Synthetic Dimensions I: dynamics in synthetic lattices

Bryce Gadway

Penn State University (& University of Illinois)

2024 Croucher Summer Course on Ultracold Atomic Physics

May 21, 2024

please ask questions!

Today: Synthetic Dimensions I

- systems, states, and capabilities
- some unifying themes & considerations

Tomorrow: Synthetic Dimensions II

- interactions in synthetic dimensions
- recent results on correlated dynamics in synthetic dynamics

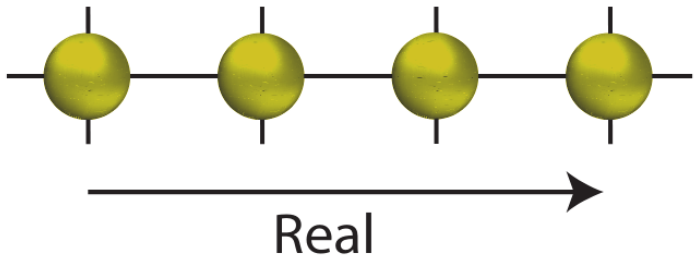
Synthetic Dimensions [sinˈTHedik dəˈmen(t)SH(ə)ns]

noun

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1. The approach to quantum simulation based on using internal states to encode an extra dimension

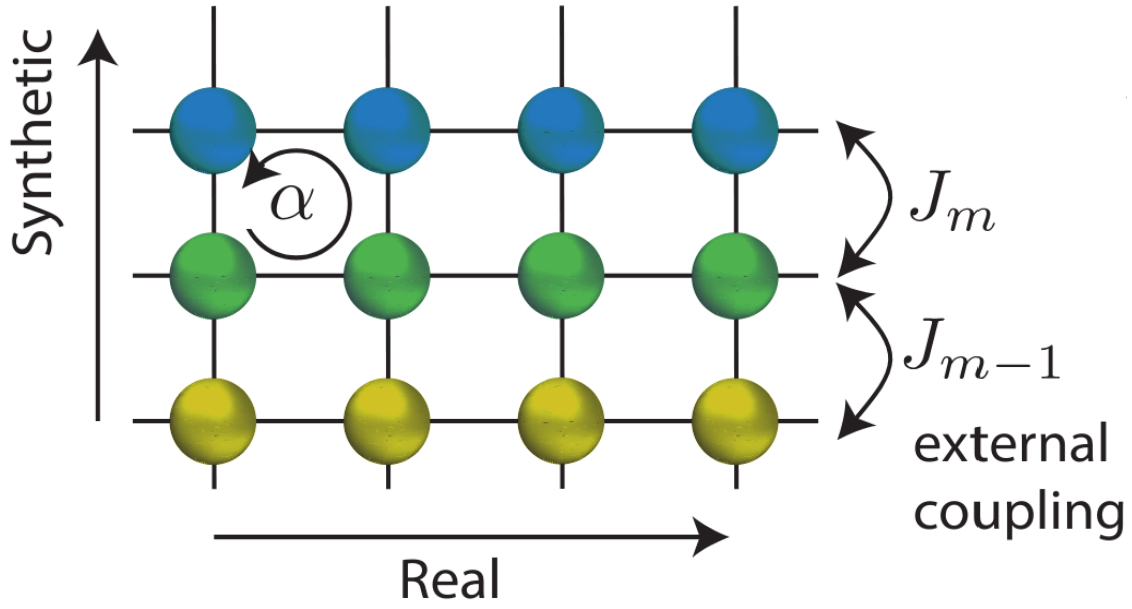


Ozawa & Price, Nat. Rev. Phys. (2019)

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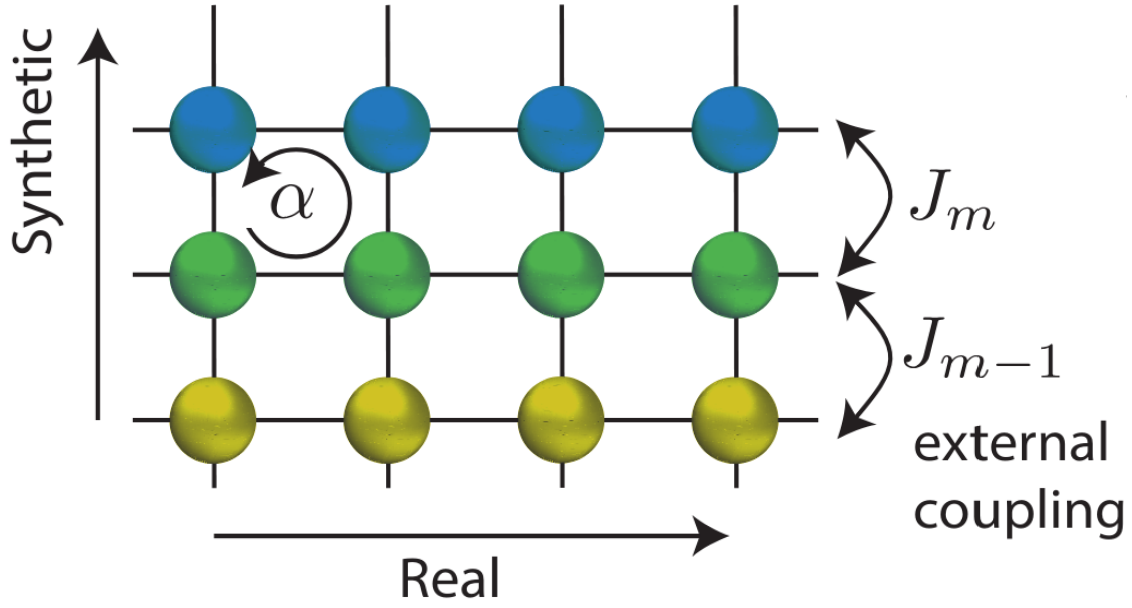


