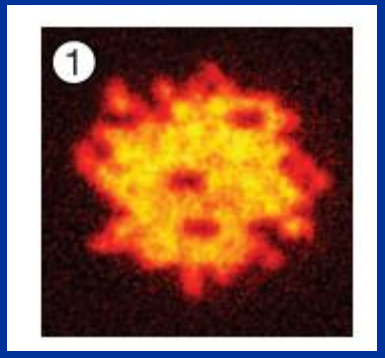
$$\mathcal{O}_{\mathrm{P}}^2 = \lim_{l\to\infty} \mathcal{O}_{\mathrm{P}}^2(l) = \lim_{l\to\infty} \left\langle \prod_{k\le j\le k+l} e^{i\pi\delta\hat{n}_j} \right\rangle$$

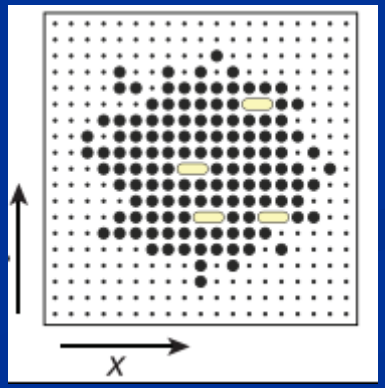
E. Berg, E. Dalla Torre, T. Giamarchi, E. Altman, *Phys. Rev. B* **77**, 245119 (2008).

Observation of Correlated Particle-Hole Pairs and String Order in Low-Dimensional Mott Insulators

M. Endres,[1]* M. Cheneau,[1] T. Fukuhara,[1] C. Weitenberg,[1] P. Schauß,[1] C. Gross,[1] L. Mazza,[1] M. C. Bañuls,[1] L. Pollet,[2] I. Bloch,[1,3] S. Kuhr[1,4]

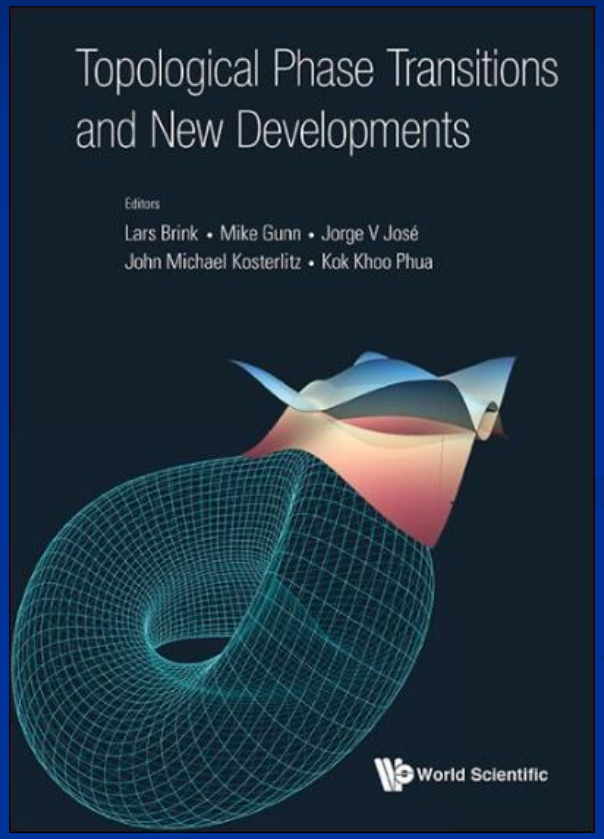
Science (2011)







Topological excitations is the norm in 1D



Topological Phase Transitions and New Developments, pp. 147-164 (2018)

Clean and dirty bosons in 1D lattices

Disorder (equilibrium)

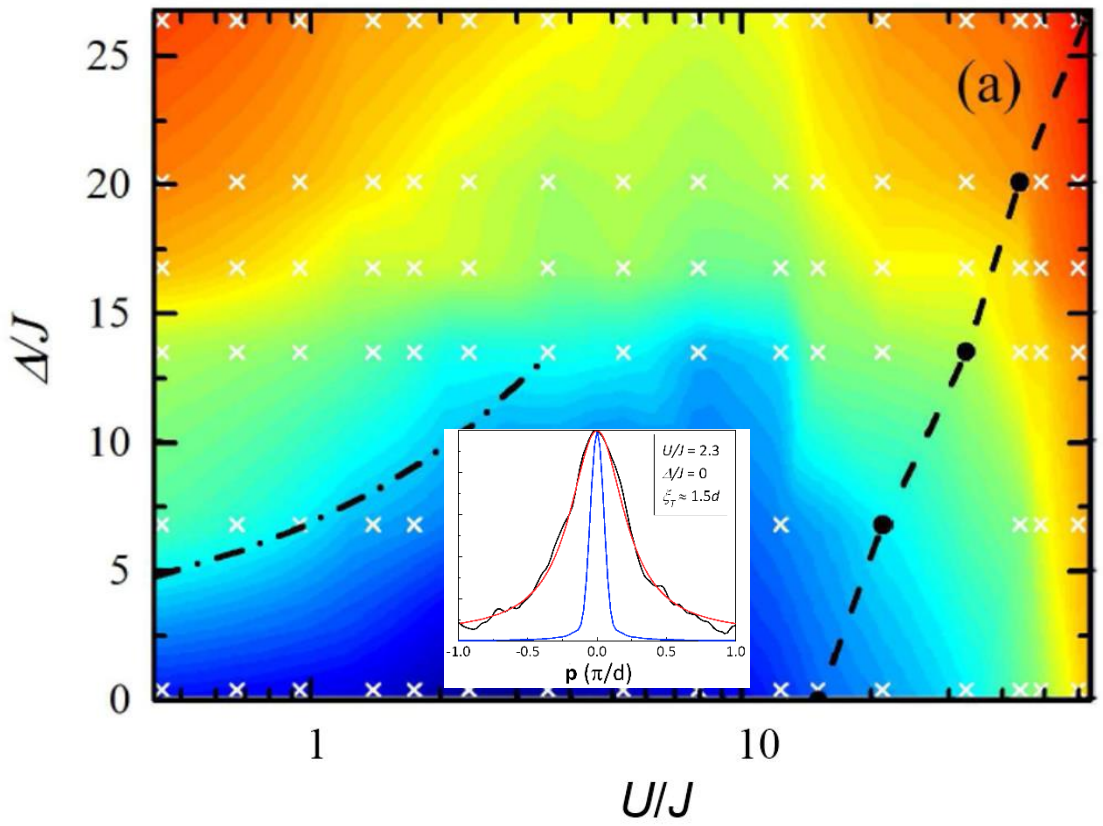
Quasi-periodics and interactions

C. D'Errico, E. Lucioni et al. PRL (2014);
L. Gori et al PRA 93 033650 (2016)







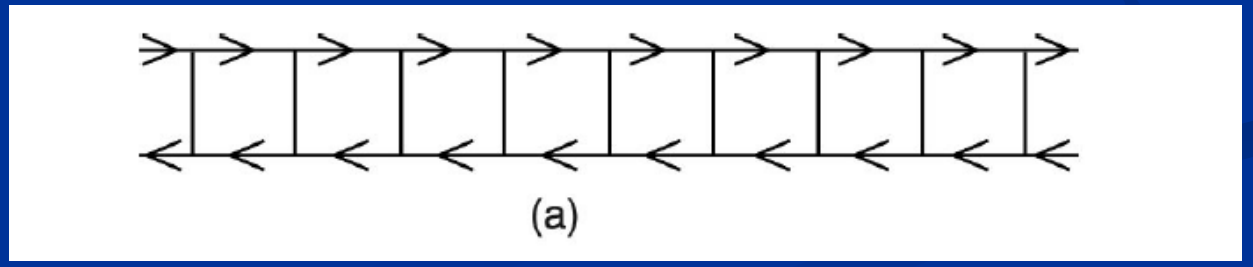



Artificial gauge fields

Meissner effect in bosonic ladders



E. Orignac, TG, PRB 64 144515 (2001)



$$H = -t_{\parallel} \sum_{i,p=1,2} \left(b^{\dagger}_{i+1,p} e^{ie^* a A_{\parallel,p}(i)} b_{i,p} + b^{\dagger}_{i,p} e^{-ie^* a A_{\parallel,p}(i)} b_{i+1,p} \right)$$

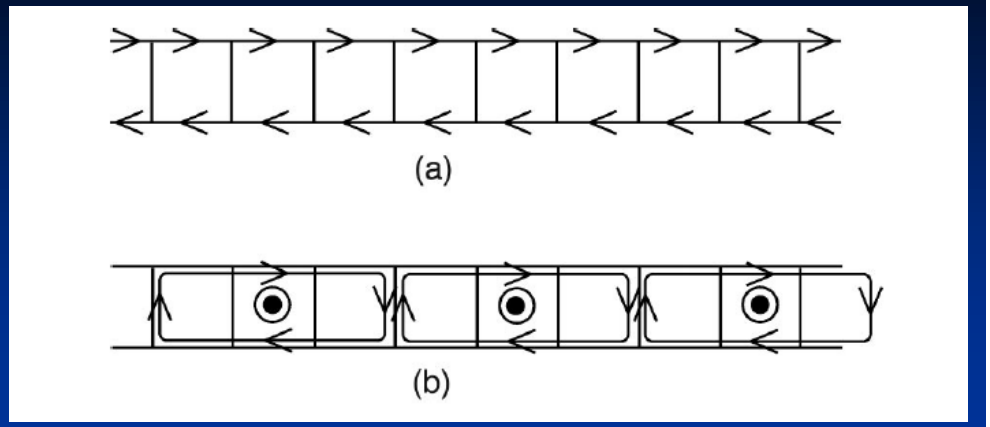
$$- t_{\perp} \sum_{i} \left(b^{\dagger}_{i,2} e^{ie^* A_{\perp}(i)} b_{i,1} + b^{\dagger}_{i,1} e^{-ie^* A_{\perp}(i)} b_{i,2} \right)$$

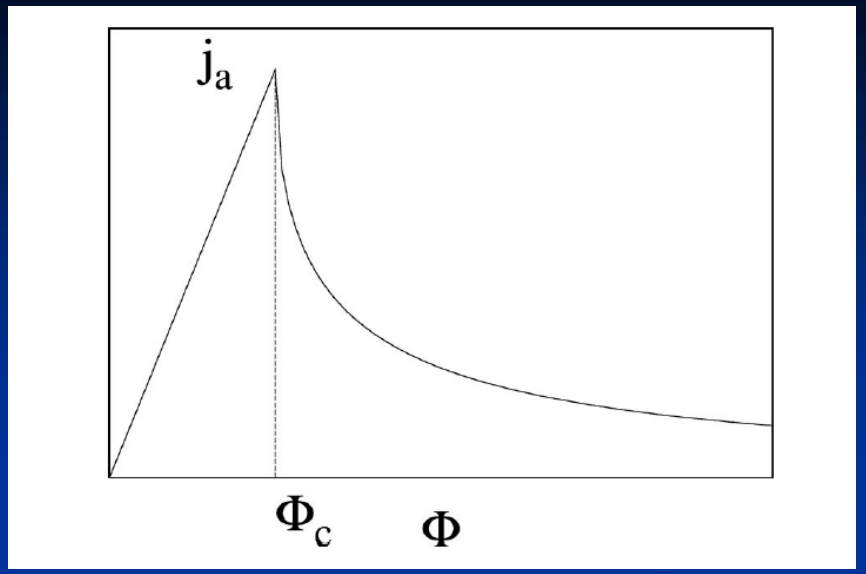
$$+ U \sum_{i,p} n_{i,p}(n_{i,p} - 1) + V n_{i,1} n_{i,2}, \qquad (1)$$

$$\int \vec{A} \cdot \vec{dl} = \Phi$$

$$H = H^0_s + H^0_a - \frac{t_{\perp}}{\pi a} \int dx \cos[\sqrt{2}\,\theta_a + e^* A_{\perp}(x)]$$

$$+ \frac{2Va}{(2\pi a)^2} \int dx \cos\sqrt{8}\,\phi_a,$$



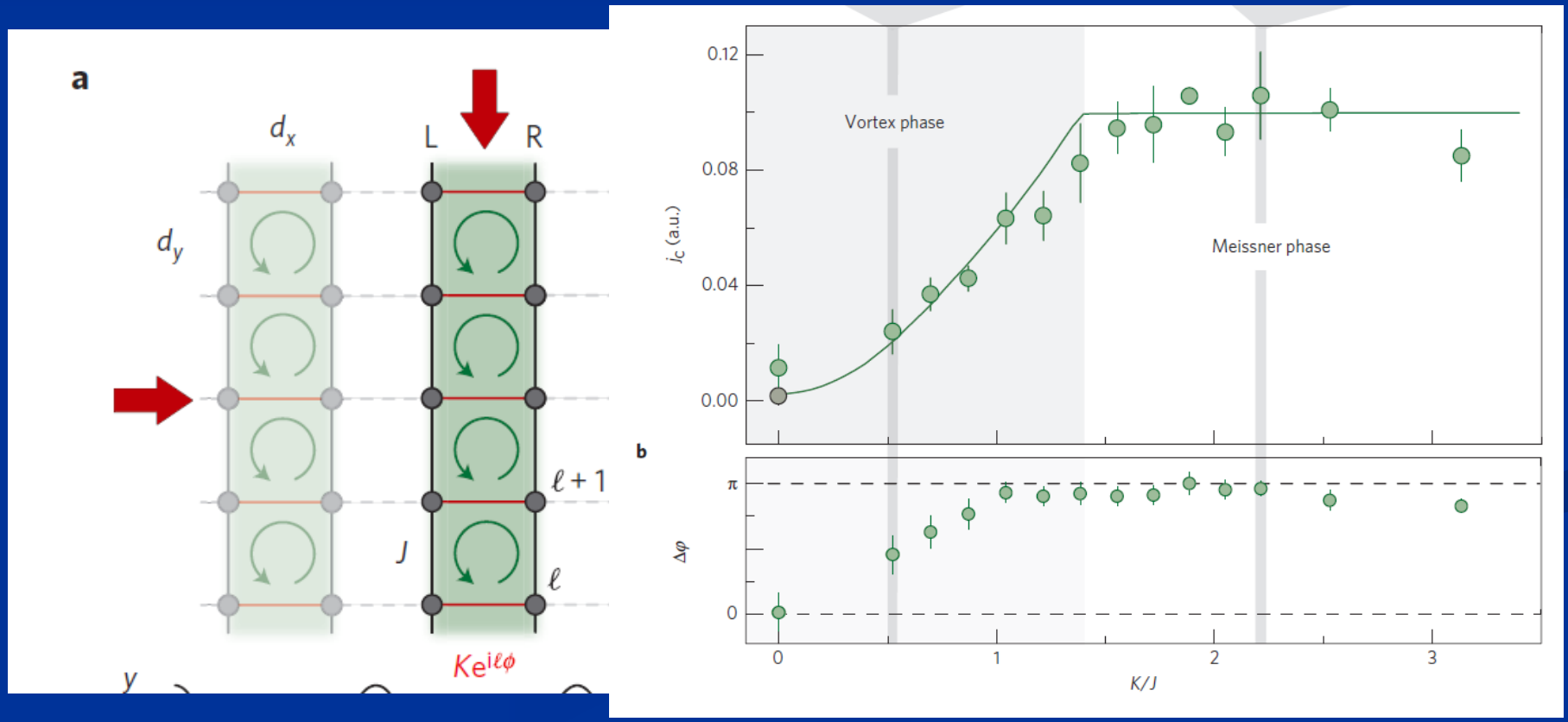


Orbital currents (``Meissner" effect)

Field ``Hc1": appearance of vortices

Artificial gauge field (cold atoms)

M Atala et al. Nat Phys, 10 588 (2014)

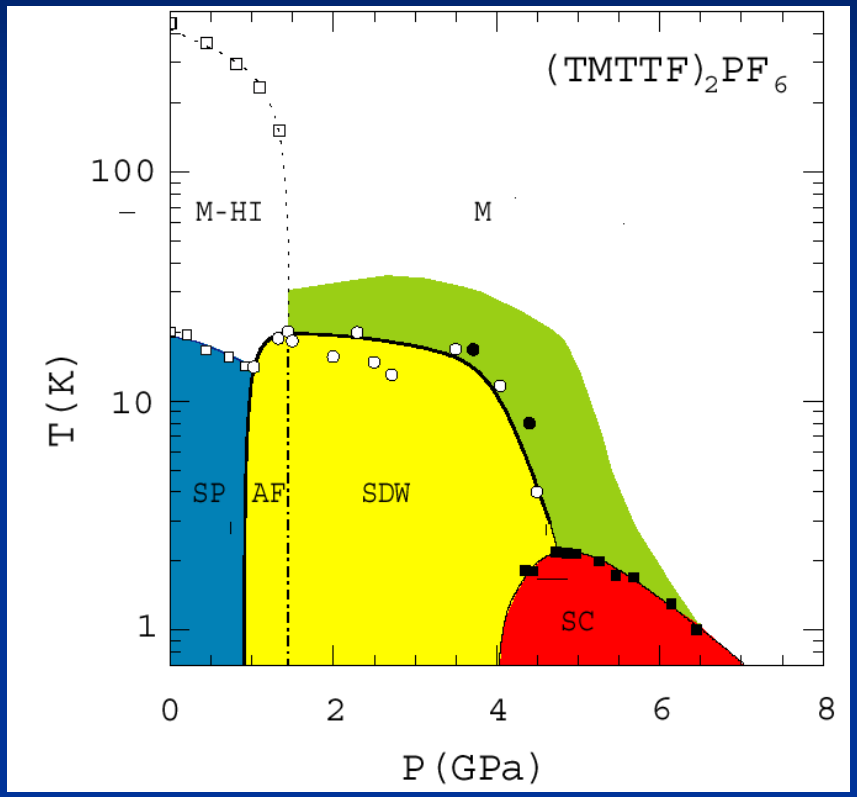


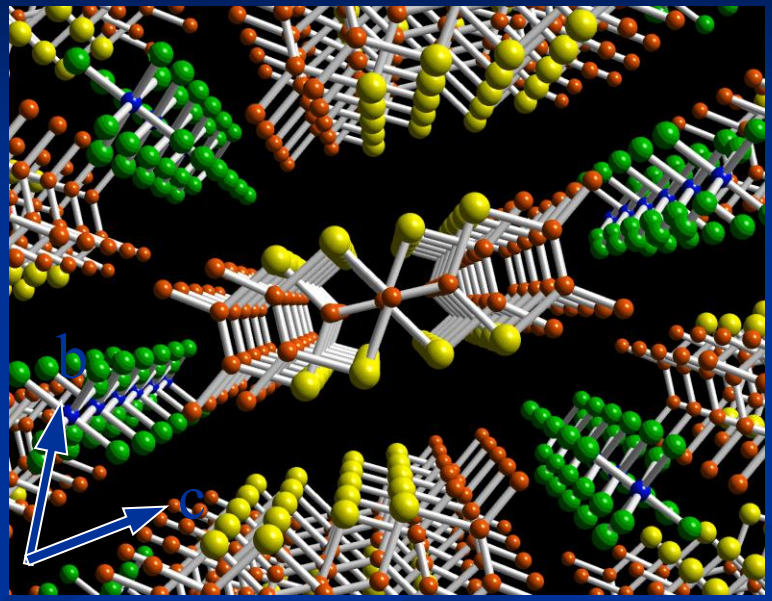
Take home message

- Good theoretical methods to deal with the case of a ``simple'' equilibrium 1d systems (analytics and numerics)
- Stepping stone to go beyond: many exciting questions and problems (out of equilibrium, disorder, many chains, etc.)
- Controled experimental realizations in condensed matter and cold atoms

Open problems

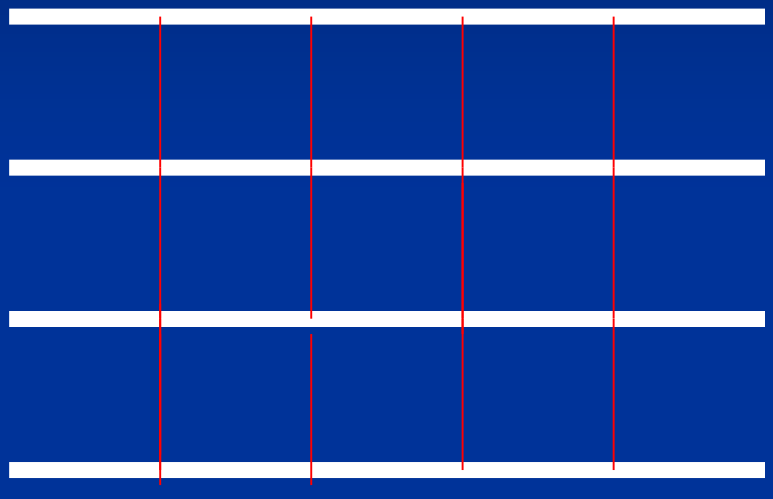
Coupled 1D systems





D. Jaccard et al., J. Phys. C, 13 L89 (2001)

- Interaction effects vary enormously with dimension



Dimensional Crossover



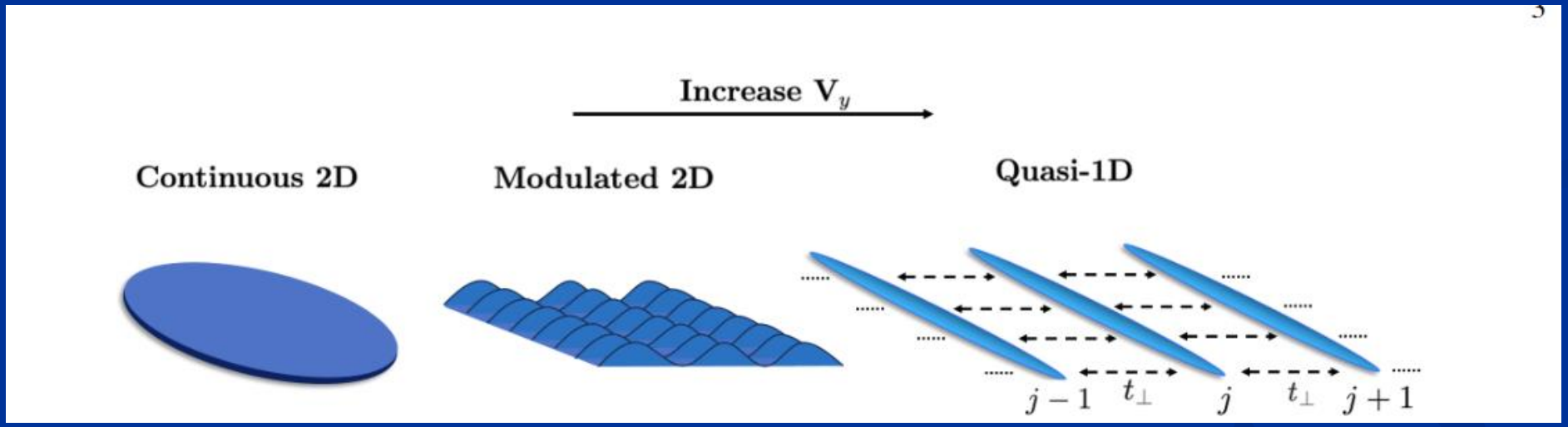
- Largely open physics
- Strong difficulties to treat (analytics, numeric)
- Allows to incorporate SOC and magnetic field effects
- Many studies limited to non-interacting case
- **Relevant for many experimental systems**

Bosons



Bosons

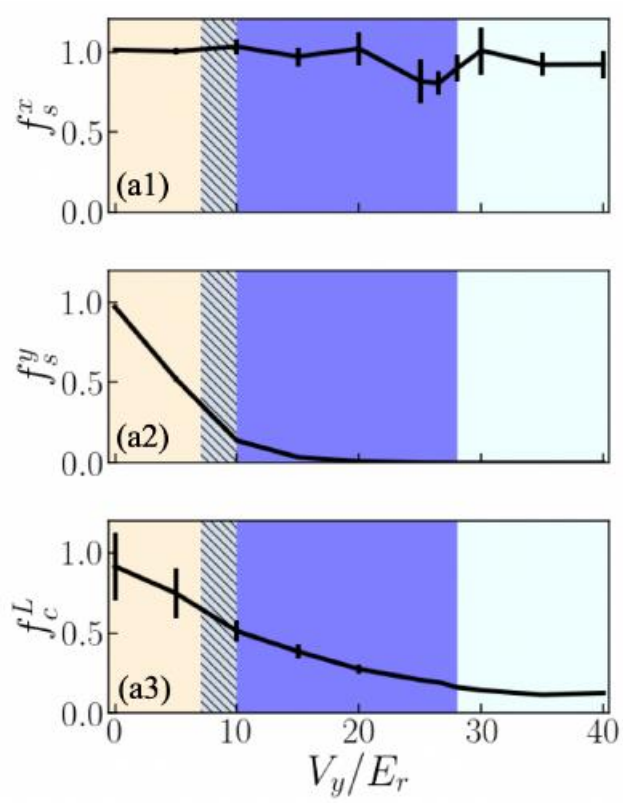


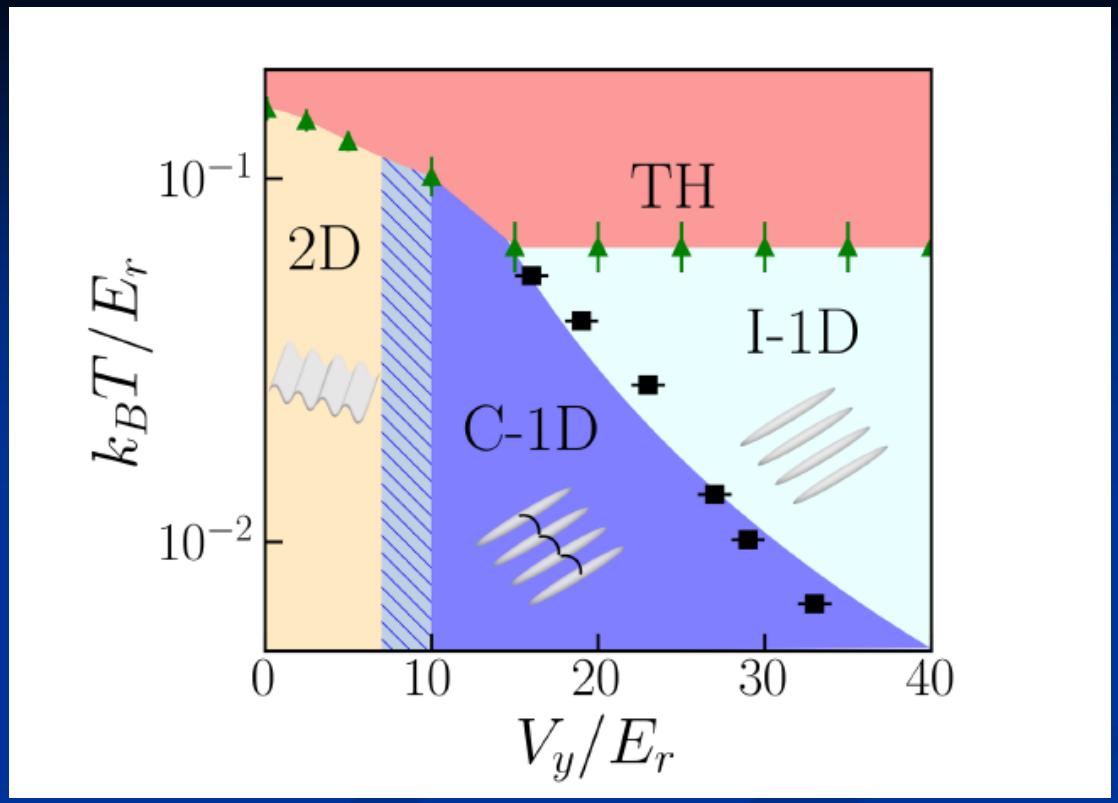


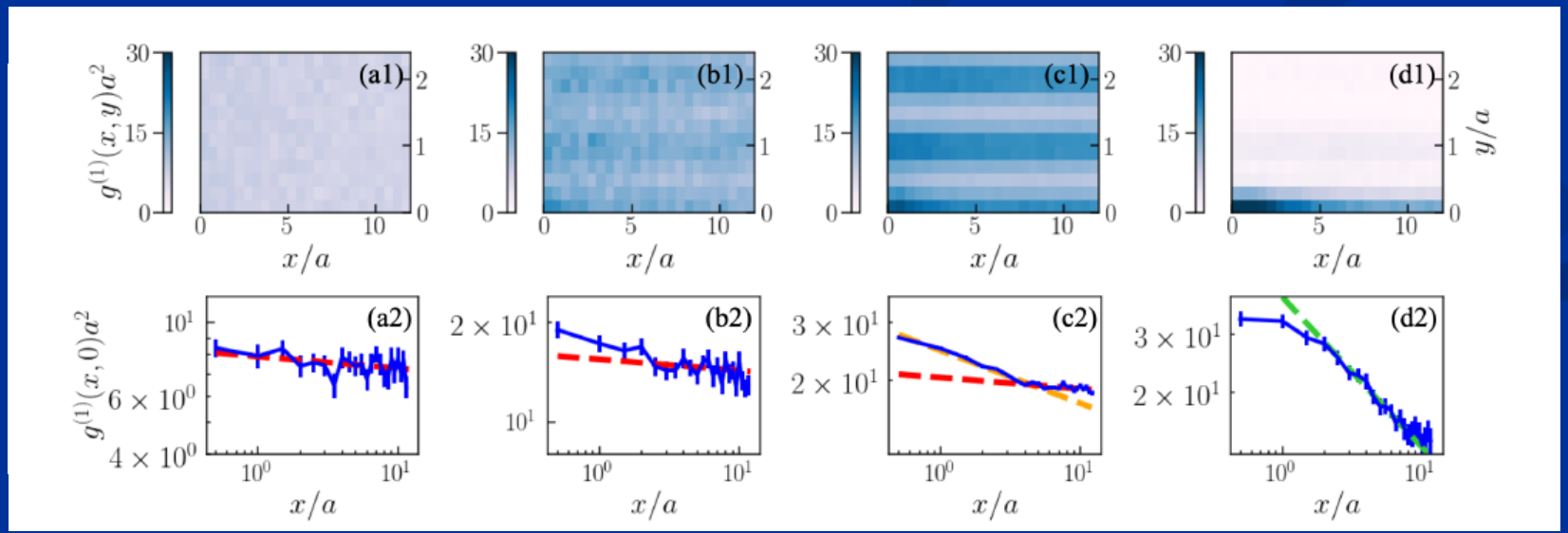
Strongly-interacting bosons at 2D-1D dimensional crossover

Hepeng Yao, Lorenzo Pizzino, Thierry Giamarchi

SciPost Phys. 15, 050 (2023) · published 7 August 2023


(a1)
(a2)
(a3)
f_s^x
f_s^y
f_c^L
V_y/E_r


TH
2D
I-1D
C-1D
k_BT/E_r
V_y/E_r


(a1)
(b1)
(c1)
(d1)
(a2)
(b2)
(c2)
(d2)
$g^{(1)}(x,y)a^2$
$g^{(1)}(x,0)a^2$
x/a
y/a





nature physics

Article

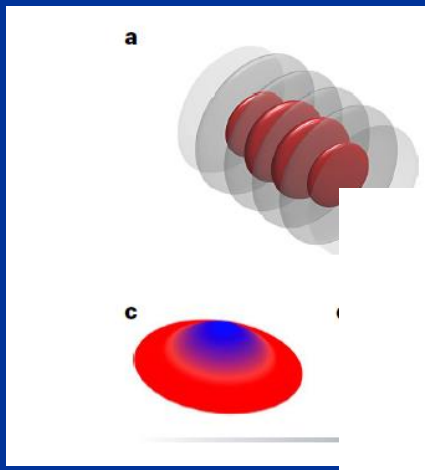
https://doi.org/10.1038/s41567-024-02459-3

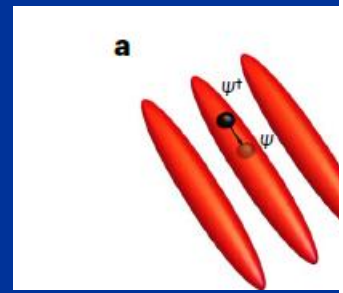
Observation of the 2D–1D crossover in strongly interacting ultracold bosons

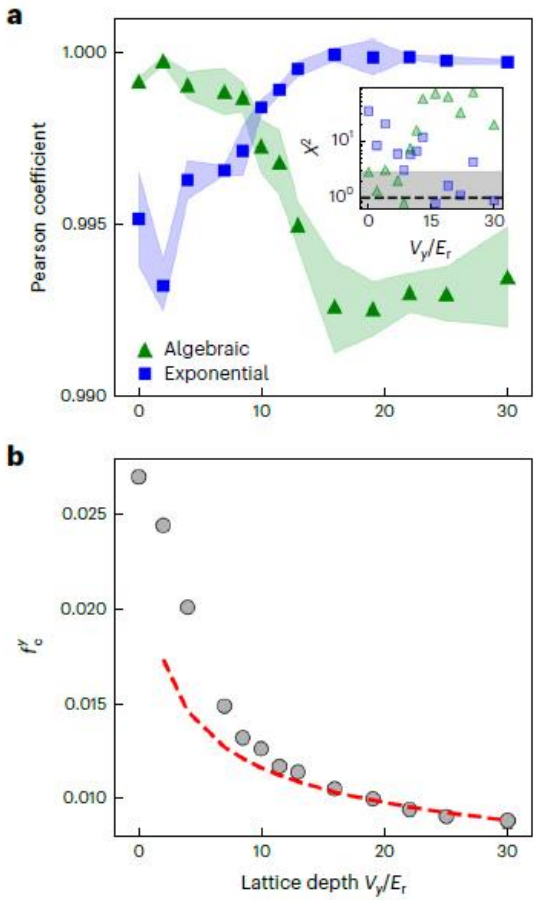
Yanliang Guo[1,3], Hepeng Yao[2,3], Satwik Ramanjanappa[1], Sudipta Dhar[1], Milena Horvath[1], Lorenzo Pizzino[2], Thierry Giamarchi[2], Manuele Landini[1] & Hanns-Christoph Nägerl[1]

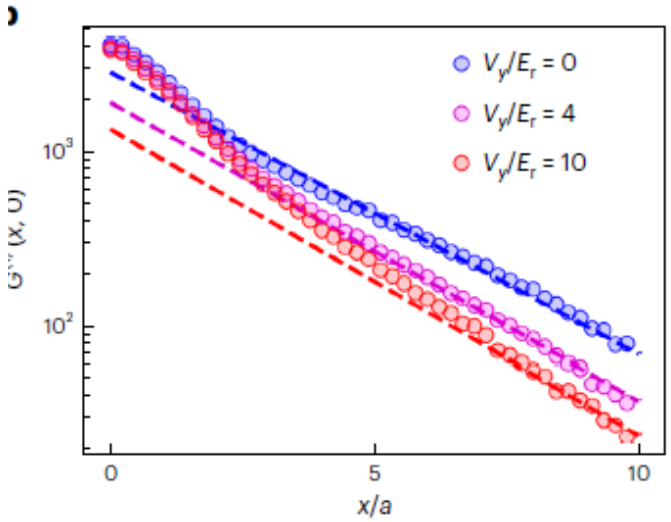
















SCIENCE ADVANCES | RESEARCH ARTICLE

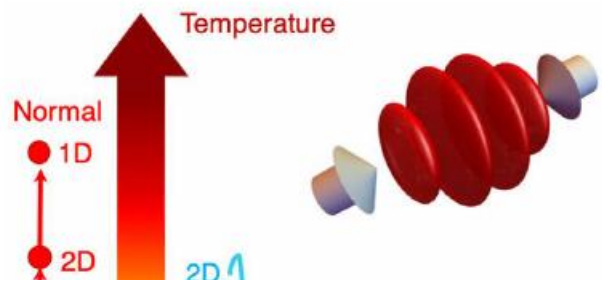
PHYSICS

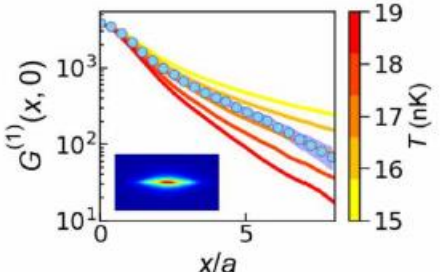
Anomalous cooling of bosons by dimensional reduction

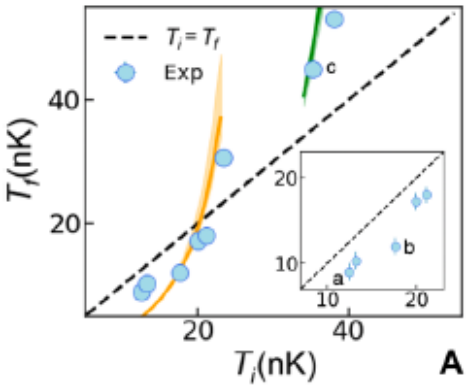
Yanliang Guo[1]†, Hepeng Yao[2]†, Sudipta Dhar[1], Lorenzo Pizzino[2], Milena Horvath[1], Thierry Giamarchi[2], Manuele Landini[1], Hanns-Christoph Nägerl[1]*