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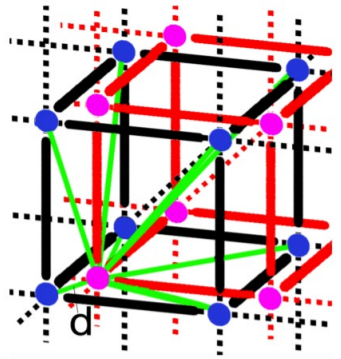
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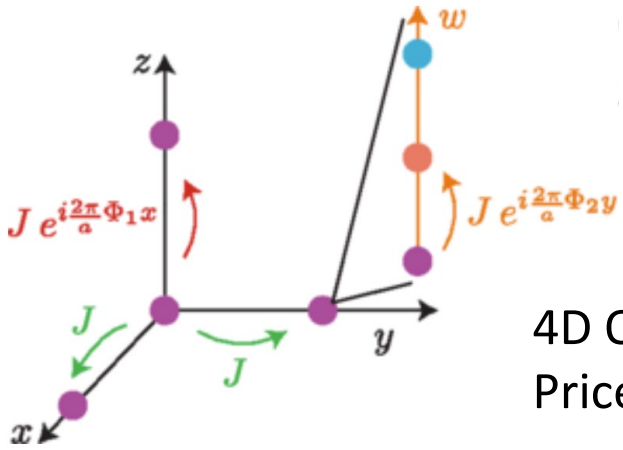
1. The approach to quantum simulation based on using internal states to encode an extra dimension



Ozawa & Price, Nat. Rev. Phys. (2019)

4-dimensional Hubbard models
Boada, *et al*. PRL (2012)





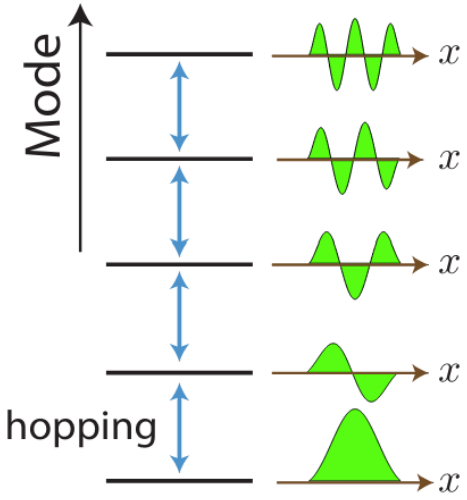
4D Quantum Hall effect
Price, *et al*. PRL (2015)

Synthetic Dimensions [sinˈTHedik dəˈmen(t)SH(ə)ns]

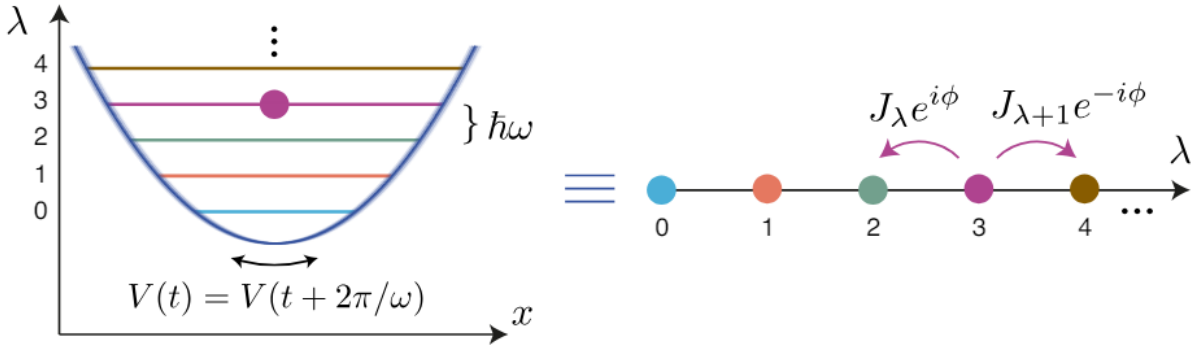
noun

1. The approach to quantum simulation based on using internal states to encode an extra dimension

2. The approach to studying discrete lattice transport problems using <u>non-standard degrees of freedom</u>



Ozawa & Price, Nat. Rev. Phys. (2019)
[cartoon of Szameit/Segev group result]



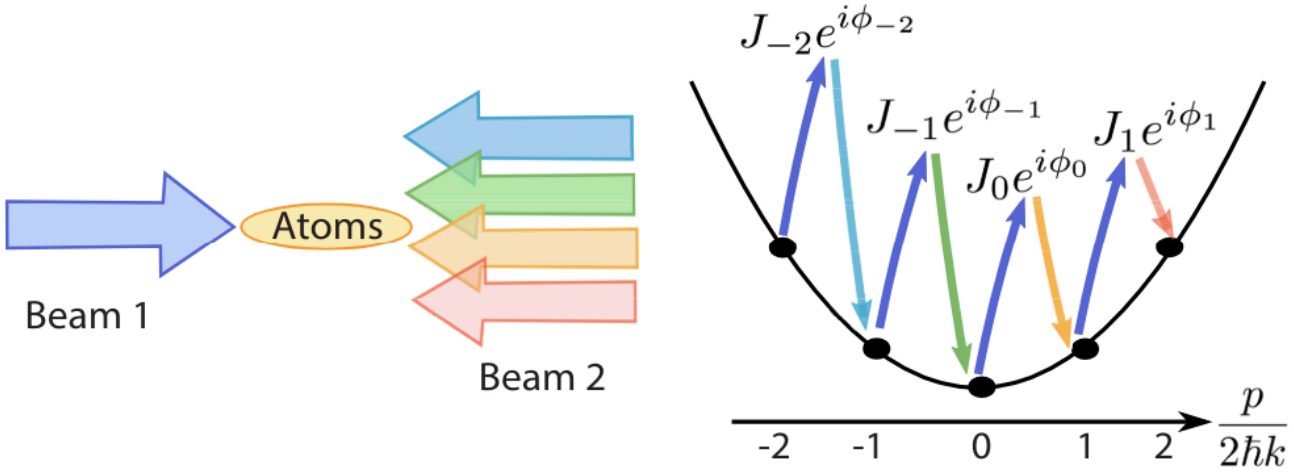
Price, Ozawa, and Goldman, PRA (2017)