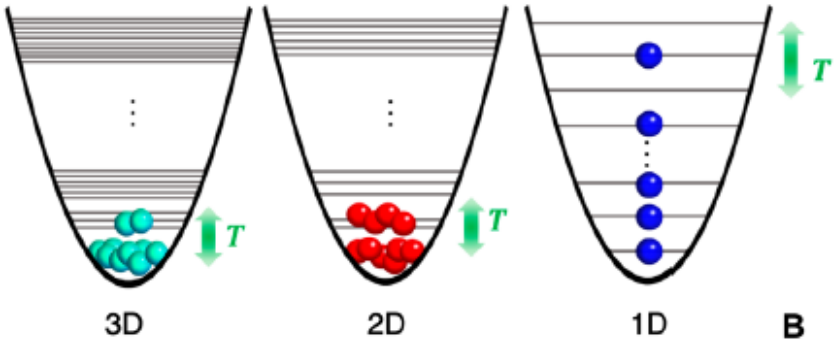


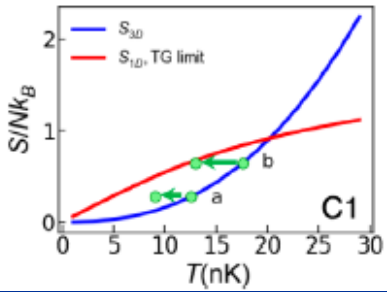


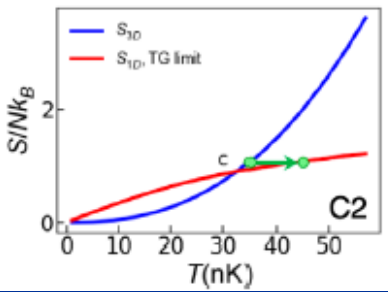






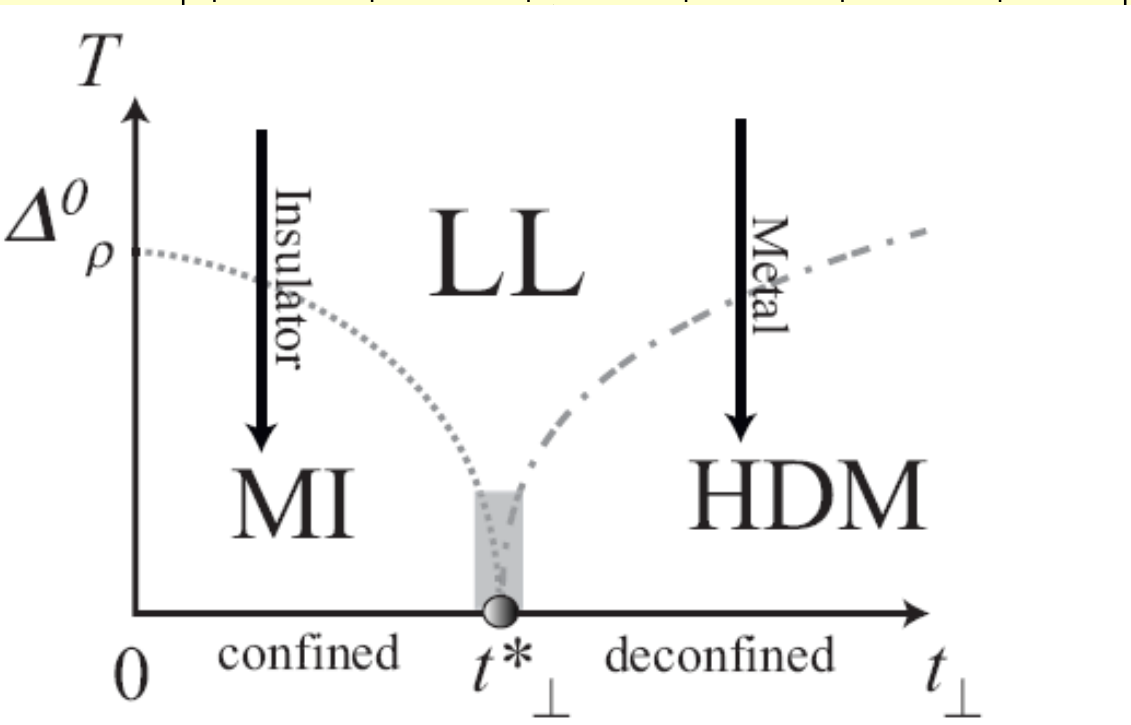


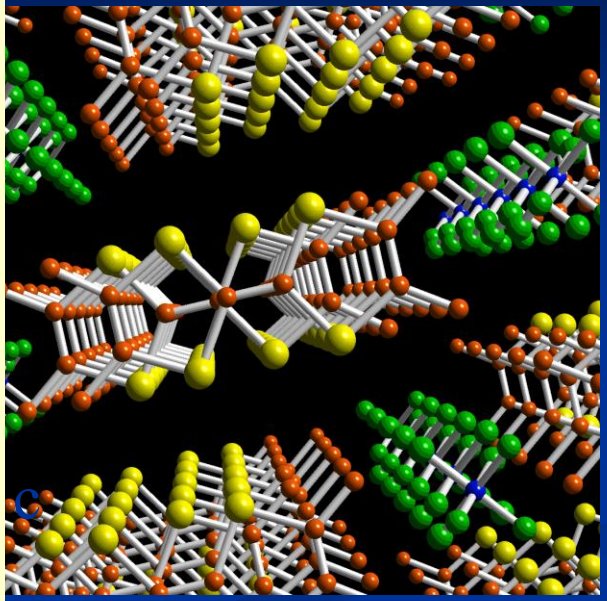




Fermions

Deconfinement



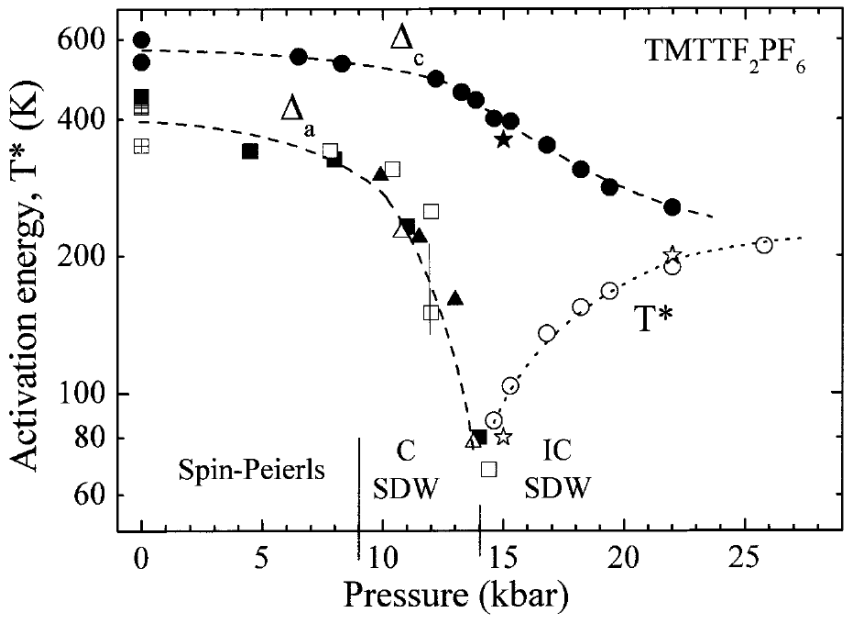


TG Chemical Review 104 5037 (2004)

P. Auban-Senzier, D. Jérome, C. Carcel and J.M. Fabre J de Physique IV, (2004)

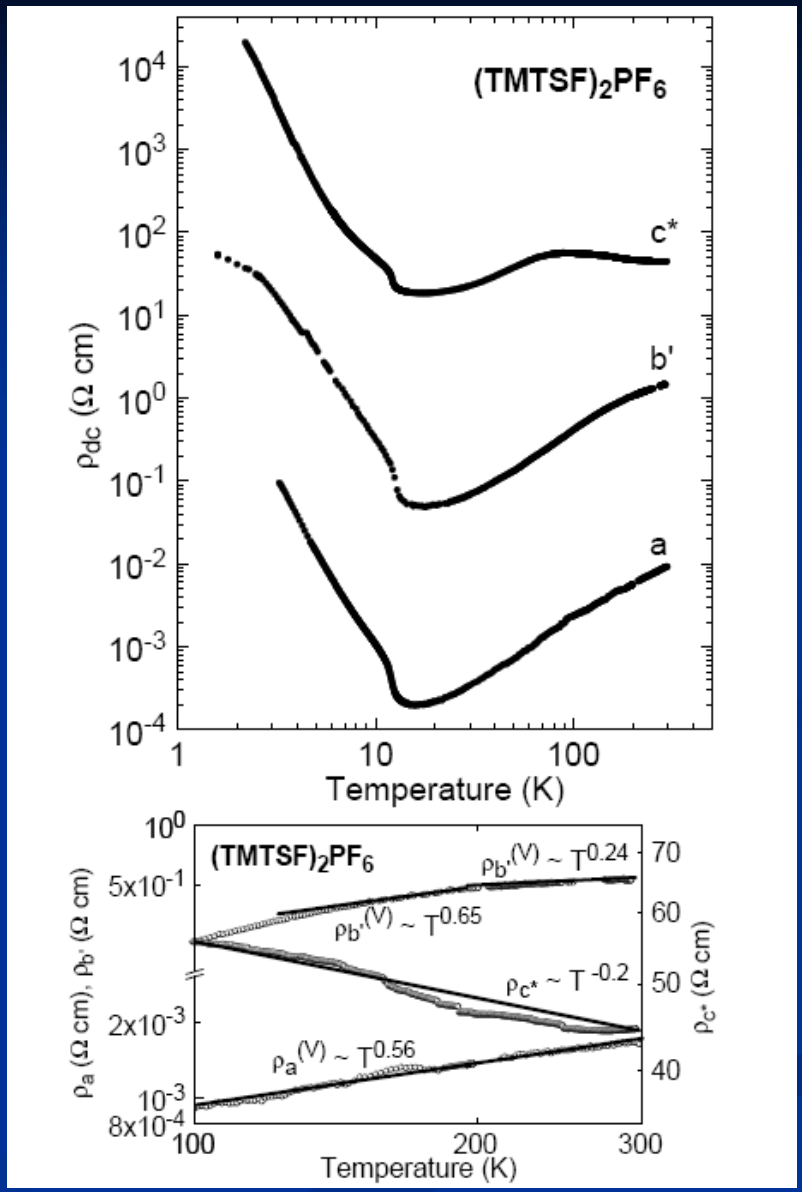
A. Pashkin, M. Dressel, M. Hanfland, C. A. Kuntscher, PRB 81 125109 (2010)

D. Jaccard et al., J. Phys. C, 13 L89 (2001)



V. Vescoli et al. Science 281 1991 (1998), Euro Phys J B 13 503 (2000)

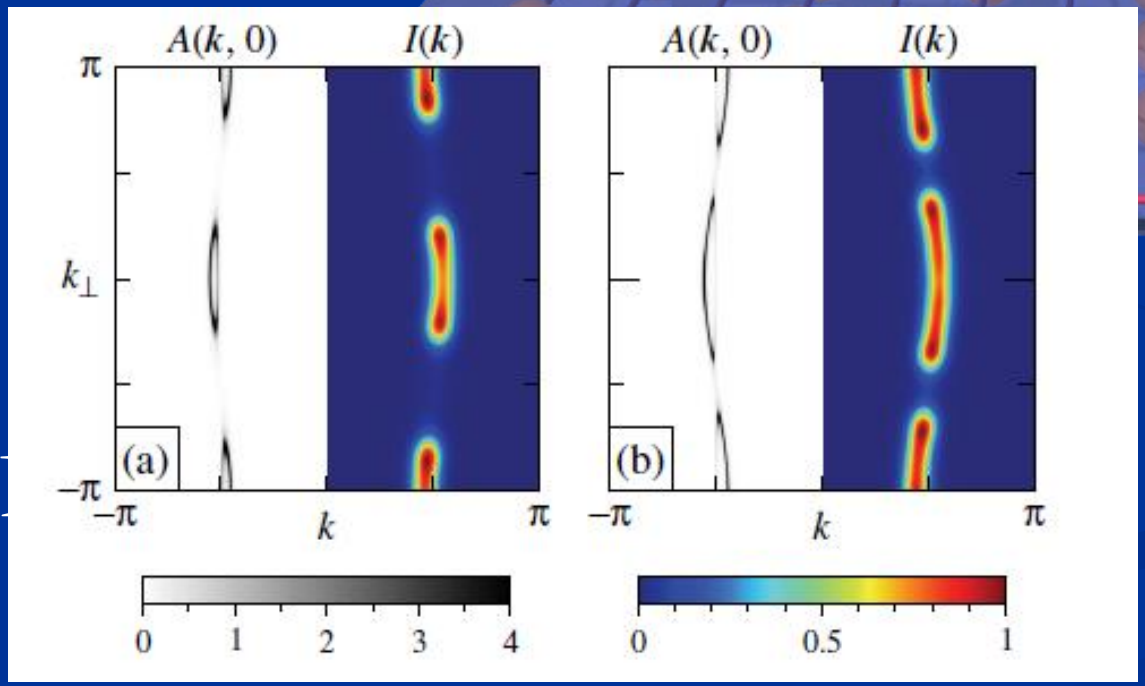
P. Auban-Senzier et al. J. Phys IV, 114 (2004)

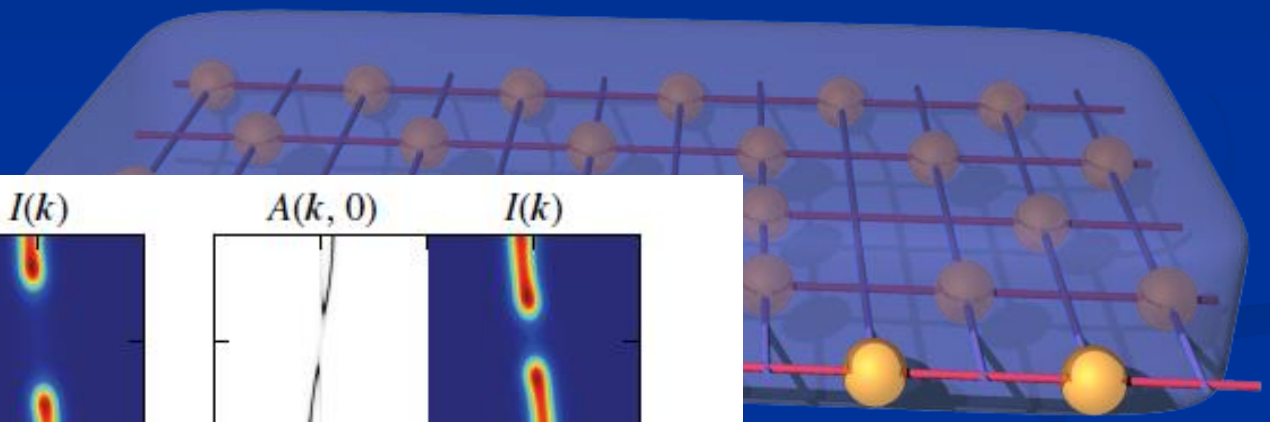


M. Dressel *et al.*, Phys. Rev. B **71**, 075104 (2005).

Chain (Cluster) - DMFT

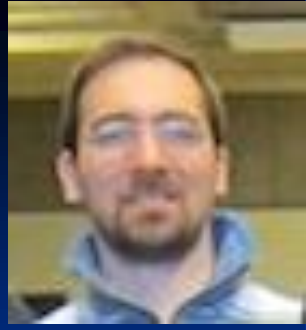
S. Biermann, A. Georges, A. Lichtenstein, TG, PRL 87 276405 (2001)
C. Berthod et al. PRL 97, 136401 (2006)





determined

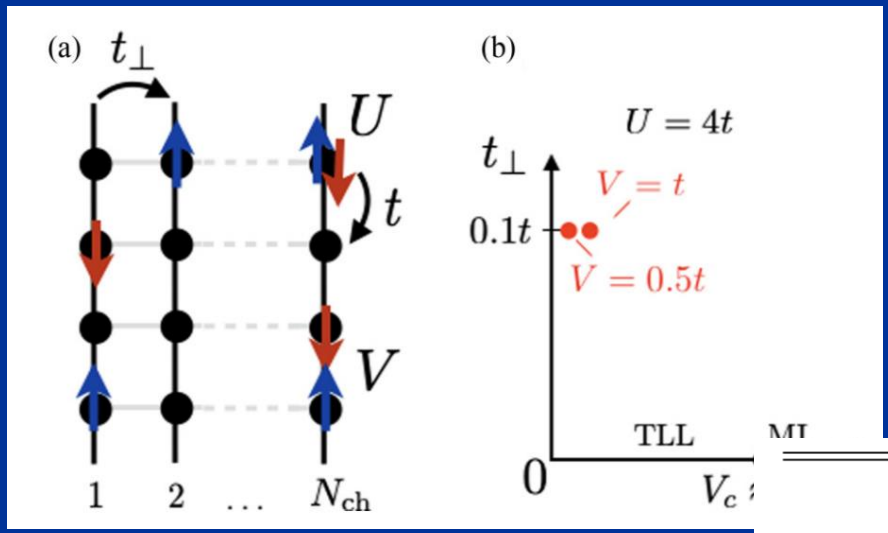
DMRG



PHYSICAL REVIEW B **100**, 075138 (2019)

Understanding repulsively mediated superconductivity of correlated electrons via massively parallel density matrix renormalization group

A. Kantian,[1] M. Dolfi,[2,3,*] M. Troyer,[2] and T. Giamarchi[4]



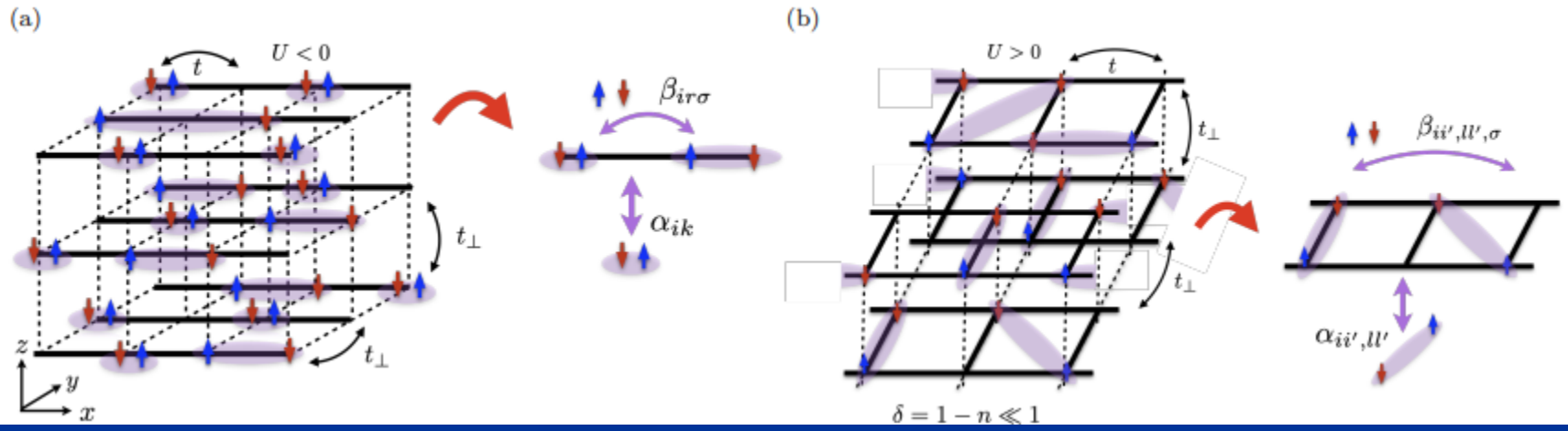
	$\overline{\Delta}_s[N_{\rm ch}]\ [t \times 10^{-3}]$			
	Straight extrapolation		Extr. + estimates	
$N_{\rm ch}$	$V/t = 0.5$	$V/t = 1$	$V/t = 0.5$	$V/t = 1$
2	5.59	7.59	5.36	7.31
4	9.68	10.2	10.77(10)[a]	10.268(11)
6	9.37	11.4	4.3(1.4)[b]	11.20(97)
8	9.45		7.6(1.3)[c]	

DMRG + Mean Field

Solving 2D and 3D lattice models of correlated fermions - combining matrix product states with mean field theory

Gunnar Bollmark,[1] Thomas Köhler,[1] Lorenzo Pizzino,[2] Yiqi Yang,[3] Johannes S. Hofmann,[4] Hao Shi,[5] Shiwei Zhang,[6] Thierry Giamarchi,[2] and Adrian Kantian[1,7]

Phys. Rev. X **13**, 011039 – Published 15 March 2023



Take home message

- Beyond TLL: largely open physics even at equilibrium (disorder, more complicated 1D models, etc)
- Big challenge: coupled 1D structures. Dimensional crossover
- A path to physics of 2D,3D systems, with new physics along the way

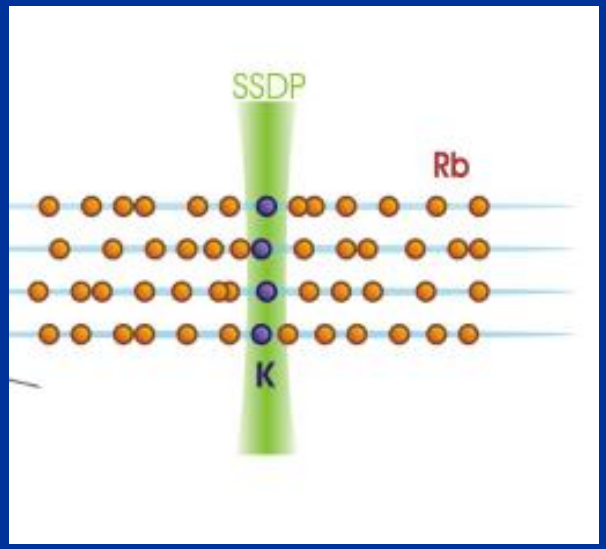
Out of equilibrium low-d systems

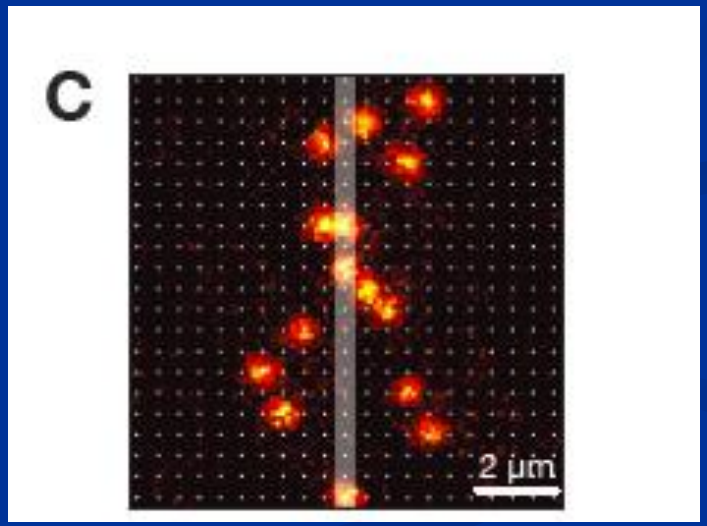


Quenches

Local quenches (impurity)

M. Zvonarev, V. Cheianov, TG PRL (2007);
J. Catani et al, PRA 85 023623 (2012) ; T. Fukuhara, A. Kantian et al. Nat Phys. (2013)
A. Kantian, U. Schollwoeck, TG PRL 113 070601 (2015)



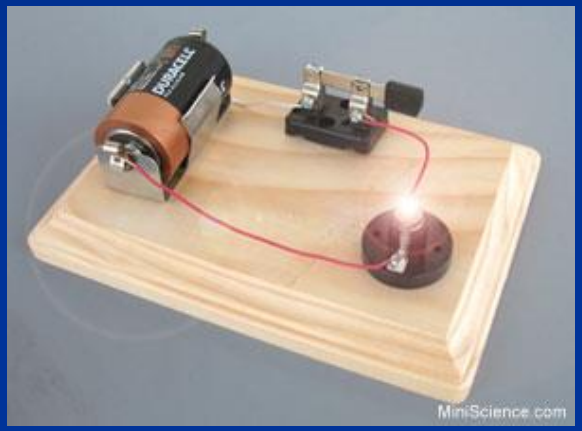




Dirac impurity:
L. Gotta, TG, Arxiv/2404.16214

Transport

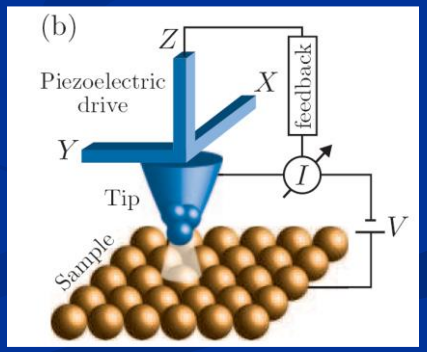
- Condensed matter: a «routine» probe for materials



- Theoretically:
  complicated !
  Out of equilibrium

- Typical situation





- Often (but not always !) : linear response I=G V

Methods to compute transport

- Kubo formula [1957]

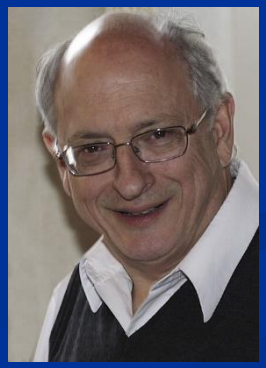


https://en.wikipedia.org/wiki/Kubo_formula

- Landauer-Büttiker formula [1957]



https://en.wikipedia.org/wiki/Rolf_Landauer



https://de.wikipedia.org/wiki/Markus_Büttiker

Bulk/Kubo (shaking, “kick”,..)

PRL 106, 205301 (2011) PHYSICAL REVIEW LETTERS week ending 20 MAY 2011

Spectroscopy for Cold Atom Gases in Periodically Phase-Modulated Optical Lattices

Akiyuki Tokuno and Thierry Giamarchi



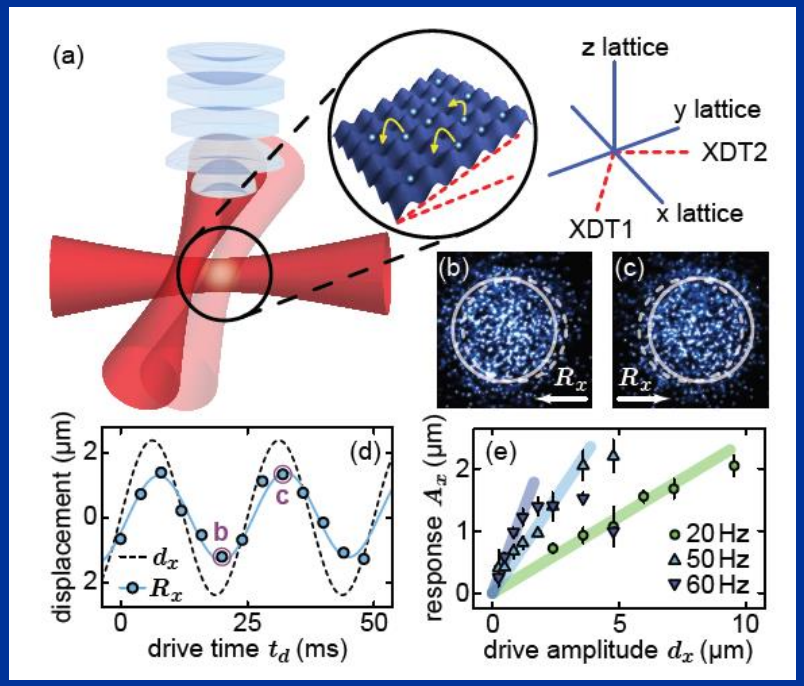
PHYSICAL REVIEW LETTERS 122, 153602 (2019)

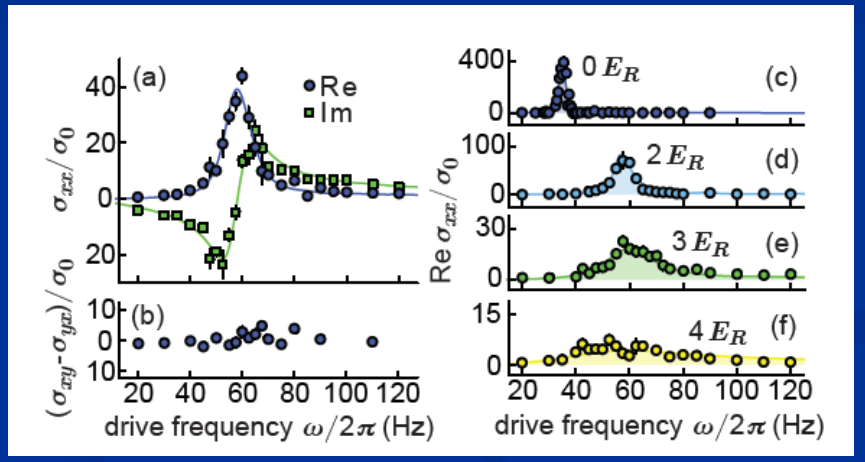
Conductivity Spectrum of Ultracold Atoms in an Optical Lattice

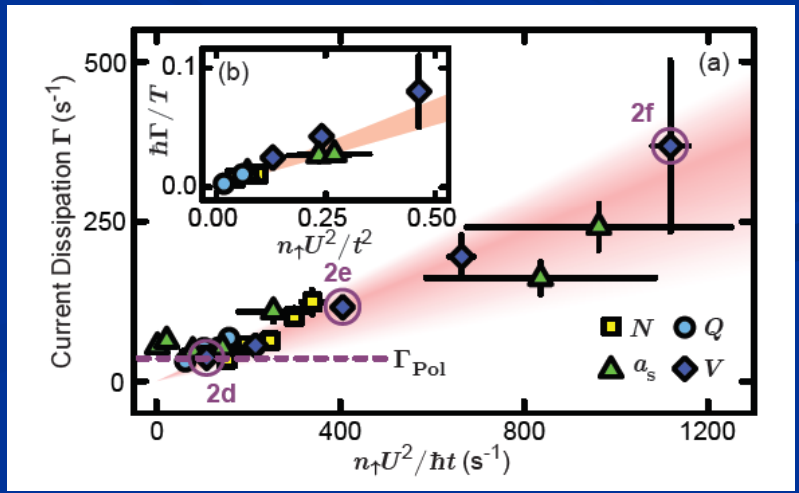
Rhys Anderson,[1] Fudong Wang,[1] Peihang Xu,[1] Vijin Venu,[1] Stefan Trotzky,[1] Frédéric Chevy,[2] and Joseph H. Thywissen[1,3]

$V_{\mathrm{op}}(x,t) = V_0 \cos^2[\varphi(x - F(t))]$

$F(t)$

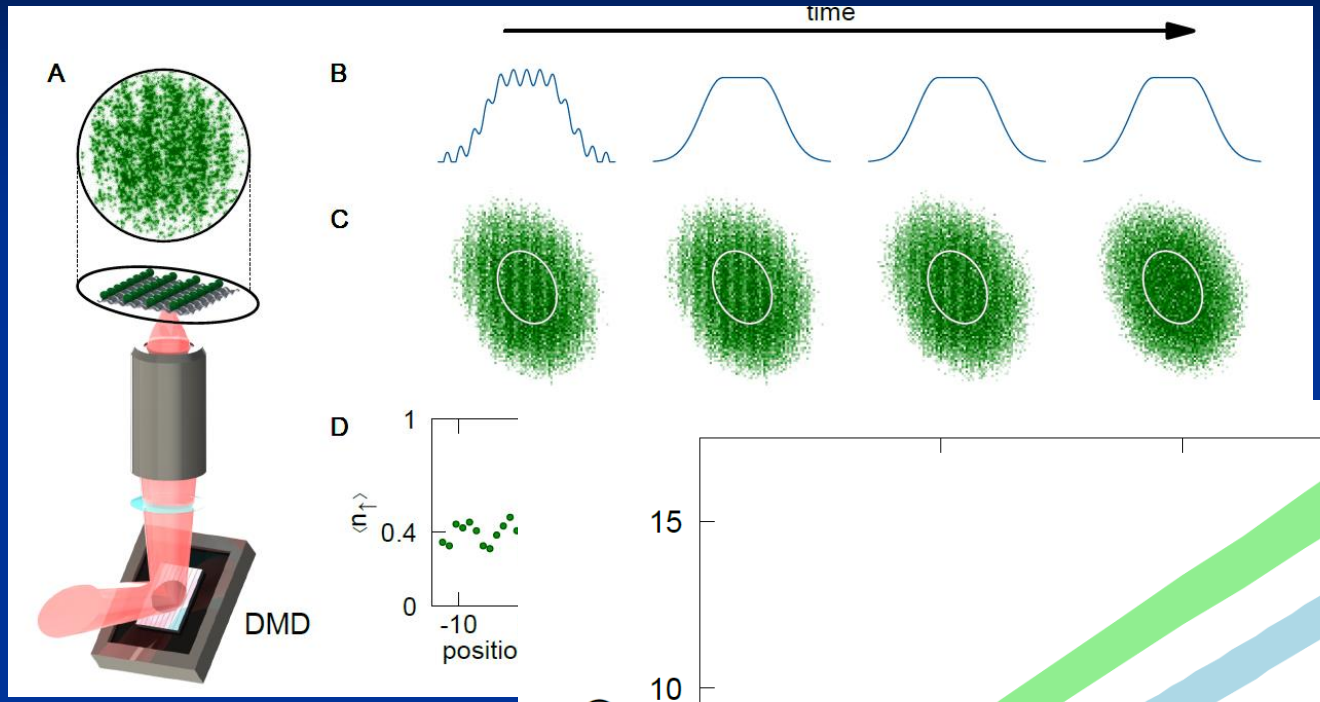


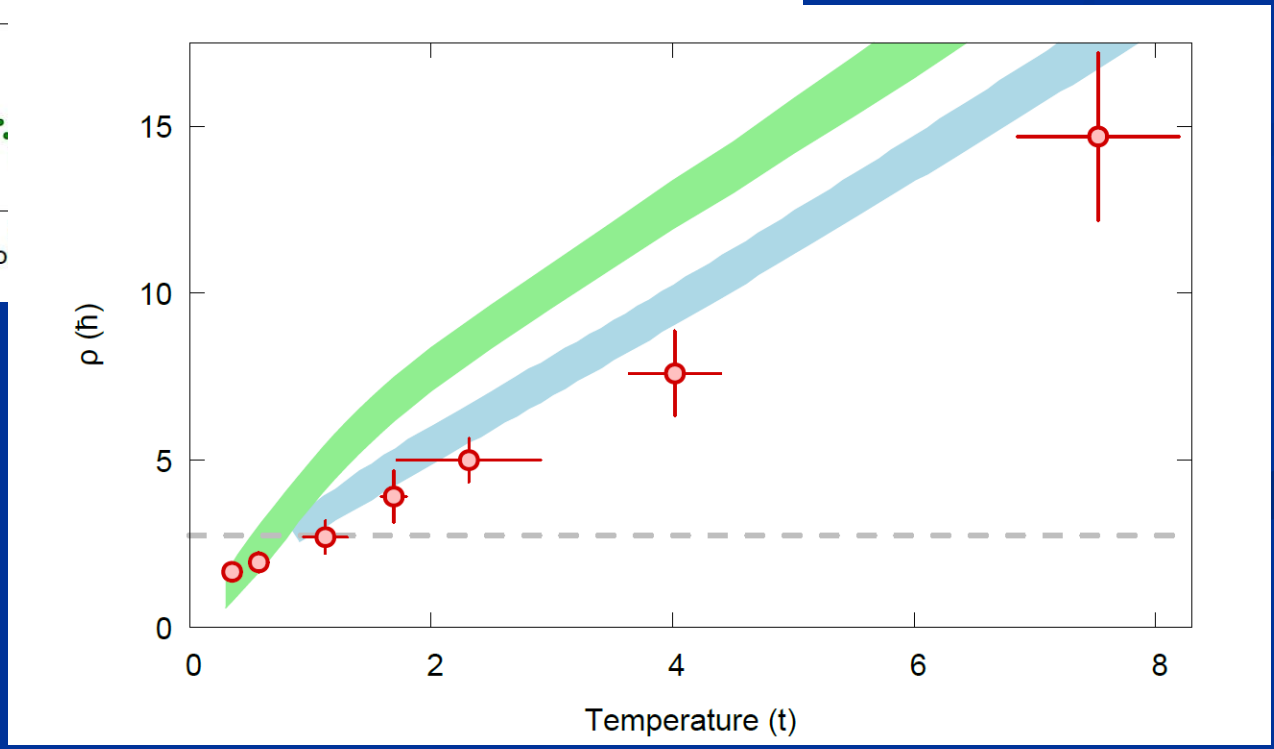




P.T. Brown et al. Science, 363, 379-382 (2019)







Mesoscopic transport



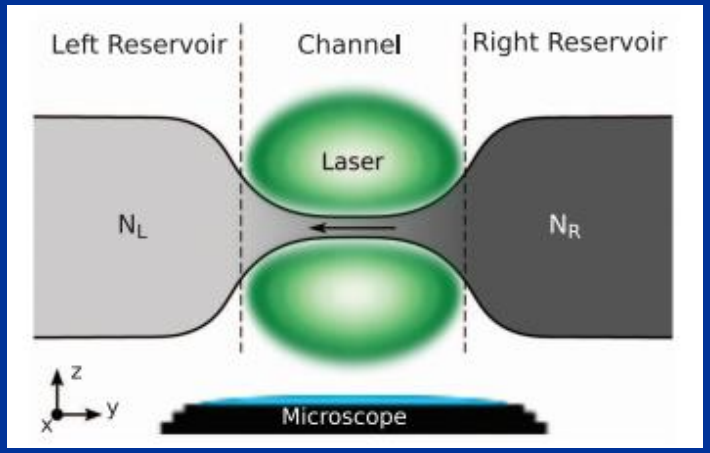
Reservoirs

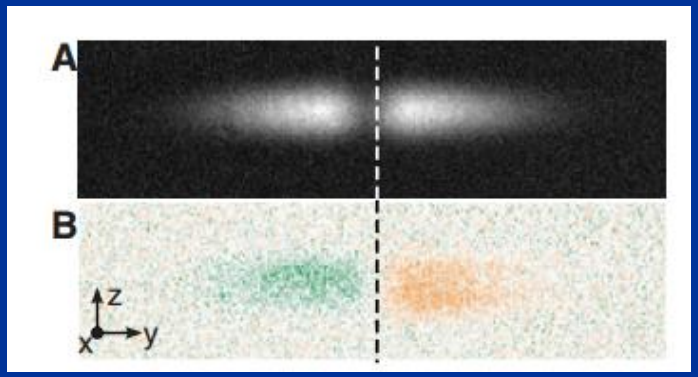


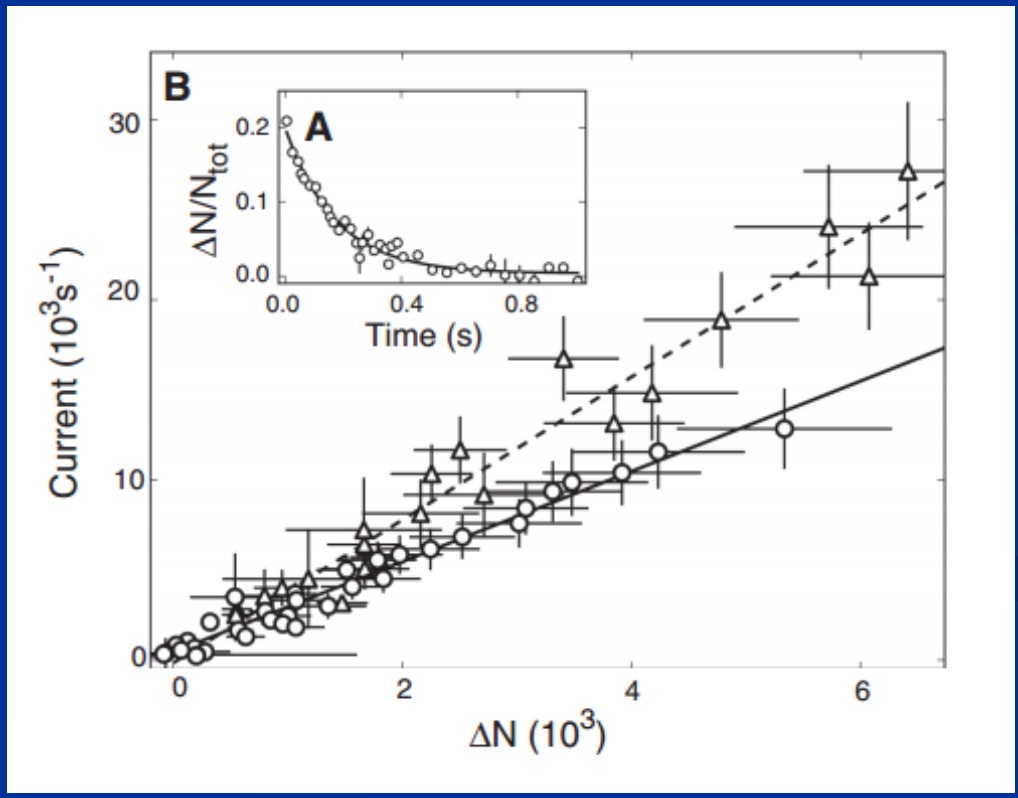
Conduction of Ultracold Fermions Through a Mesoscopic Channel

Jean-Philippe Brantut, Jakob Meineke, David Stadler, Sebastian Krinner, Tilman Esslinger*

SCIENCE VOL 337 31 AUGUST 2012







Quantum point contact

1D ballistic: quantized conductance

VOLUME 60, NUMBER 9 PHYSICAL REVIEW LETTERS 29 FEBRUARY 1988

Quantized Conductance of Point Contacts in a Two-Dimensional Electron Gas

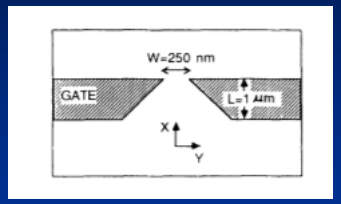
B. J. van Wees
Department of Applied Physics, Delft University of Technology, 2628 CJ Delft, The Netherlands

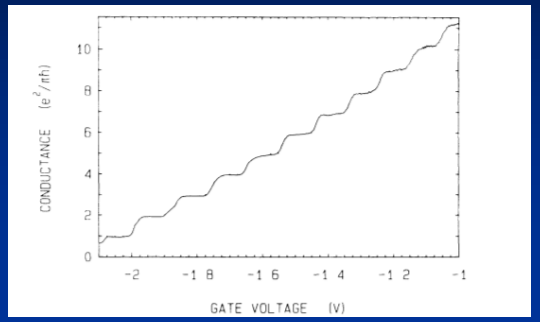
H. van Houten, C. W. J. Beenakker, and J. G. Williamson,
Philips Research Laboratories, 5600 JA Eindhoven, The Netherlands

L. P. Kouwenhoven and D. van der Marel
Department of Applied Physics, Delft University of Technology, 2628 CJ Delft, The Netherlands

and

C. T. Foxon
Philips Research Laboratories, Redhill, Surrey RH1 5HA, United Kingdom
(Received 31 December 1987)

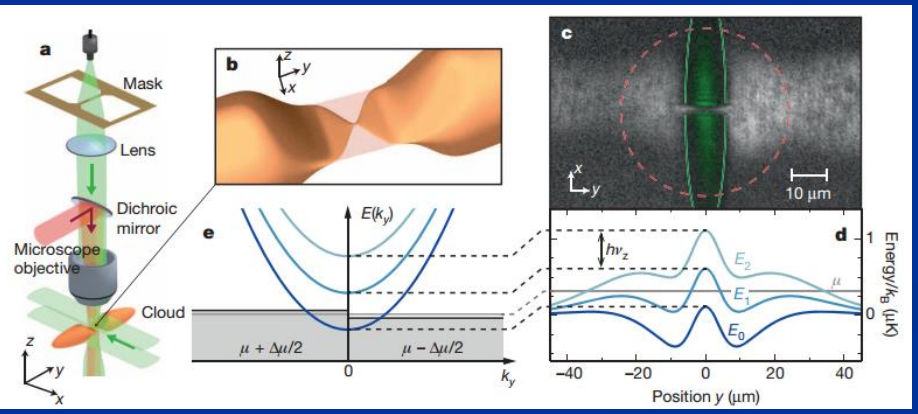


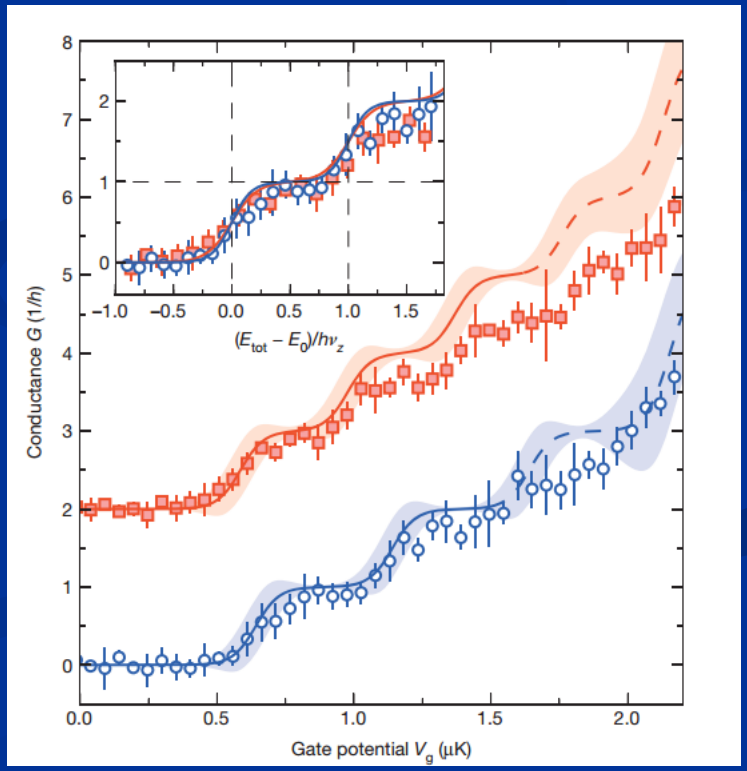


Observation of quantized conductance in neutral matter

Sebastian Krinner[1], David Stadler[1], Dominik Husmann[1], Jean-Philippe Brantut[1] & Tilman Esslinger[1]