Synthetic Dimensions [sinˈTHedik dəˈmen(t)SH(ə)ns]

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2. The approach to studying discrete lattice transport problems using non-standard degrees of freedom



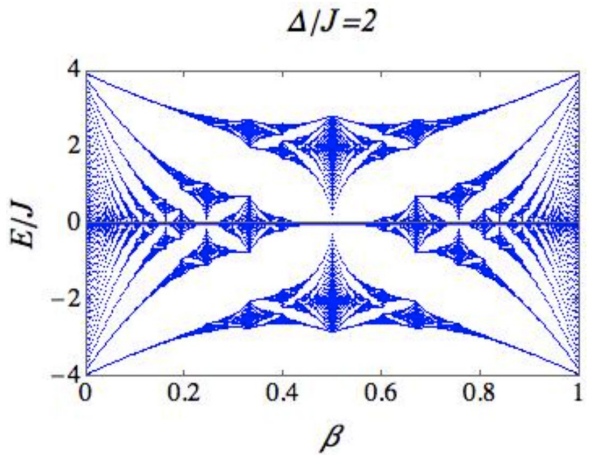
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Synthetic Dimensions [sinˈTHedik dəˈmen(t)SH(ə)ns]

noun

1. The approach to quantum simulation based on using internal states to encode an extra dimension
2. The approach to studying discrete lattice transport problems using non-standard degrees of freedom
3. The approach to embedding extra dimensions in a problem by <u>parametric</u> encoding (inverse of dimensional reduction)

quasicrystals ~ higher-dimensional lattice models



energy spectrum of Aubry-André-Harper model
vs. incommensurability parameter

Domínguez-Castro & Paredes, EJP (2019)

Synthetic Dimensions [sinˈTHedik dəˈmen(t)SH(ə)ns]

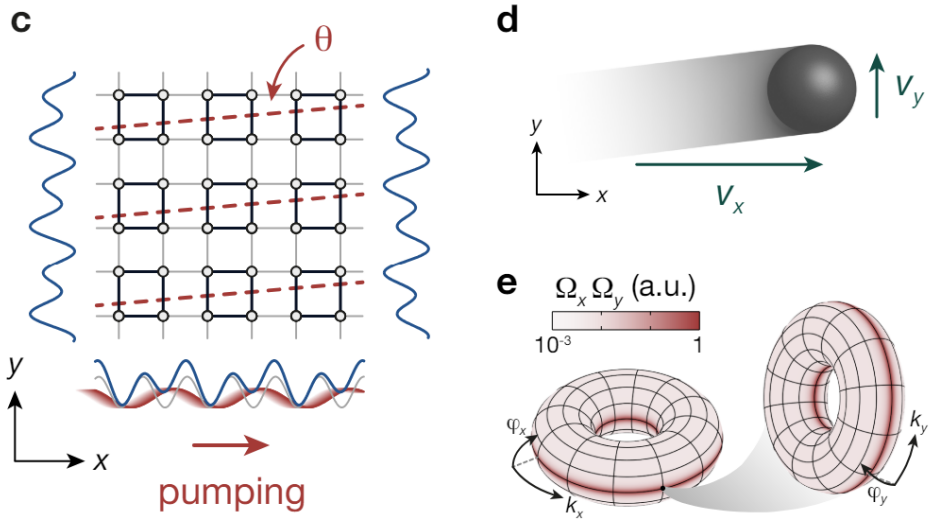
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quasicrystals ~ higher-dimensional lattice models

pumps ~ higher-dimensional lattices

Exploring 4D quantum Hall physics with a 2D topological charge pump



Lohse, *et al*. Nature (2018)

Synthetic Dimensions [sinˈTHedik dəˈmen(t)SH(ə)ns]

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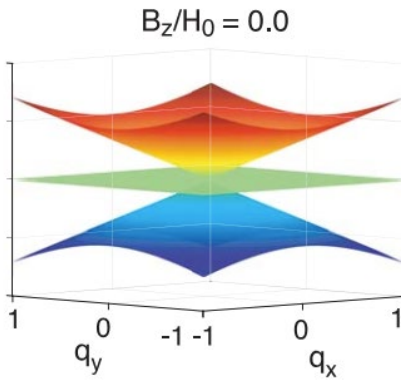
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quasicrystals ~ higher-dimensional lattice models

pumps ~ higher-dimensional lattices

parameterized realizations of higher-dimensional models in k-space

$$\hat{H}_{4D} = \begin{pmatrix} 0 & q_x - iq_y & 0 \\ q_x + iq_y & 0 & q_z + iq_w \\ 0 & q_z - iq_w & 0 \end{pmatrix}$$



REPORT

TOPOLOGICAL PHYSICS

A synthetic monopole source of Kalb-Ramond field in diamond

Mo Chen (陈墨)[1,2]†‡, Changhao Li[1,3]‡, Giandomenico Palumbo[4,5], Yan-Qing Zhu[4]§, Nathan Goldman[4], Paola Cappellaro[1,3,6]*

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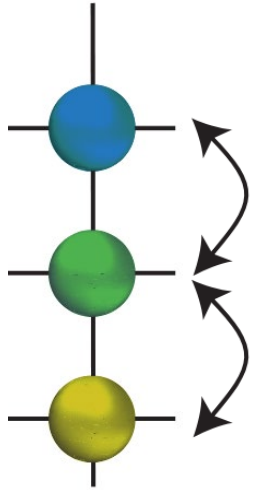
focus of my talks

1. The approach to quantum simulation based on using internal states to encode an extra dimension

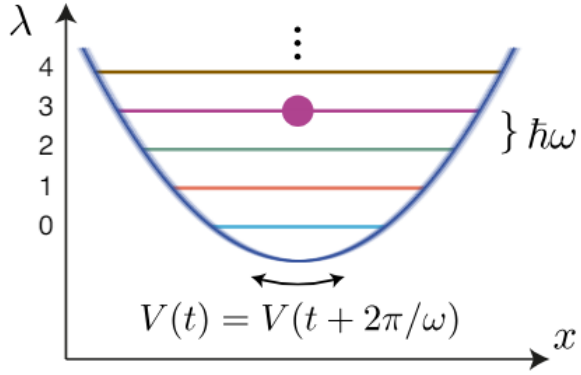
2. The approach to studying discrete lattice transport problems using non-standard degrees of freedom