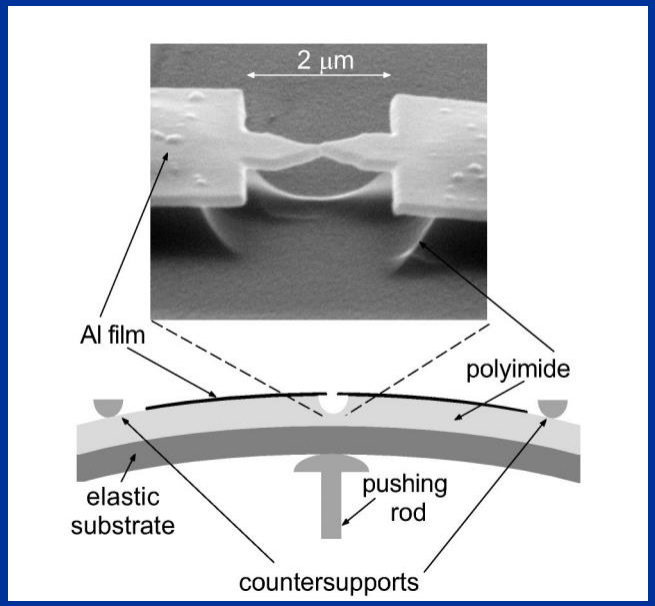
64 | NATURE | VOL 517 | 1 JANUARY 2015

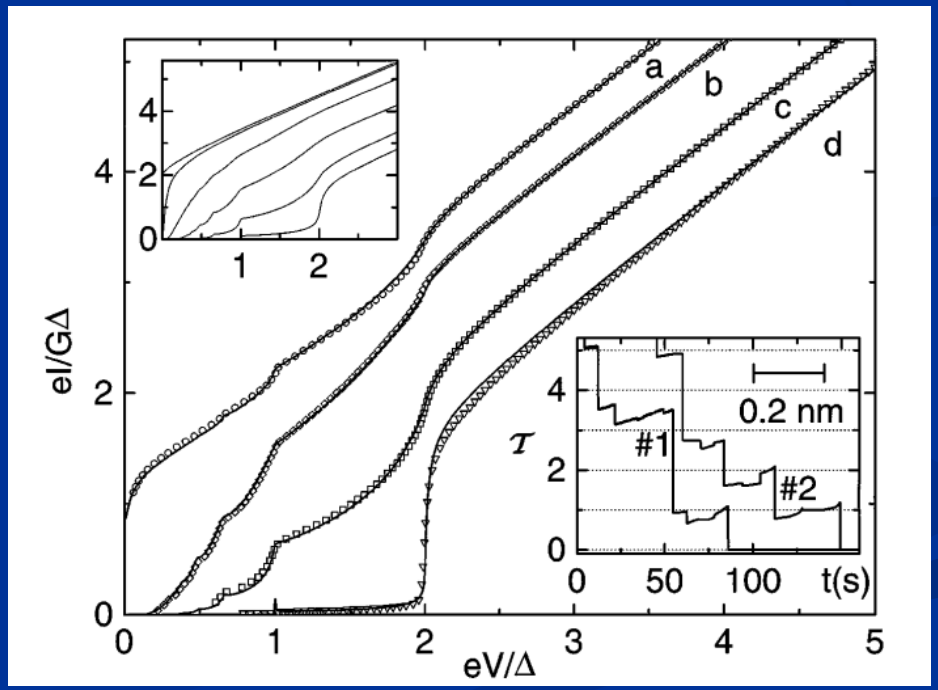




S-S Quantum point contact

- Theory: Blonder, Tinkham, Klapwijk (1982); Averin, Bardas (1995); Cueva, Martin-Rodero, Yetati (1996)





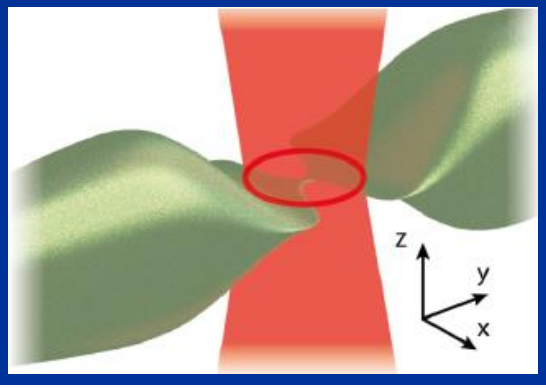
E. Scheer et al. PRL 78 3535 (1997)

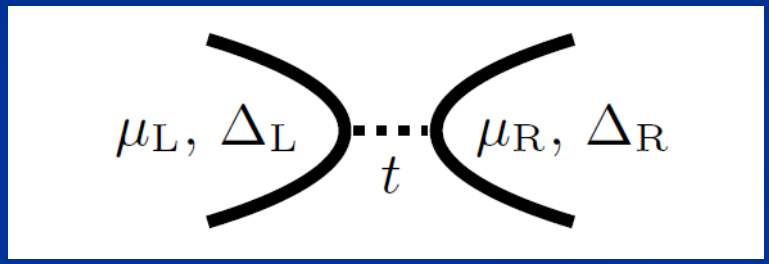


Quantum point contact

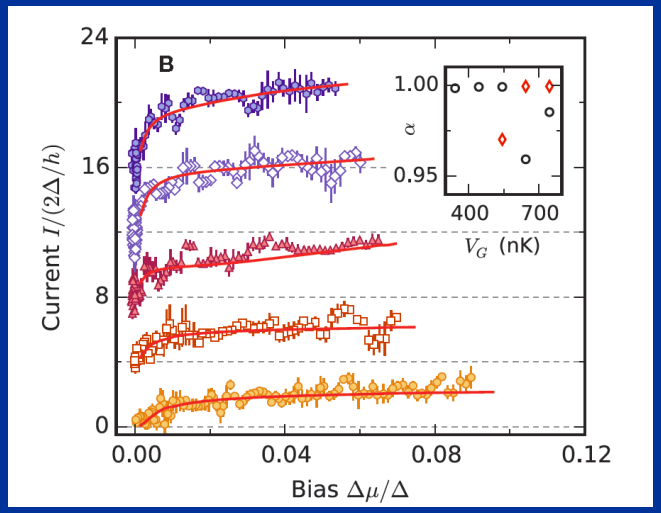


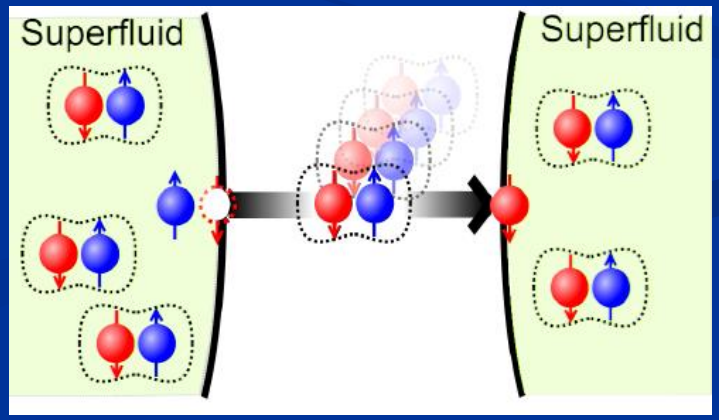
D. Husmann, S. Uchino, et al. Science 350 62667 (2015).





- Quantum point contact: multiple Andreev reflexions





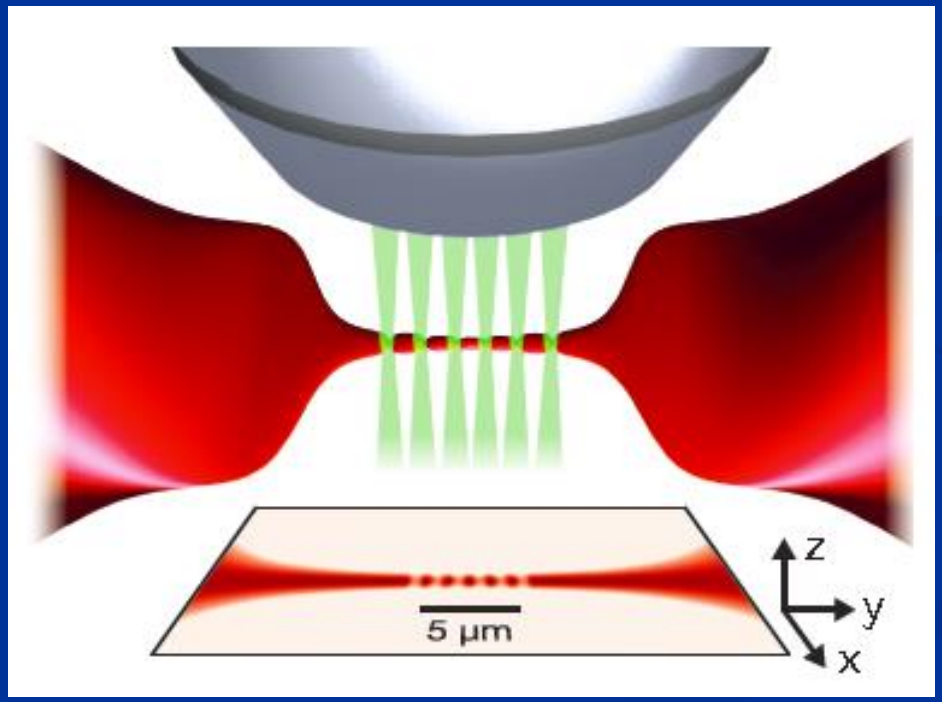
One dimensional structures

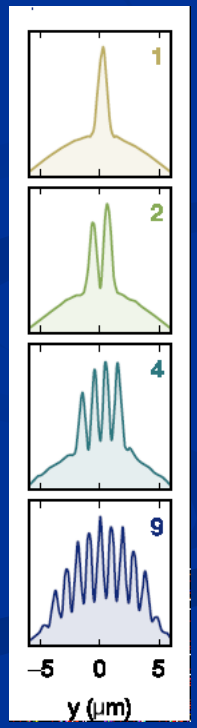


Atomtronic

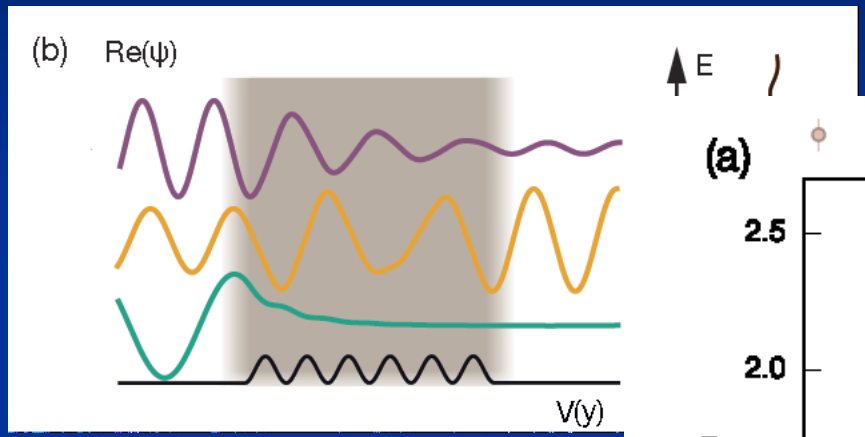


M. Lebrat, P. Grisins et al., PRX 8 011053 (18)

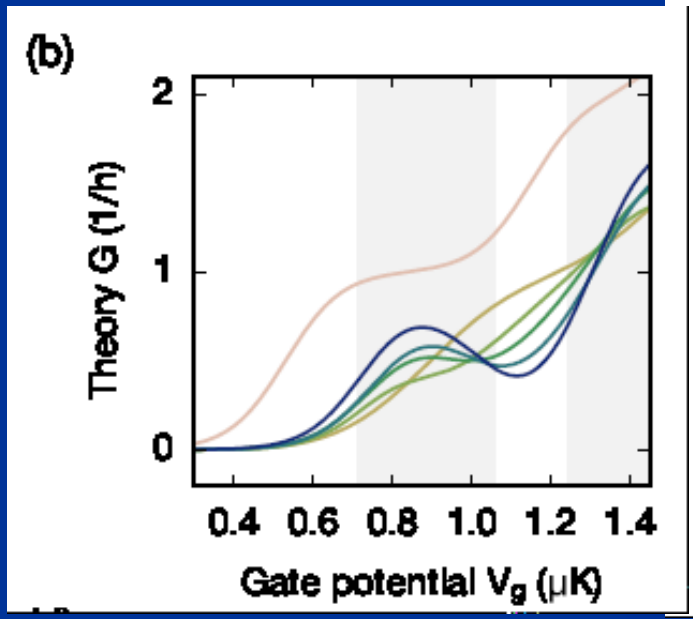


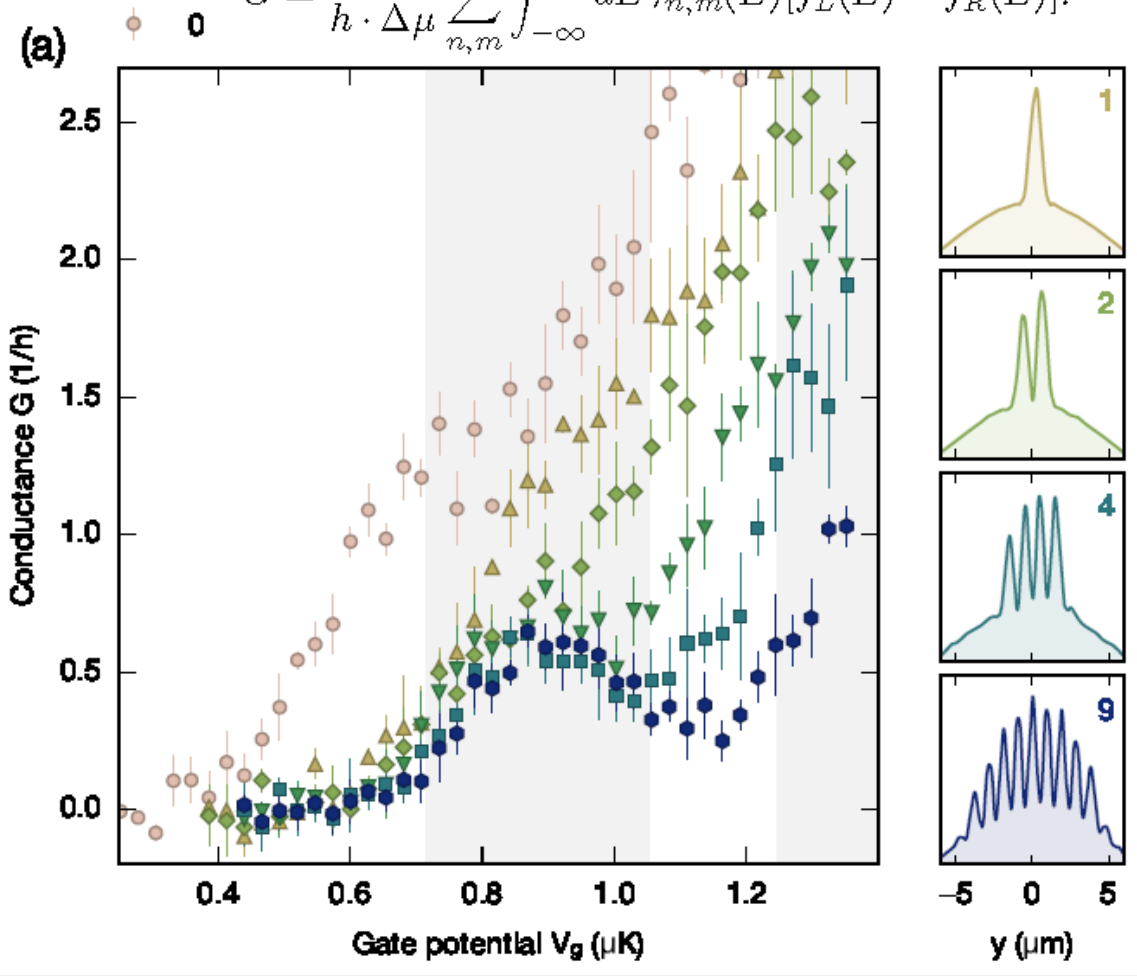


No interactions: band insulator



$$G = \frac{1}{h \cdot \Delta\mu} \sum_{n,m} \int_{-\infty}^{\infty} dE T_{n,m}(E)[f_L(E) - f_R(E)].$$





Weak periodic lattice in 1D

- No interactions: band insulator
- Attraction: singlet superconductor
- Superconductor resists "scattering" (disorder, potentials, etc.)
- Expect a competition band insulator-superconductivity

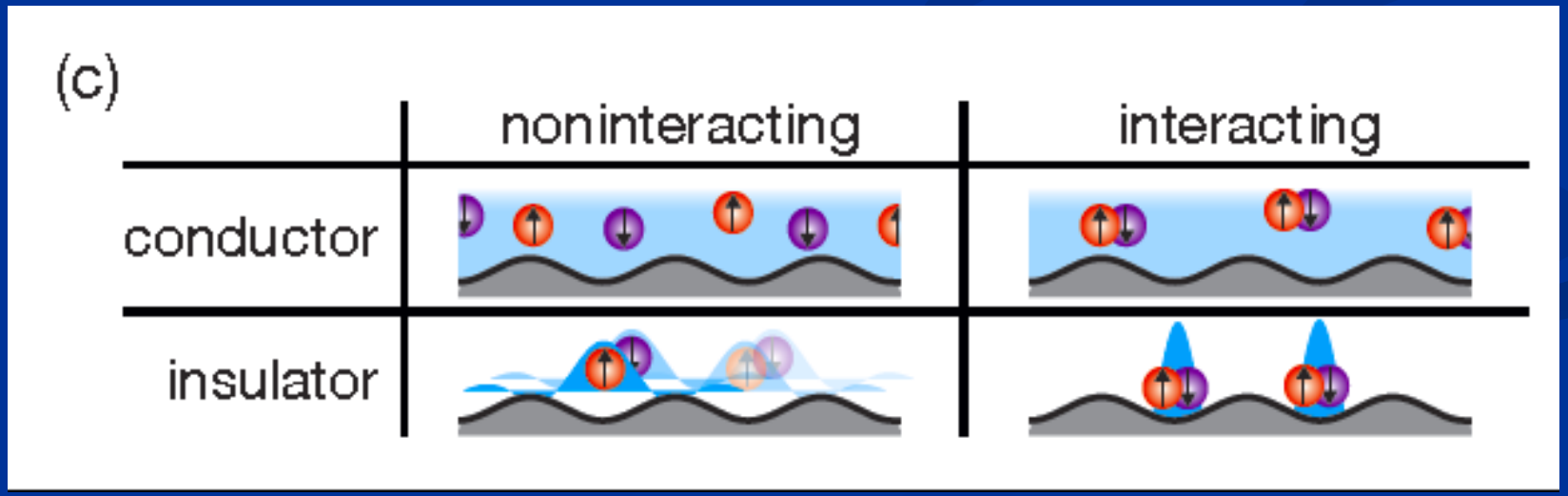


Luther-Emery liquid

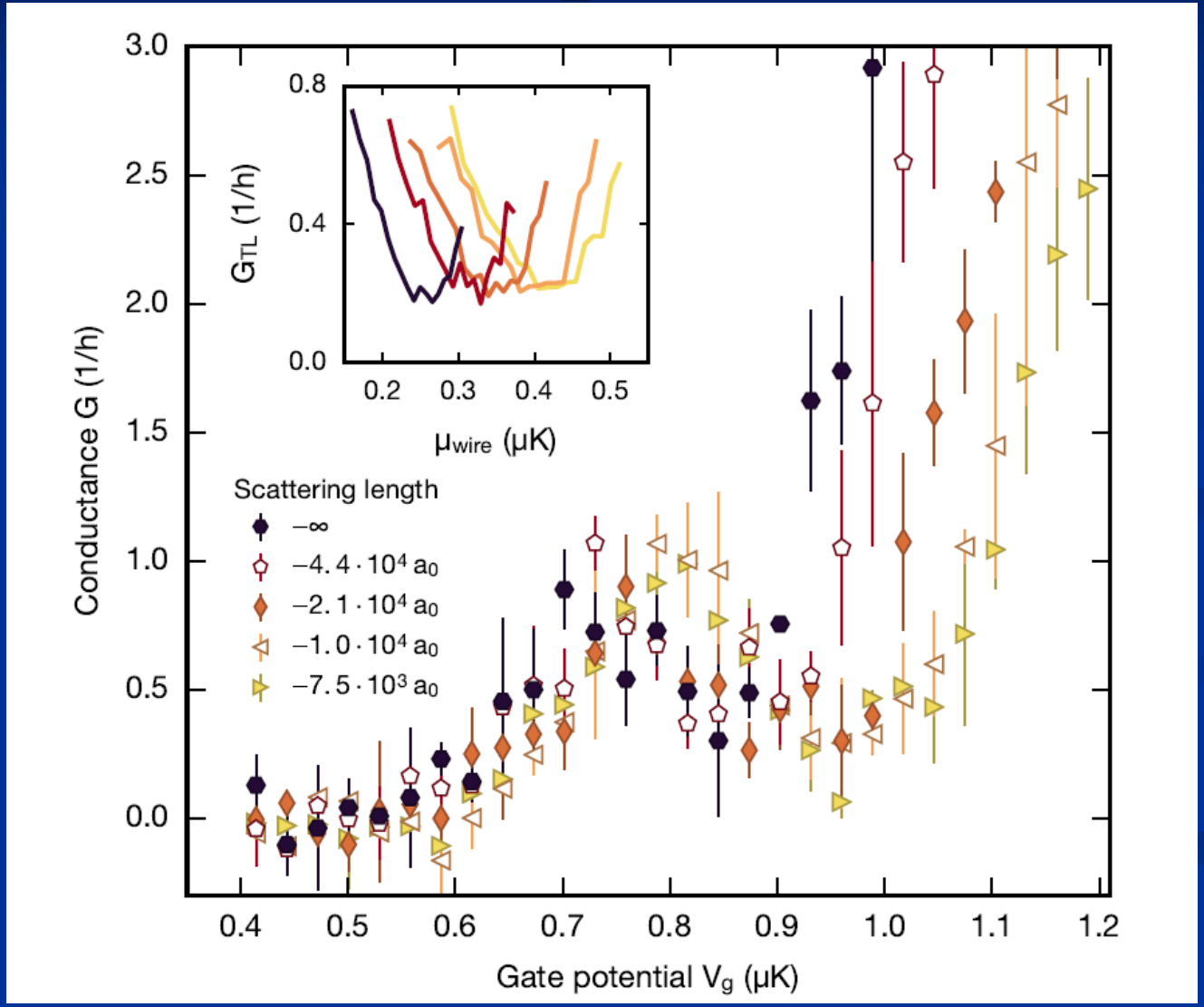


Gap in spin sector; Charge Massless (hard core boson/spinless fermions)

Many-body insulator ``pinned'' L.E. liquid



Experimental evidence for L.E. liquid

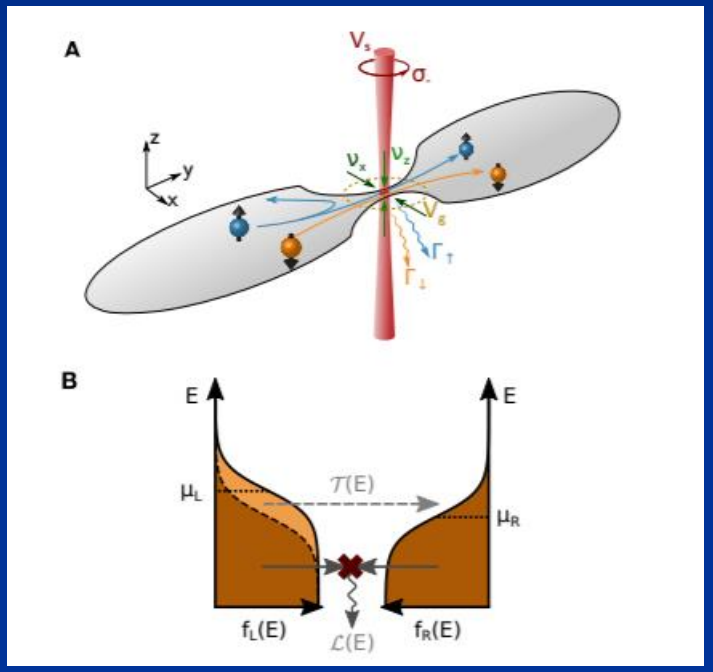


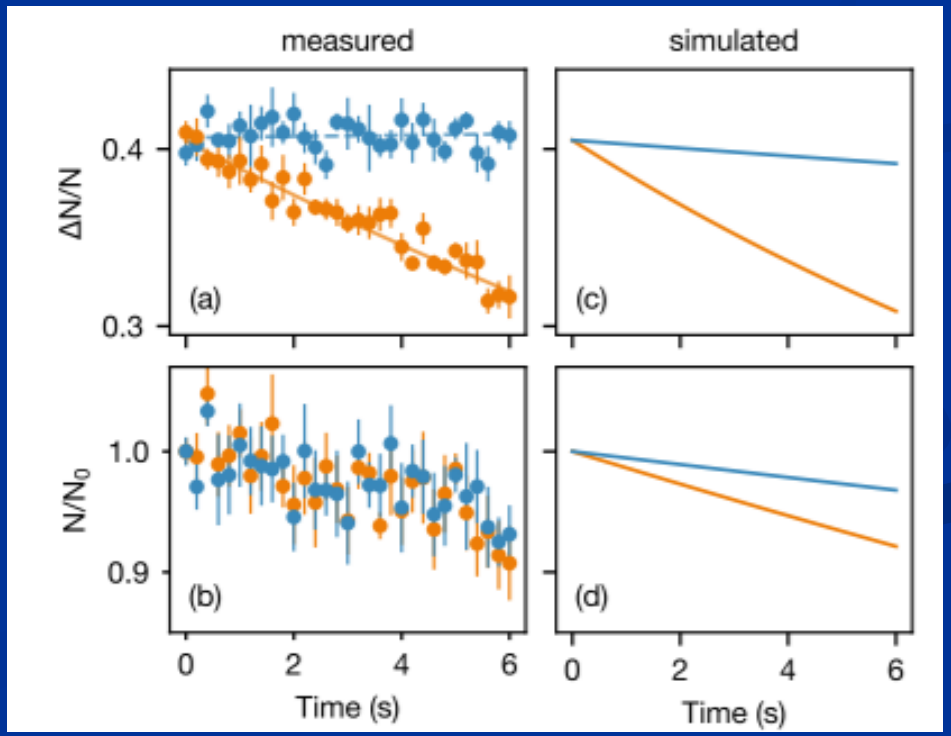
Lossy systems

Dissipative Atomic point contact

L. Corman et al. PRA 100 053605 (2019)







Transport with losses

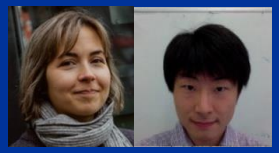


Normal réservoirs:



A.M. Visuri, TG, C. Kollath,
PRL 129, 056802 (2022)
PRR 5, 013195 (2023)

Superconducting réservoirs:



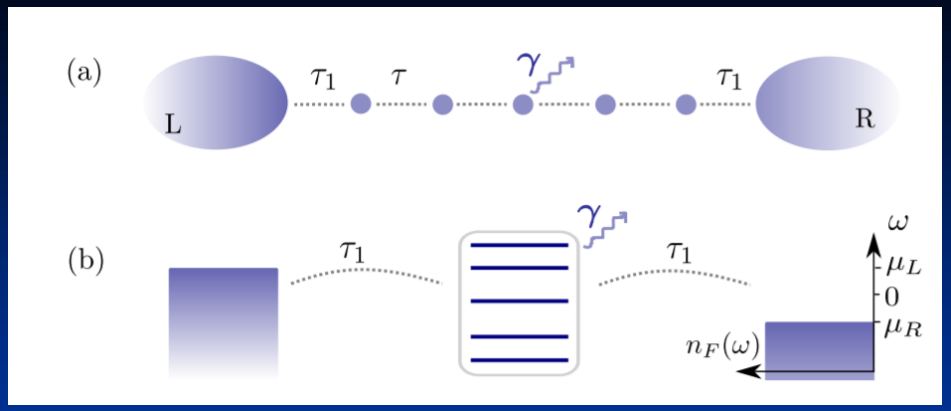
A.-M. Visuri, S. Uchino

Coll



T. Esslinger

M.Z Huang et al.
PRL 130, 200404 (2023)



$$\int \frac{d\omega}{2\pi} \bar{\mathbf{\Psi}}(\omega) \mathcal{G}^{-1}(\omega) \mathbf{\Psi}(\omega)$$

$$\mathbf{\Psi} = \left(\psi_L^1 \;\; \psi_L^2 \;\; d_{-l}^1 \;\; d_{-l}^2 \;\; \dots \;\; d_l^1 \;\; d_l^2 \;\; \psi_R^1 \;\; \psi_R^2\right)^T$$

$$H_{\text{chain}} = \epsilon \sum_{m=-l}^{l} d_m^\dagger d_m - \tau \sum_{m=-l}^{l-1} \left(d_{m+1}^\dagger d_m + \text{H.c.} \right),$$

$$\mathcal{G}^{-1} = \begin{pmatrix} L & T_1 & 0 & & \dots & & 0 \\ T_1 & D_{-l} & T & 0 & & & \\ 0 & T & \ddots & T & & & \\ & 0 & T & D_0 & T & & \\ \vdots & & & T & \ddots & T & \\ & & & & T & D_l & T_1 \\ 0 & & & & & T_1 & R \end{pmatrix}.$$

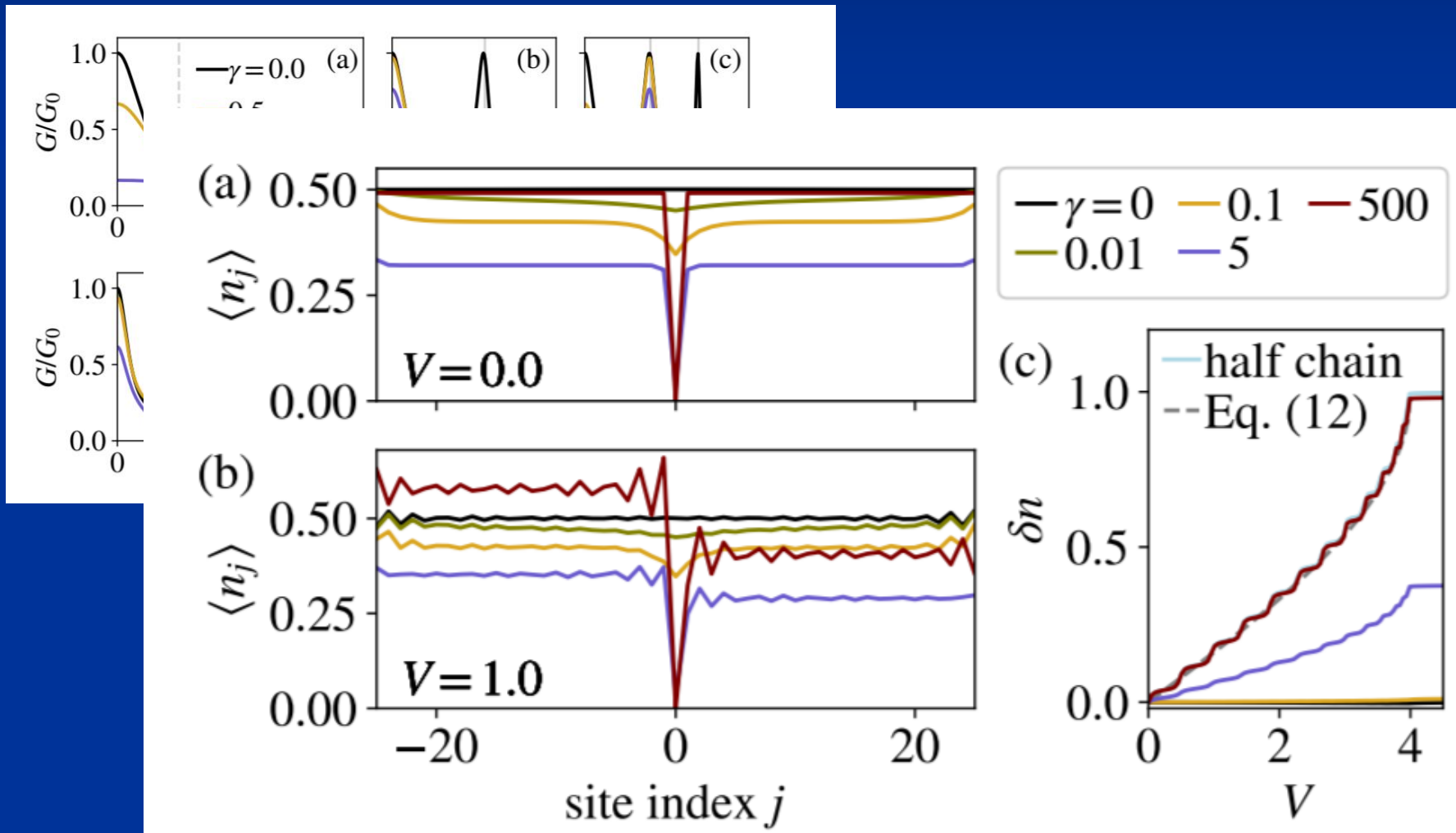
$$R/L = \begin{pmatrix} 0 & [\mathcal{G}^A_{L/R}]^{-1} \\ [\mathcal{G}^R_{L/R}]^{-1} & [\mathcal{G}^K_{L/R}]^{-1} \end{pmatrix},$$

$$\mathcal{G}^{R,A}_{L/R}(\mathbf{r}=0,\omega) = \frac{1}{\mathcal{V}} \sum_{\mathbf{k}=-\Lambda/v_F}^{\Lambda/v_F} \frac{1}{\omega - \epsilon_{\mathbf{k}} \pm i\eta}.$$

$$D_{j\neq 0} = \begin{pmatrix} 0 & \omega - \epsilon - i\eta \\ \omega - \epsilon + i\eta & 2i\eta \tanh\left(\frac{\omega-\epsilon}{2T}\right) \end{pmatrix}.$$

$$D_0 = \begin{pmatrix} 0 & \omega - \epsilon - i\gamma/2 \\ \omega - \epsilon + i\gamma/2 & i\gamma \end{pmatrix}.$$

Normal reservoirs: quantum Zeno effect



Two particle losses

Modeling particle loss in open systems using Keldysh path integral and second order cumulant expansion

Chen-How Huang,[1] Thierry Giamarchi,[2] and Miguel A. Cazalilla[1,3]

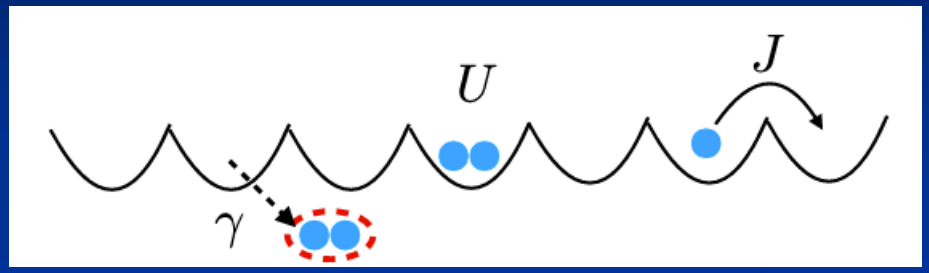
PHYSICAL REVIEW RESEARCH **5**, 043192 (2023)



$$H_c = -J \sum_i [c_j^\dagger c_{j+1} + \text{H.c.}].$$

$$H_d = U \sum_j d_j^\dagger d_j.$$

$$H_{cd} = -J \sum_j [(c_j^\dagger c_{j+1}^\dagger + c_{j-1}^\dagger c_j^\dagger) d_j + \text{H.c.}].$$

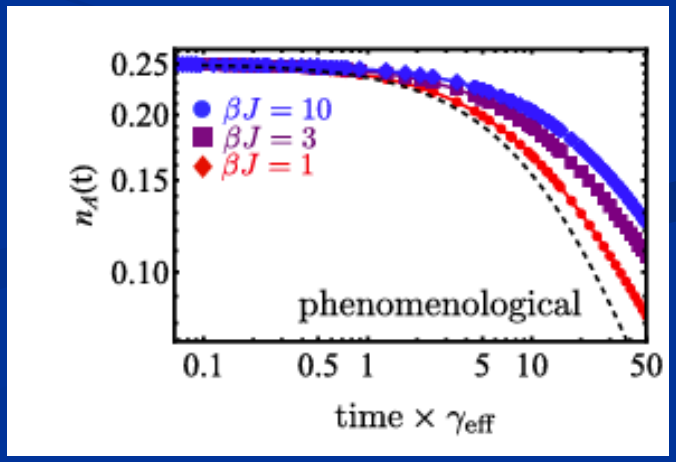


$$\frac{dn_A(t)}{dt} = -\gamma_{\text{eff}} \int \frac{dpdk}{(2\pi)^2} C_{kp}^2 \, n_p(t) n_k(t).$$

$$\frac{dn_A(t)}{dt} = -\gamma_T \, n_A^2(t),$$

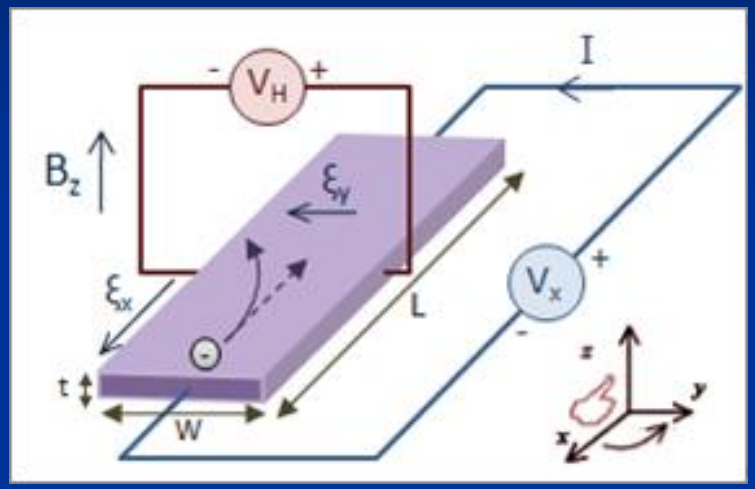
$$C_{kp} = \sin\left(\frac{p-k}{2}\right) \cos\left(\frac{p+k}{2}\right)$$

$$\gamma_{\text{eff}} = \frac{16J^2\gamma}{U^2 + \gamma^2/4}.$$



Hall effect

Hall effect 101



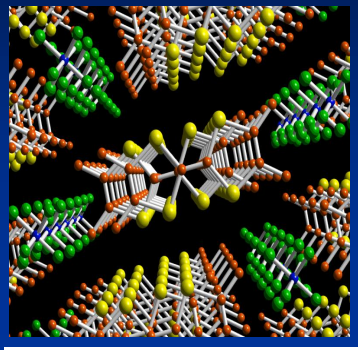
Non interacting ``simple"