3. The approach to embedding extra dimensions in a problem by parametric encoding (inverse of dimensional reduction)

4. Any related approaches/formulations that expand the capabilities of quantum simulation or lead to new insights



Ozawa & Price, Nat. Rev. Phys. (2019)

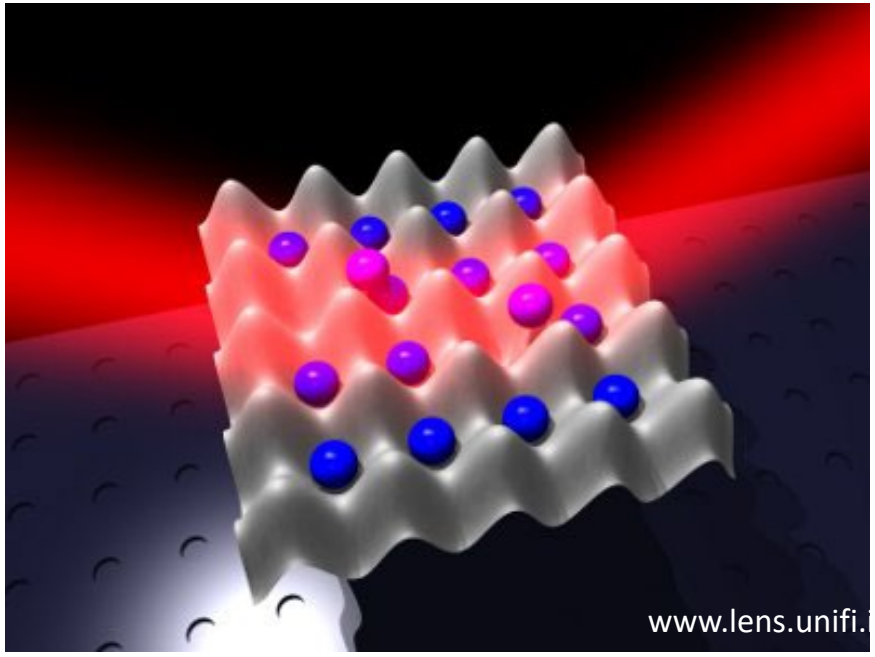


Price, Ozawa, and Goldman, PRA (2017)

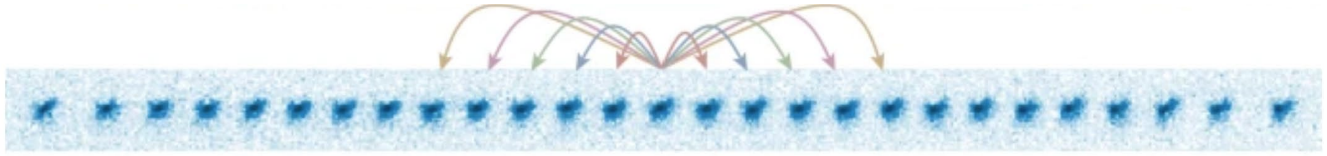
outline for Synthetic Dimensions I

- background and motivation
- basic building blocks: states & “tunneling” by resonant driving
- a synthetic zoo: commonalities and differences
- the suite of synthetic capabilities
- summary

background: analog simulators of discrete lattice systems



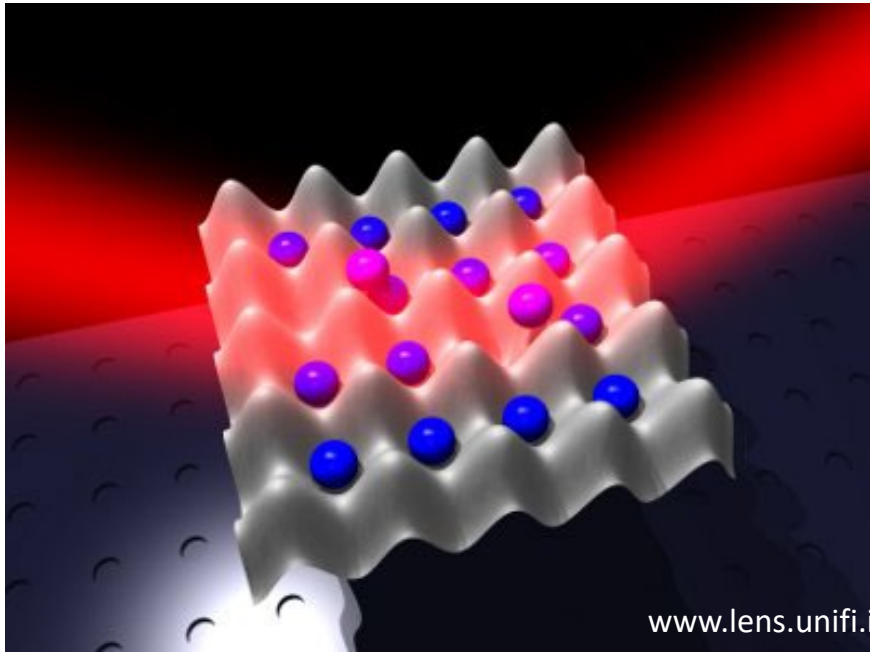
neutral atoms in optical lattices



Feng, *et al*. Nature (2023)

spin excitations on a lattice of ions
(or Rydbergs/molecules/qubits)