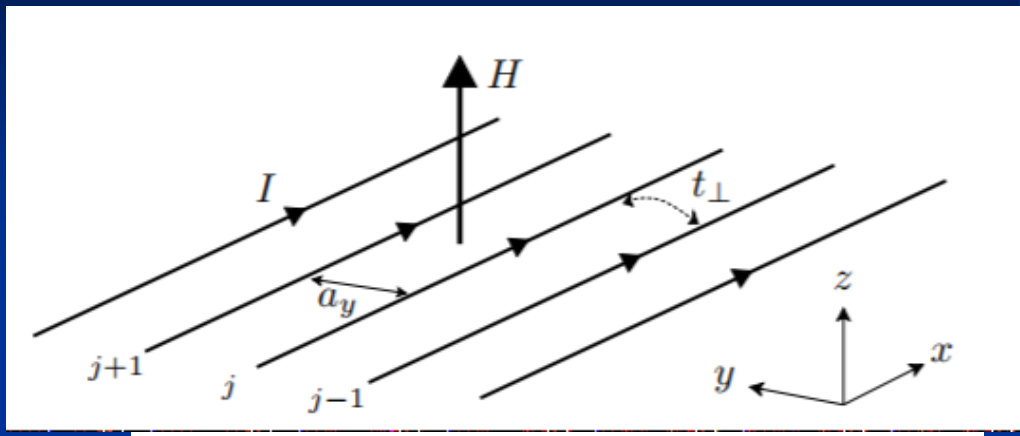
$$R_h = \frac{V_\perp}{I_\parallel B} \propto \frac{1}{n}$$

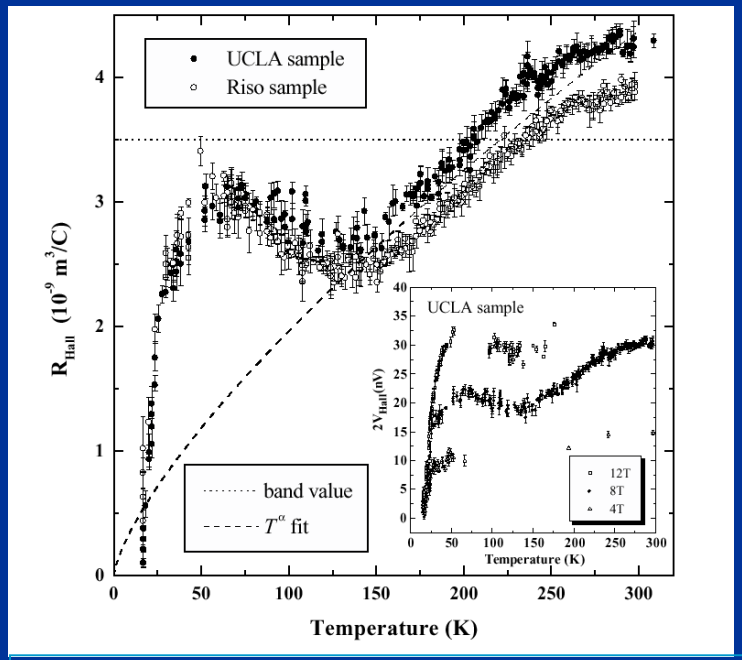
No interactions: curvature of fermi surface - topological formula (Thouless-Kohmoto)

With **interactions**: open question

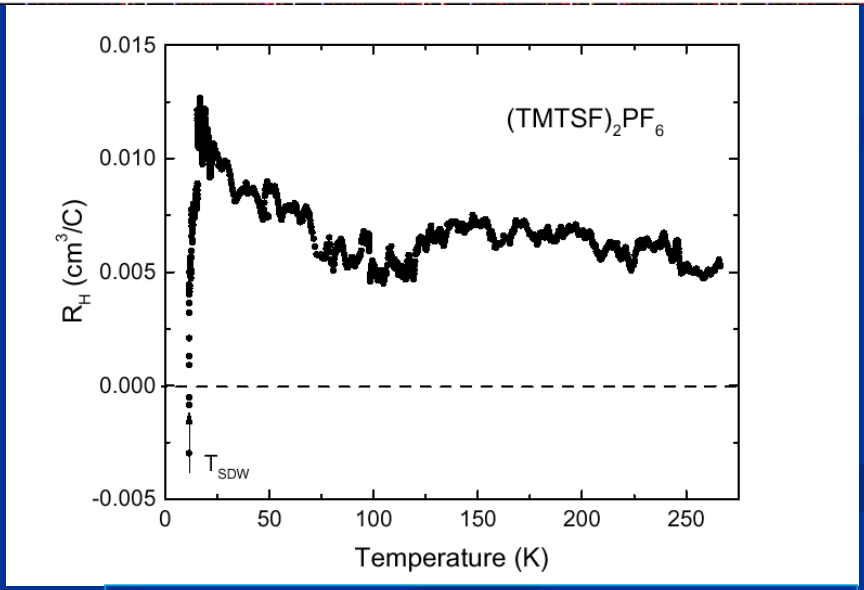
Experiments ?





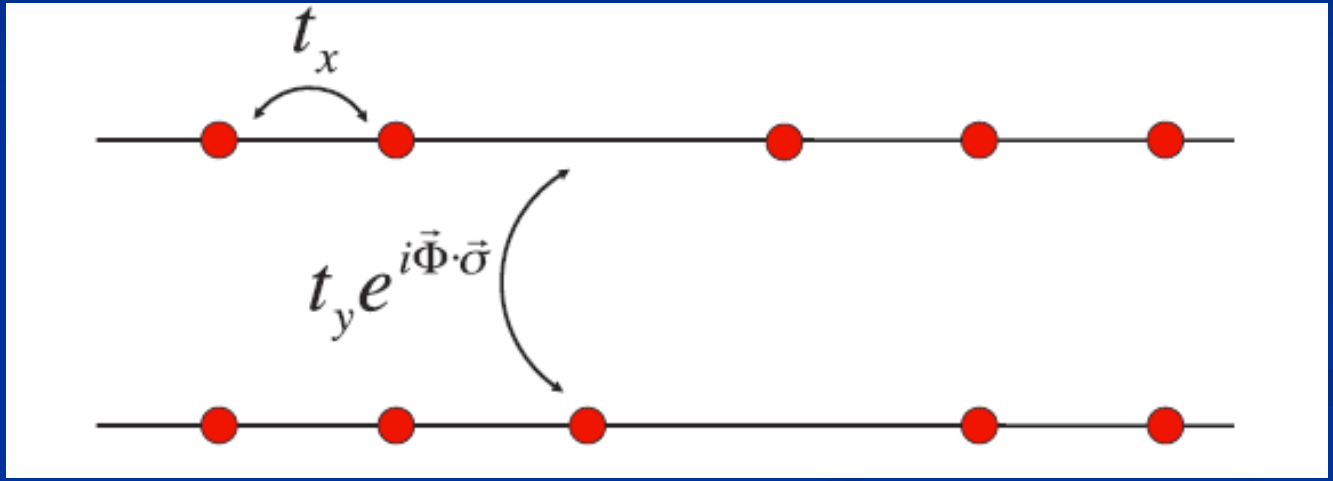


J. Moser et al., PRL 84 2674 (00)

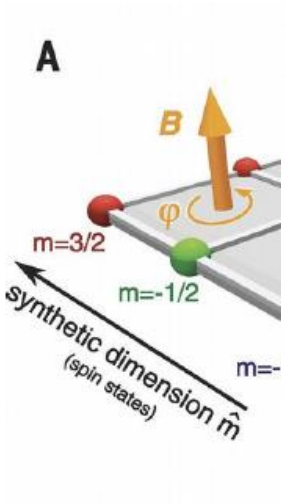


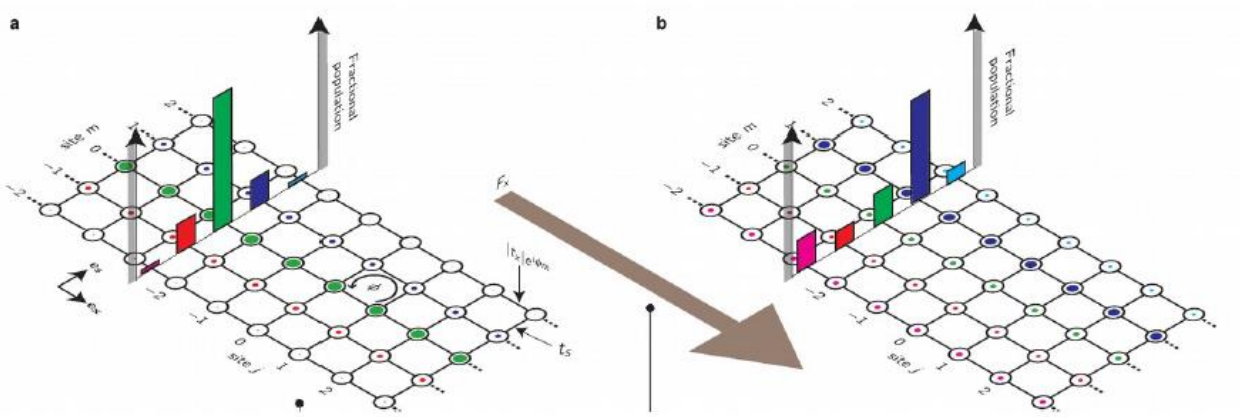
G. Mihaly et al., PRL 84 2670 (00)

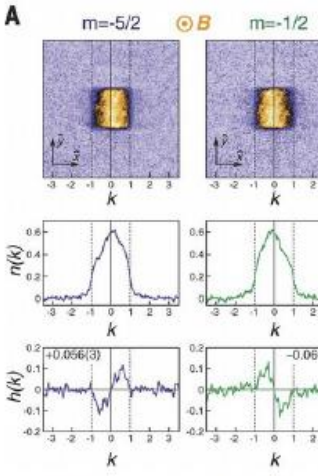
Other geometries: ladders

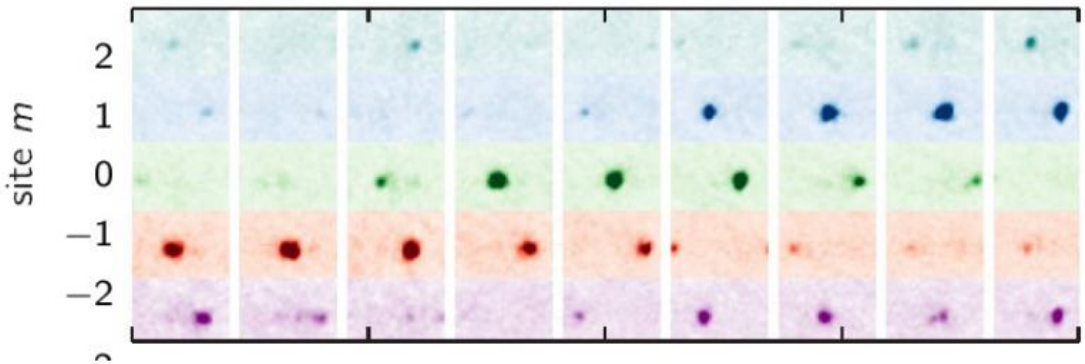


Synthetic GF / dimensions









Dina Genkina et al 2019 New J. Phys. 21 053021

Fallani's group (Florence) Scien

Spielman's group (Maryland) arXiv:1804.06345

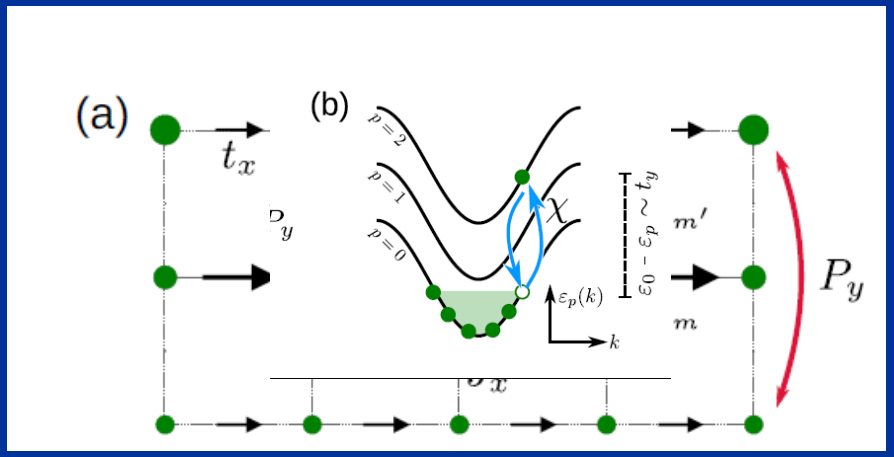
Hall effect with interactions



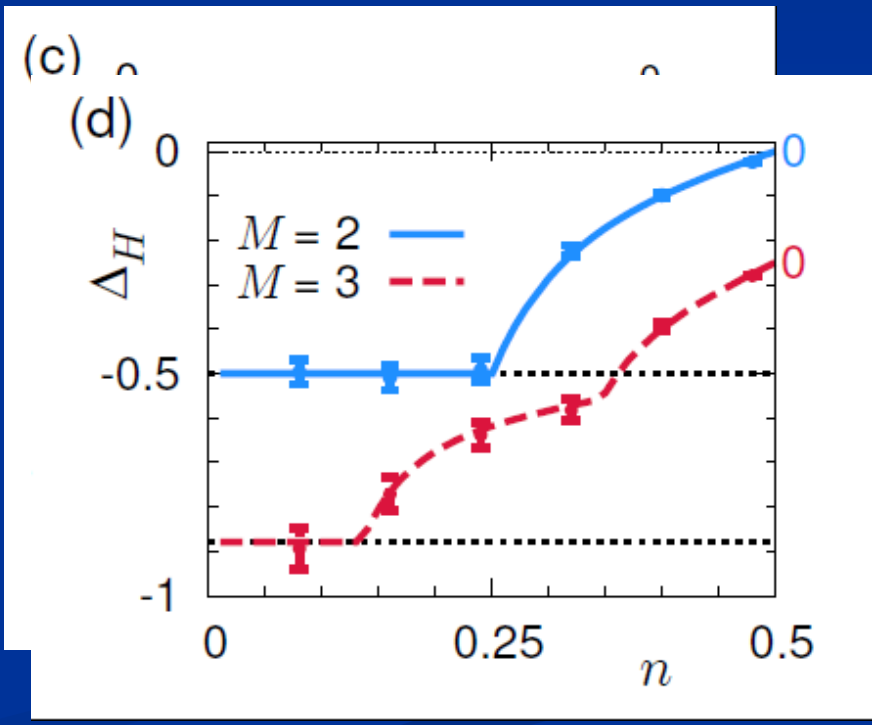
Hall effect on ladders



S. Greshner, M. Filippone, TG, PRL 122, 083402 (2019)



$$\Delta_H = q\mathcal{I}_0(M, t_\perp) \quad \text{and} \quad R_H = -1/n.$$



Universal value of Hall resistance

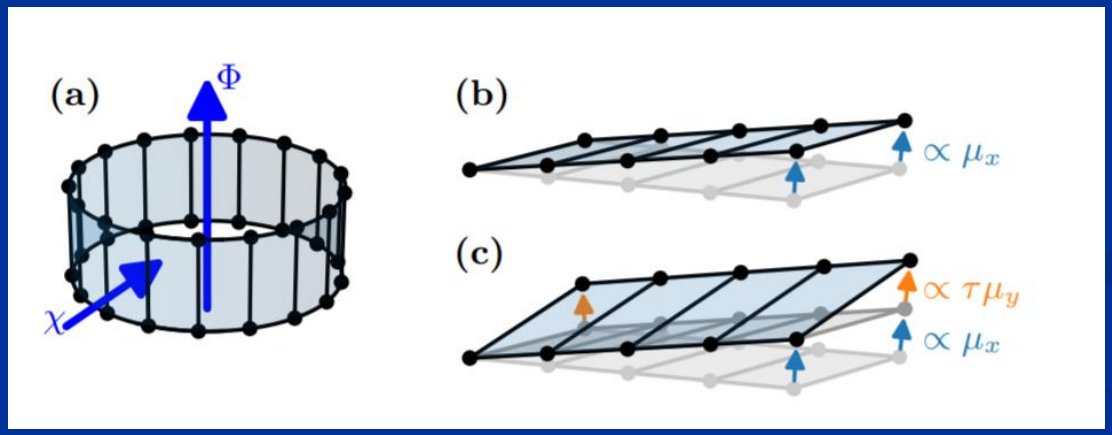


Ring vs boxes





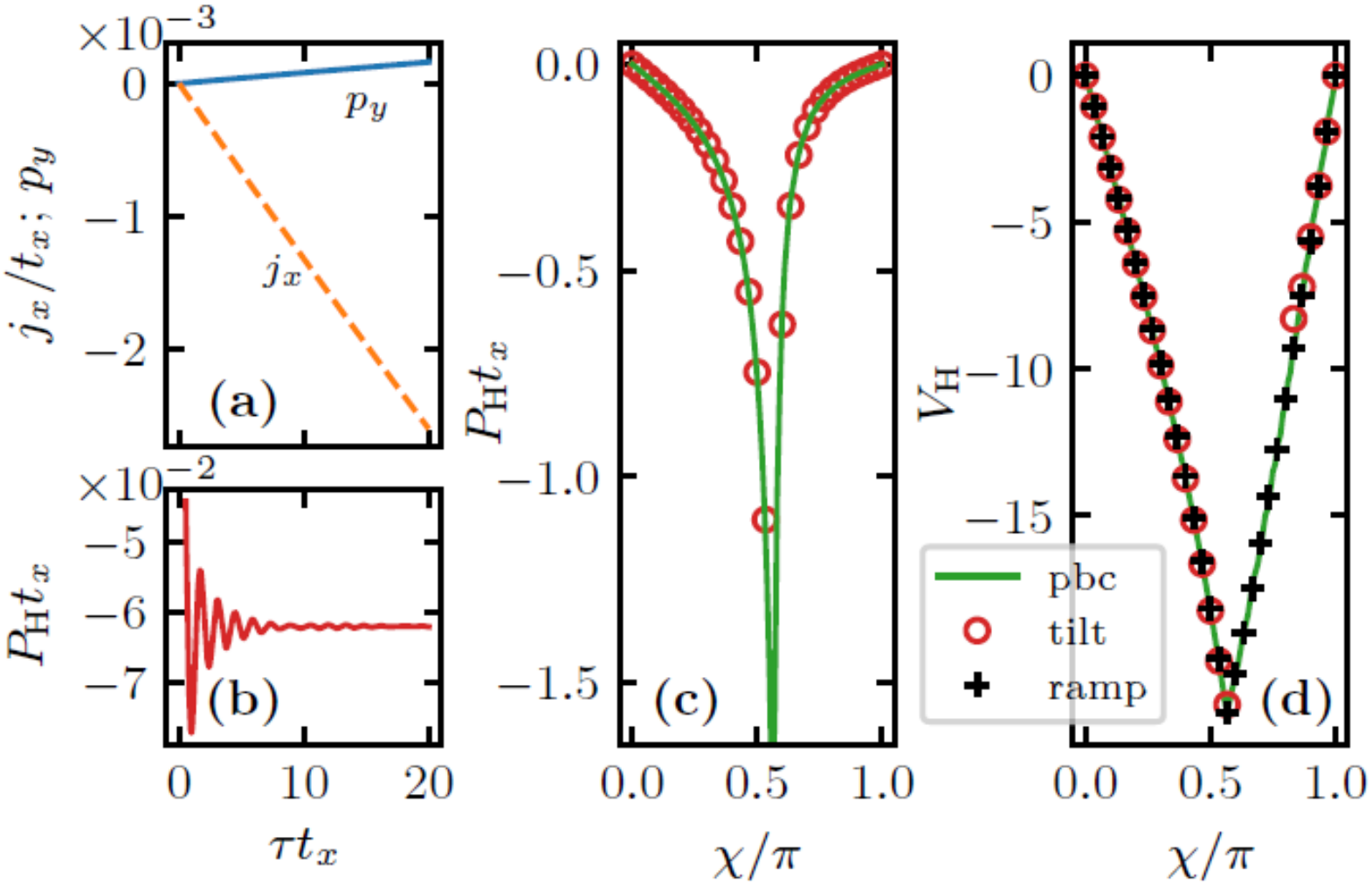
M. Buser, S. Greshner, U. Schollwöck,
TG, PRL 126, 030501 (21)