lattice tight-binding models



neutral atoms in optical lattices

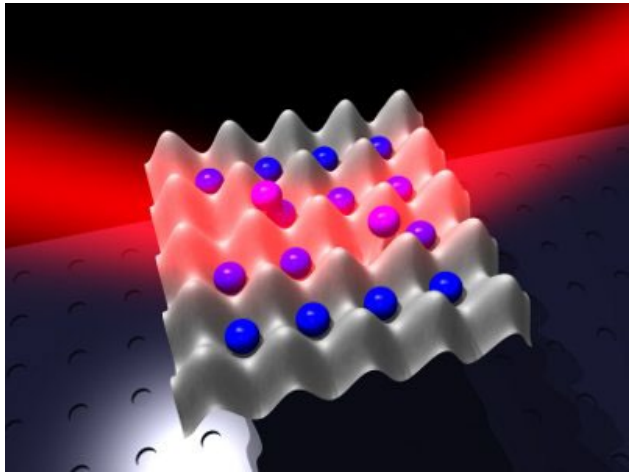
$$\hat{H} = \sum_i \epsilon_i \hat{c}_i^\dagger \hat{c}_i - t \sum_i \left(\hat{c}_{i+1}^\dagger \hat{c}_i + \hat{c}_i^\dagger \hat{c}_{i+1} \right)$$

site orbital energies (diagonal)

tunneling terms (off-diagonal)

$$H = \begin{pmatrix} \epsilon & -t & 0 & 0 & & \cdots & 0 \\ -t & \epsilon & -t & 0 & & \cdots & 0 \\ 0 & -t & \epsilon & -t & & \cdots & 0 \\ 0 & 0 & -t & \epsilon & -t & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdots & & & & \epsilon & -t \\ 0 & \cdots & & & & -t & \epsilon \end{pmatrix}$$

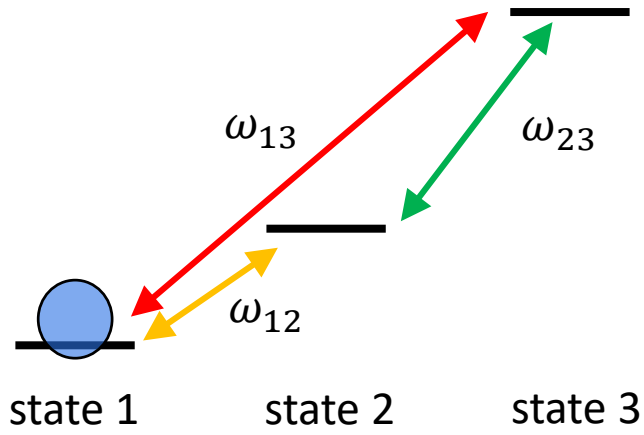
lattice tight-binding models – some real-space limitations

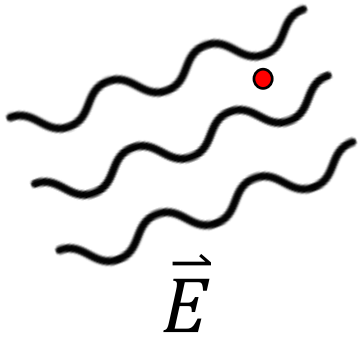


$$\hat{H} = \sum_i \epsilon_i \hat{c}_i^\dagger \hat{c}_i - t \sum_i \left(\hat{c}_{i+1}^\dagger \hat{c}_i + \hat{c}_i^\dagger \hat{c}_{i+1} \right)$$

- atoms / excitations are neutral – hard to implement **complex tunneling & artificial magnetic fields**
- "hopping" graphs have some physical constraints
- residual motional entropy (finite temperature)

synthetic (internal state) tight-binding models





particles pick up the phase of driving fields when undergoing transitions

- atoms / excitations are neutral – hard to implement **complex tunneling & artificial magnetic fields**

→ **can *naturally* implement complex hopping and artificial gauge fields**

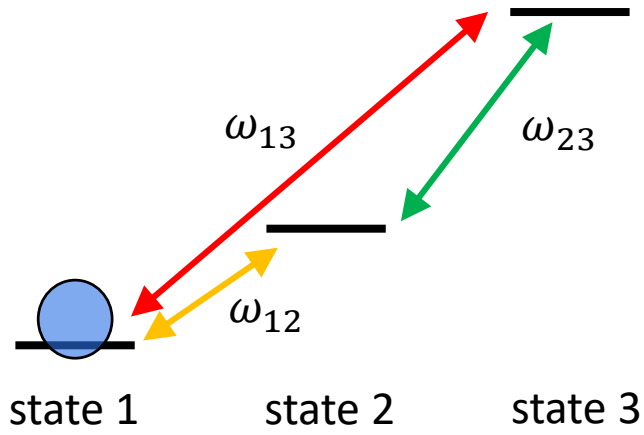
- "hopping" graphs have some physical constraints

→ **some constraints (e.g., dipole selection rules), but more flexibility**

- residual motional entropy (finite temperature)

→ **easy to "polarize" atoms in a single state**

synthetic (internal state) tight-binding models



$$\hat{H} = -\vec{d} \cdot \vec{E}$$

$$\vec{E} = \hat{\epsilon}\, E(t) \propto \sum_{i<j} E_{ij} \cos\left(\left(\omega_{ij} - \delta_{ij}\right)t + \varphi_{ij}\right)$$

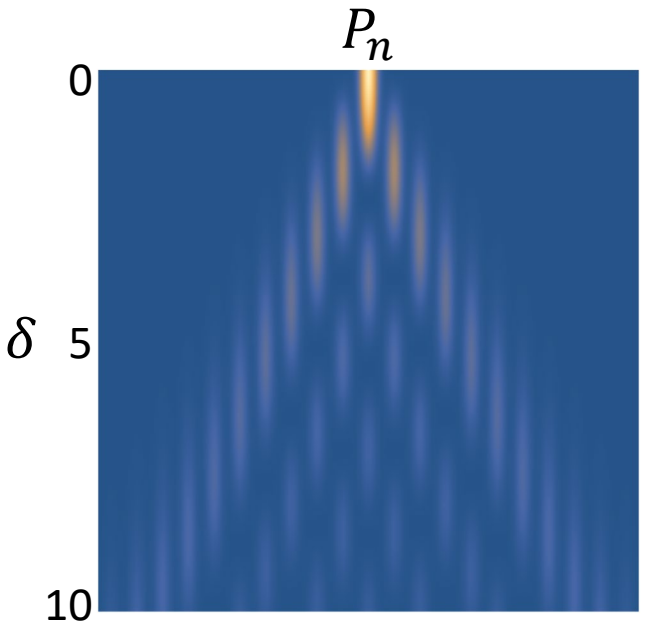
RWA…

spectroscopic Hamiltonian control!

$$\hat{H} = \sum_i \epsilon_i \hat{c}_i^\dagger \hat{c}_i - \sum_{i<j} \left(t_{ij} e^{i\varphi_{ij}} \hat{c}_j^\dagger \hat{c}_i + \text{h.c.}\right)$$

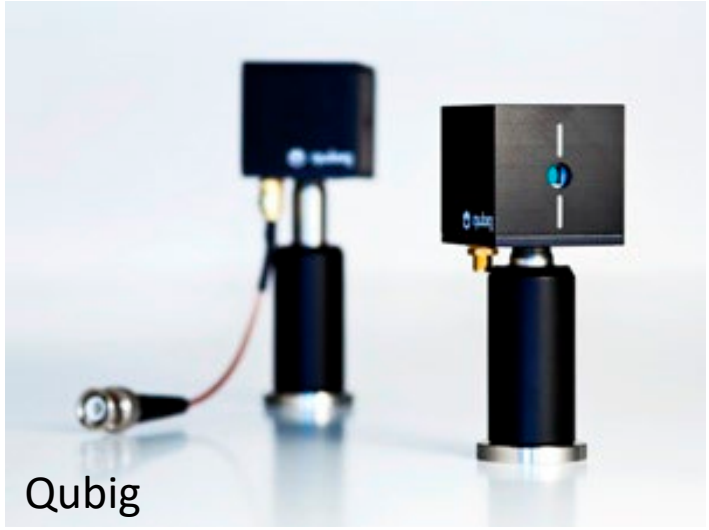
key ingredients for synthetic dimensions

- (many) states with unequal energies
  - "bare" hopping suppressed
- time-dependent "driving" to induce transitions (hopping)
  - modulation-assisted hopping



"hopping" between frequency modes
(or diffraction in frequency space)

a helpful analogy:


Qubig

Input: CW single-frequency light

$$E e^{i(kz-\omega t)}$$

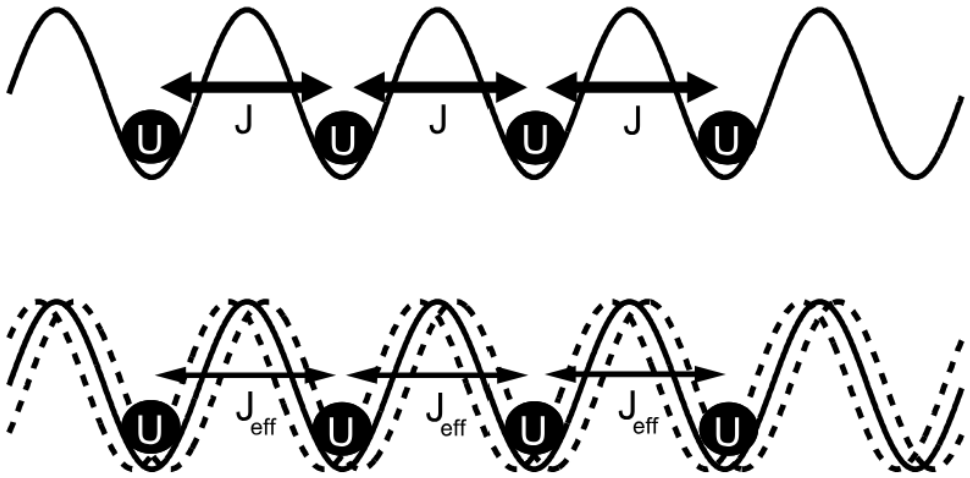
Output: phase-modulated light

$$E e^{i(kz-\omega t+\delta \sin(\omega_d t+\varphi))} \equiv E \sum_{n=-\infty}^{\infty} J_n(\delta)\, e^{in\varphi} e^{i(kz-(\omega+n\omega_d)t)}$$

Jacobi-Anger expansion

indirect analogs in cold atoms – some history

renormalized tunneling in **shaken lattices** [Lignier, et al. PRL (2007)]

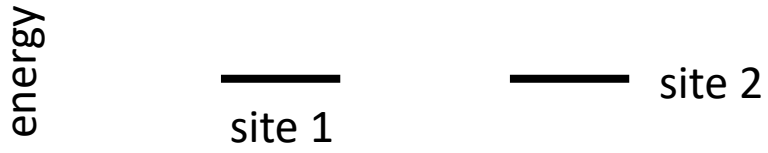


two-site system after frame transformation

$$\begin{bmatrix} -(\delta/2)\cos(\omega t+\varphi) & -J \\ -J & (\delta/2)\cos(\omega t+\varphi) \end{bmatrix}$$