Identical result on a ring and in a box

Practical procedure to compute Vh for a box

Hall imbalance / Hall voltage



Quantum simulations

L. Fallani's group



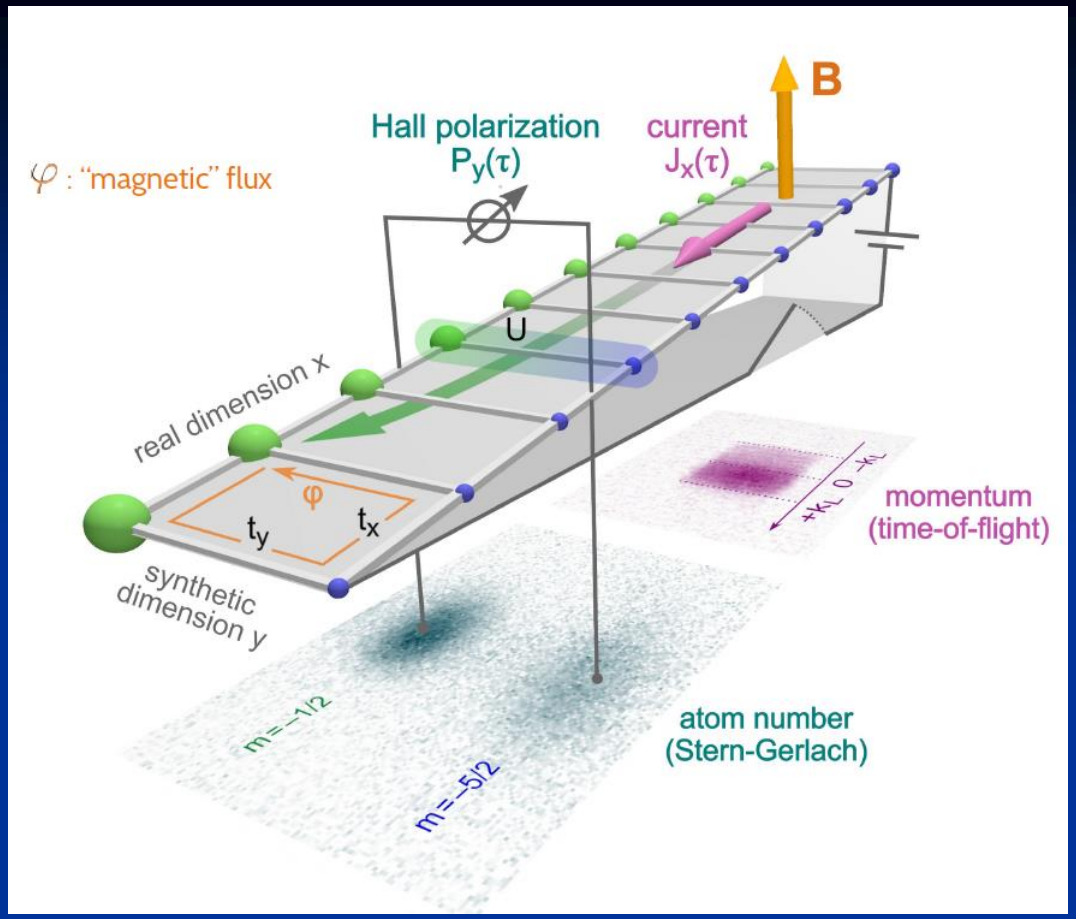


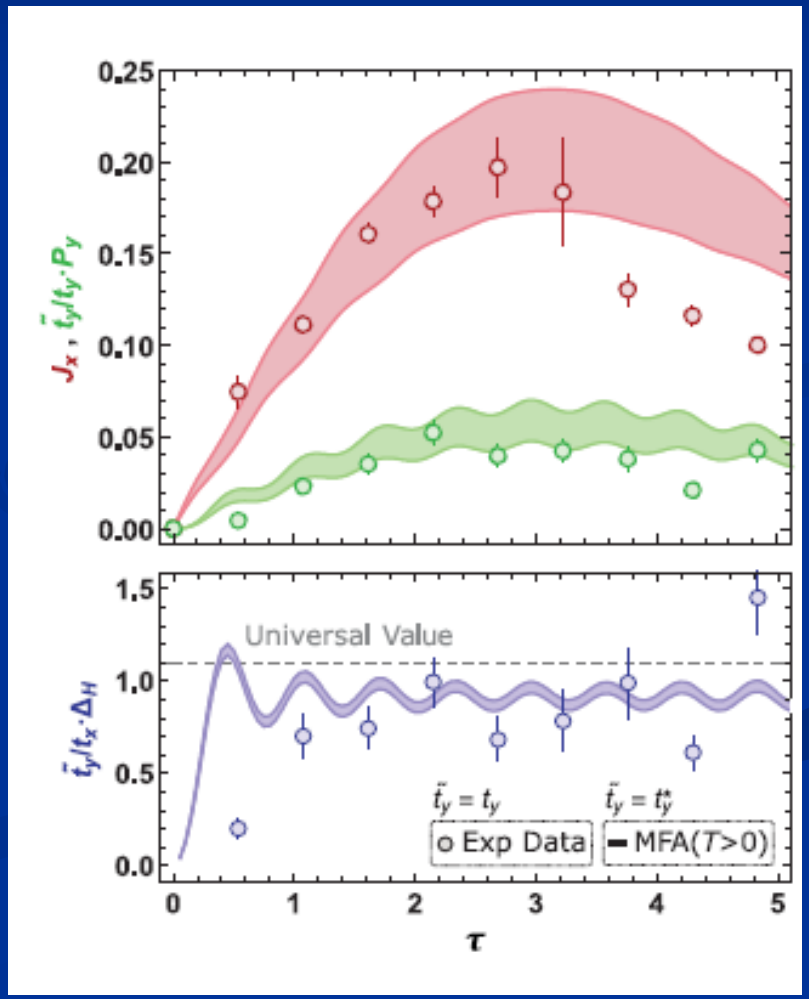


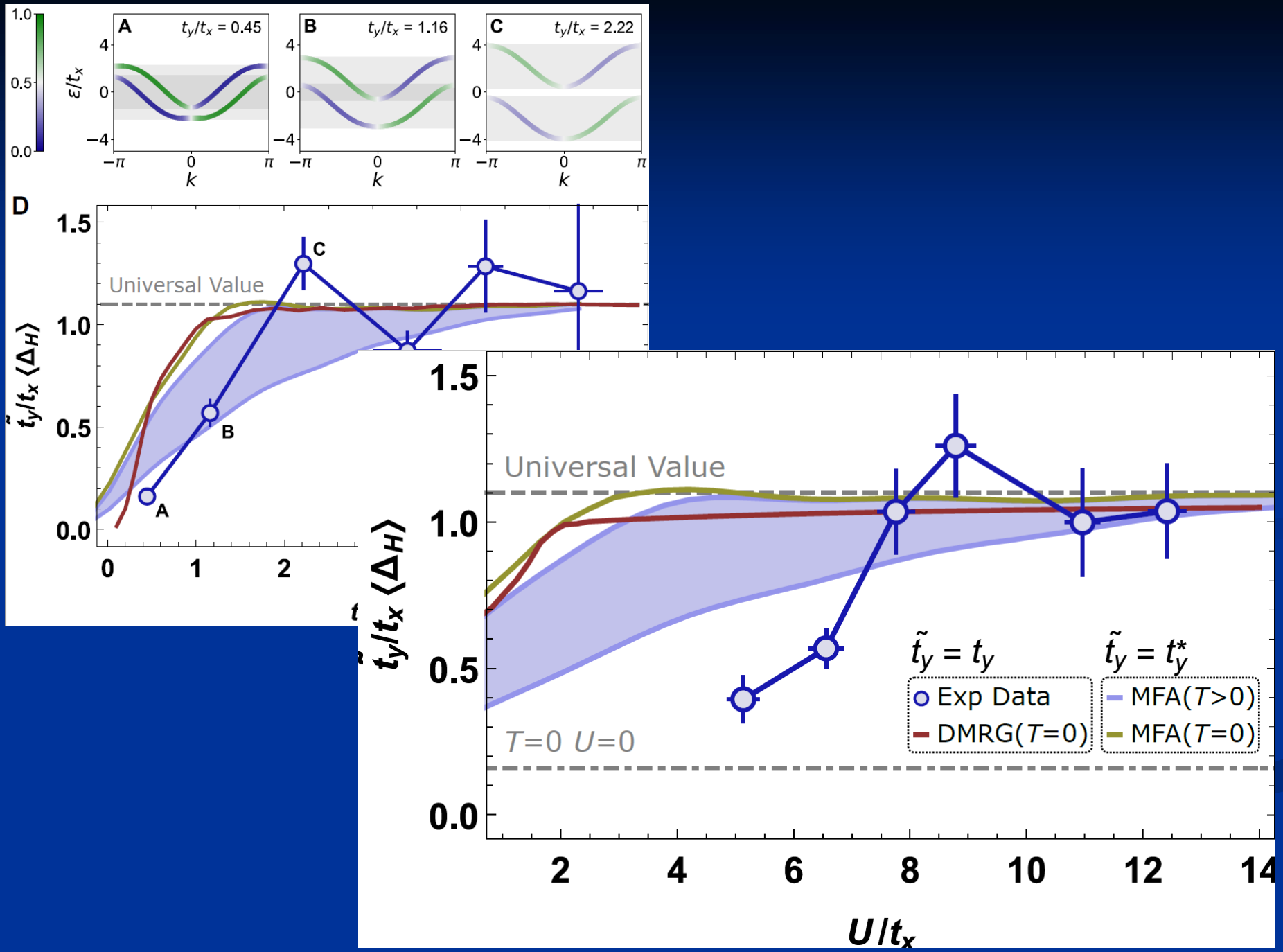


T.-W. Zhou, G. Cappellini, D. Tusi, L. Franchi, J. Parravicini, C. Repellin, S. Greschner, M. Inguscio, TG, M. Filippone, J. Catani, L. Fallani, Science 381 427 (2023).



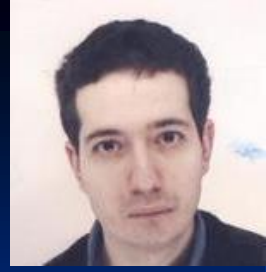

B
Hall polarization
$P_y(\tau)$
current
$J_x(\tau)$
φ : "magnetic" flux
U
real dimension x
φ
t_y
t_x
synthetic
dimension y
momentum
(time-of-flight)
$+k_L$ 0 $-k_L$
atom number
(Stern-Gerlach)
m=−1/2
m=−5/2


J_x , $\tilde{t}_y/t_y \cdot P_y$
0.25
0.20
0.15
0.10
0.05
0.00
$\tilde{t}_y/t_x \cdot \Delta_H$
1.5
1.0
0.5
0.0
Universal Value
$\tilde{t}_y = t_y$
$\tilde{t}_y = t_y^*$
o Exp Data
MFA(T>0)
0
1
2
3
4
5
τ


A
t_y/t_x = 0.45
B
t_y/t_x = 1.16
C
t_y/t_x = 2.22
ε/t_x
k
D
Universal Value
$\tilde{t}_y/t_x\ \langle\Delta_H\rangle$
$\tilde{t}_y = t_y$
$\tilde{t}_y = t_y^*$
Exp Data
DMRG(T=0)
MFA(T>0)
MFA(T=0)
T=0 U=0
U/t_x

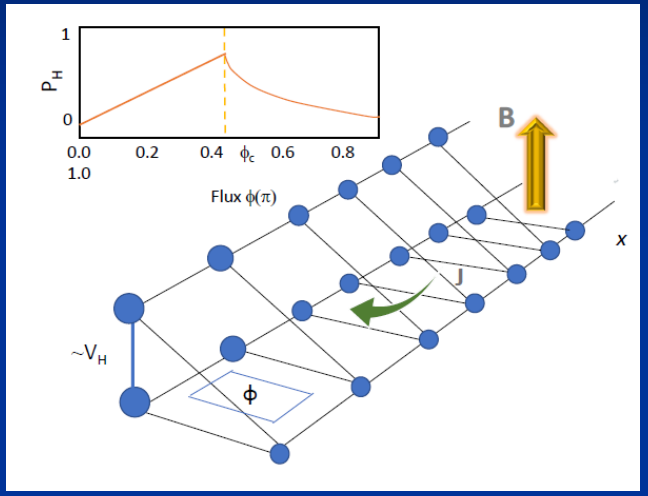


Analytical derivation



R. Citro, TG, E. Orignac, arxiv/2404.16973

Bosonization: small ty



$$\psi(x) = \sqrt{\rho(x)}e^{i\theta(x)}$$

$$\pi\Pi(x) = \nabla\theta(x)$$

$$\rho(x) = -\frac{1}{\pi}\nabla\phi(x)$$

$$\alpha = -\frac{\partial}{\partial\rho_0}\left(\frac{u}{6\pi^2 K}\right),$$

$$\gamma = -\frac{\partial}{\partial\rho_0}\left(\frac{uK}{2\pi^2}\right).$$

$$H = \sum_{p=1,2} H_p + H_{tunn.}$$

$$H_p = \int \frac{dx}{2\pi}\left[uK(\pi\Pi_p)^2 + \frac{u}{K}(\partial_x\phi_p)^2\right] + \int dx\left[\alpha(\partial_x\phi_p)^3 + \gamma(\pi\Pi_p)^2\partial_x\phi_p\right],$$

$$H_{tunn.} = -\frac{t_\perp}{\pi\alpha}\int dx\cos(\theta_1 - \theta_2),$$

Hall voltage

$$V_H = -\frac{\pi^2 \gamma \langle j_\rho \rangle}{e u_\rho K_\rho} Ba = -\langle j_\rho \rangle \frac{Ba}{2e} \frac{1}{u_\rho K_\rho} \frac{\partial}{\partial \rho_0} (u_\rho K_\rho),$$

- Hall resistance

$$R_H = \frac{a}{e u_s K_s} \frac{\partial}{\partial \rho_0} (u_s K_s).$$

Charge stiffness !

- Galilean system

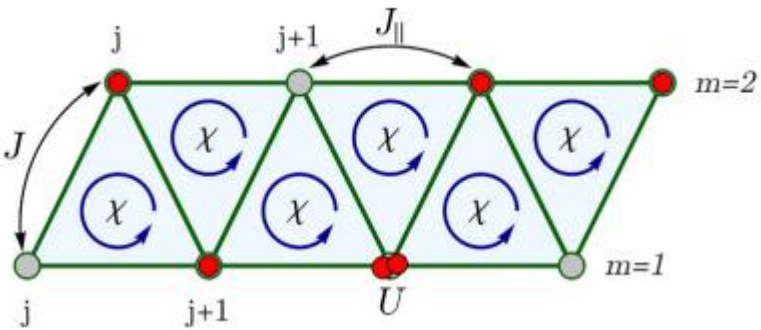
$$R_H = \frac{aB}{e\rho_0},$$

- Near Mott

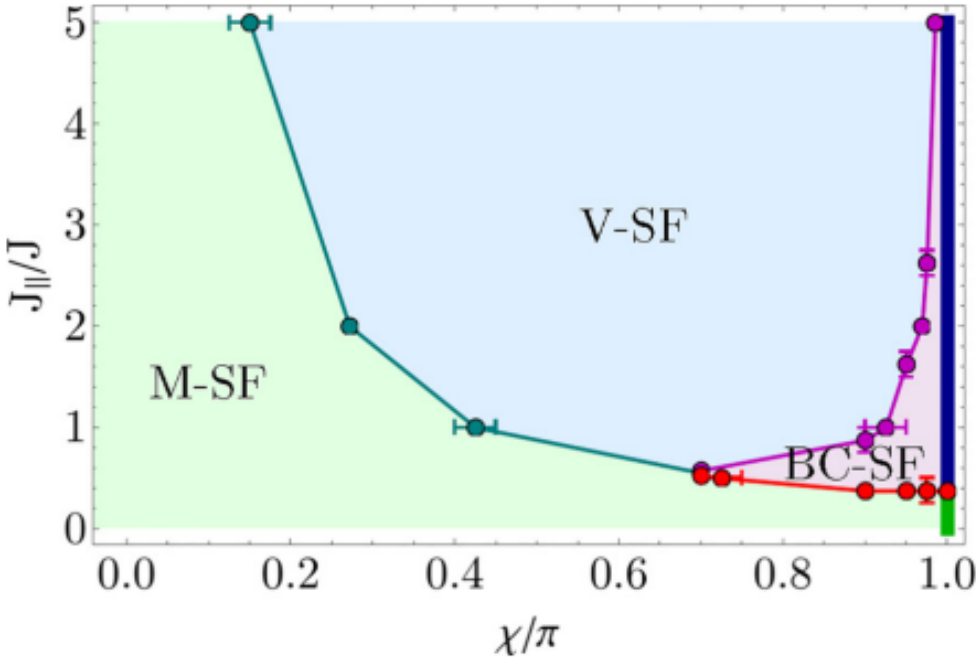
$$R_H \sim \frac{a}{e} \frac{\mathrm{sign}(\rho_0 - \rho_{0,c})}{|\rho_0 - \rho_{0,c}|},$$

Triangular ladders

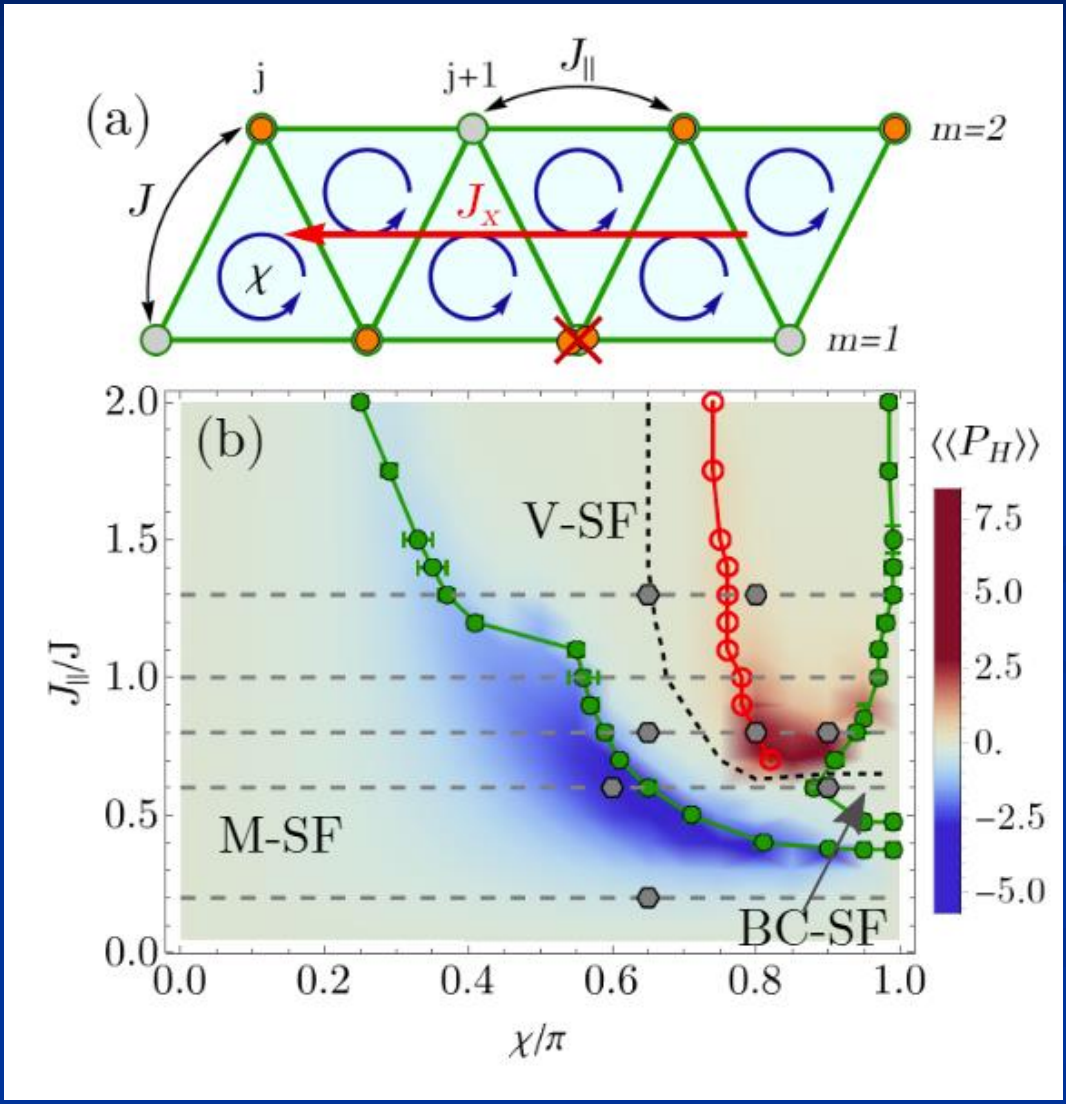




C. Halati and TG
PRR 5, 013126 (2023)



Hall effect



Harcore bosons

Conclusions/Perspectives



- Out of equilibrium: very challeni problems; several incarnations (impurities, quenches, transport, etc.)
- Novel situations and effects (Hall effect, time dependent noise, losses, etc.)
- Help needed from quantum simulations in cold atomic gases/new theories !