$$\begin{bmatrix} 0 & -J \\ -J & \delta\cos(\omega t+\varphi) \end{bmatrix}$$

indirect analogs in cold atoms – some history



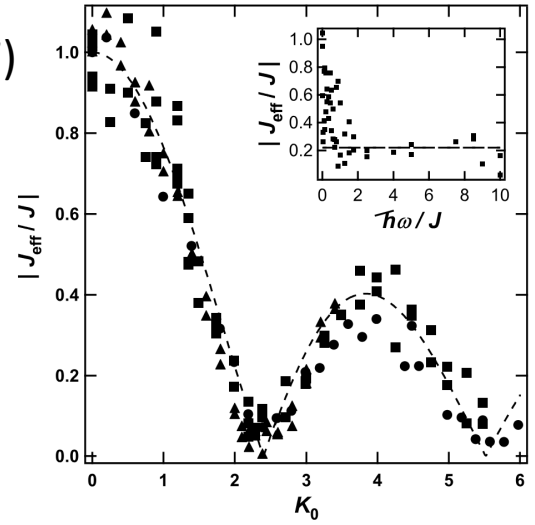
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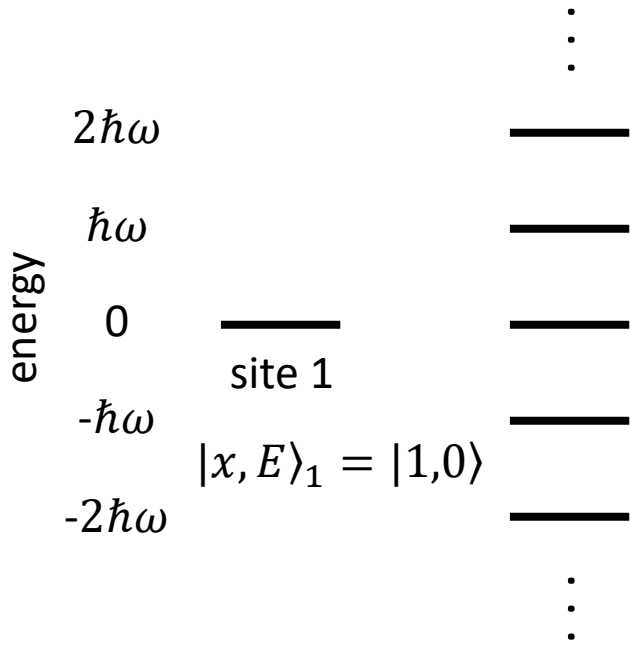
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indirect analogs in cold atoms – some history

Lignier, et al. PRL (2007)





→ expand as superposition of **fictitious** "energy states"

$$|x,E\rangle_2 = \sum_{n=-\infty}^{\infty} J_n(\delta/\hbar\omega)e^{in\varphi}\,|1,n\hbar\omega\rangle$$

two-site system after frame transformation

$$\begin{bmatrix} -(\delta/2)\cos(\omega t+\varphi) & -J \\ -J & (\delta/2)\cos(\omega t+\varphi) \end{bmatrix}$$

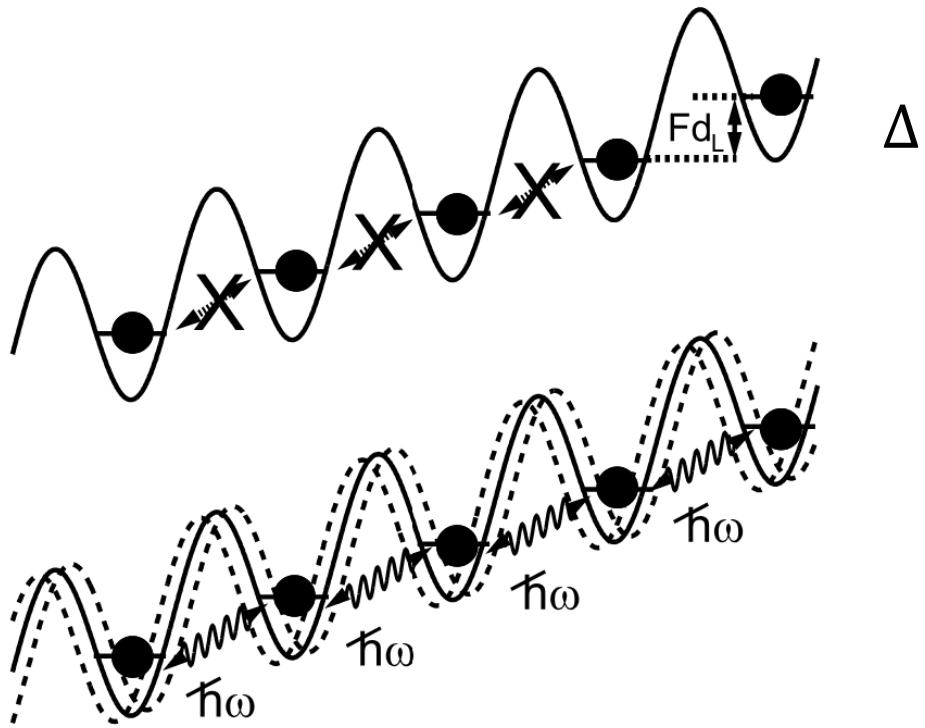
$$\begin{bmatrix} 0 & -J \\ -J & \delta\cos(\omega t+\varphi) \end{bmatrix}$$

$$\begin{bmatrix} 0 & -J_{\text{eff}} \\ -J_{\text{eff}} & 0 \end{bmatrix}$$

$$J_{\text{eff}} = J \times J_0(\delta/\hbar\omega)$$

indirect analogs in cold atoms – some more history

modulation-assisted tunneling in **tilted & shaken lattices**
[Sias, et al. PRL (2008)]

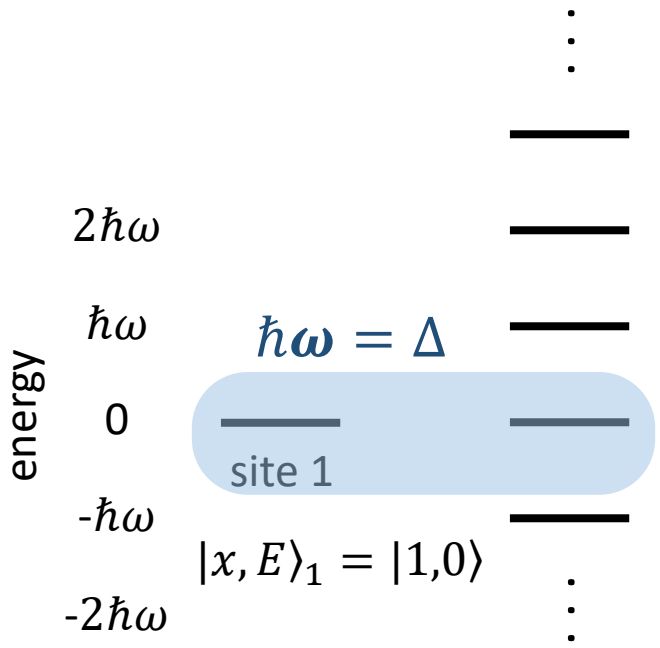


two-site system after frame transformation

$$\begin{bmatrix} -(\delta/2)\cos(\omega t+\varphi) & -J \\ -J & \Delta+(\delta/2)\cos(\omega t+\varphi) \end{bmatrix}$$

$$\begin{bmatrix} 0 & -J \\ -J & \Delta+\delta\cos(\omega t+\varphi) \end{bmatrix}$$

indirect analogs in cold atoms – some more history



→ expand as superposition of **fictitious** "energy states"

$$|x,E\rangle_2 = \sum_{n=-\infty}^{\infty} J_n(\delta/\hbar\omega)e^{in\varphi}\,|1,\Delta+n\hbar\omega\rangle$$

two-site system after frame transformation

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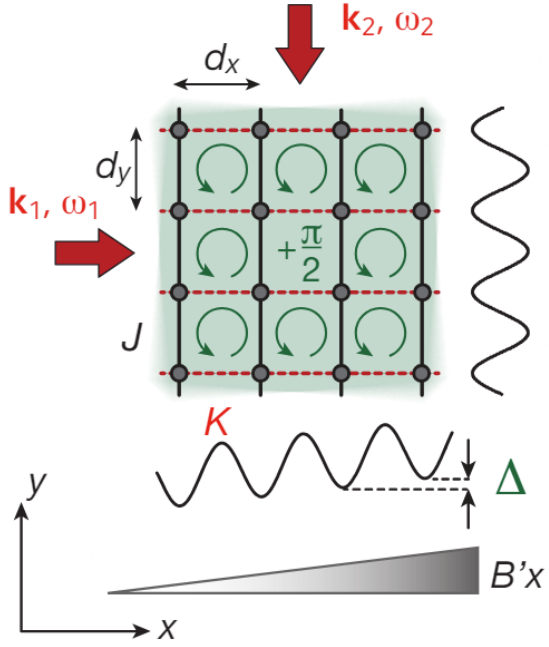
$$\begin{bmatrix} 0 & -J_{\text{eff}} \\ -J_{\text{eff}} & 0 \end{bmatrix}$$

$$J_{\text{eff}} = J \times J_{-1}(\delta/\Delta)e^{-i\varphi}$$

<u>connection</u>:
the "sites" in <u>synthetic dimensions</u> typically have unequal energies,
so one naturally needs to drive/shake the system to induce "hopping"

indirect analogs in cold atoms – some more history

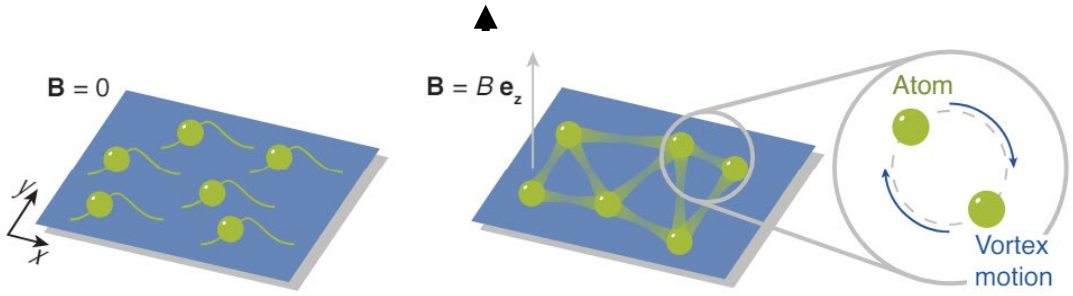
Raman-assisted tunneling



Jaksch & Zoller (2003)
Aidelsburger, *et al* /Miyake, *et al* (2013)

Fang, *et al* (2012) – photons
Mukherjee, *et al* (2015) - photons

Léonard, *et al* (2023)



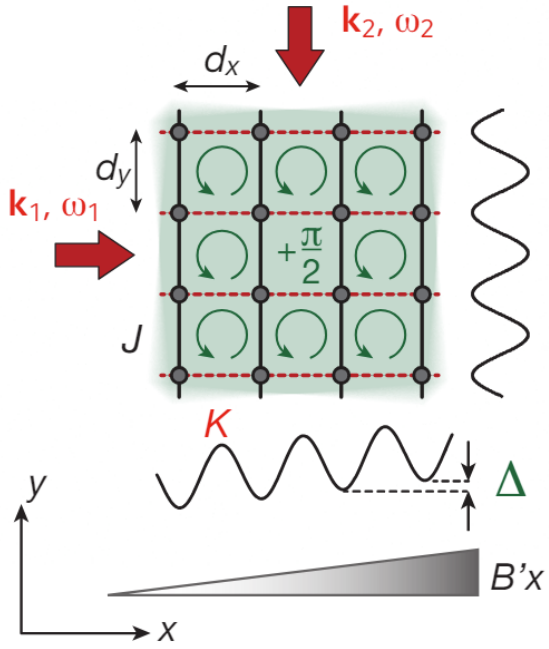
internal states
[big gaps, no higher "bands"]

<u>connection</u>:
the "sites" in <u>synthetic dimensions</u> typically have unequal energies,
so one naturally needs to drive/shake the system to induce "hopping"

$$J_{\text{eff}} = J \times J_{-1}(\delta/\Delta)e^{-i\varphi}$$

indirect analogs in cold atoms – some more history

Raman-assisted tunneling



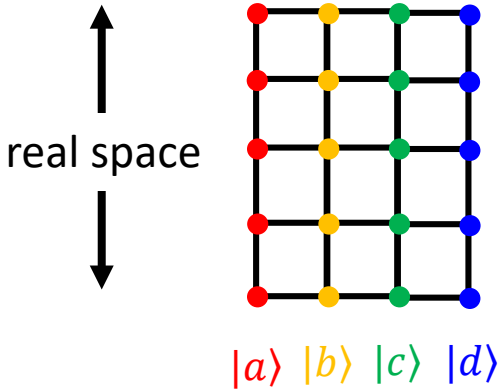
← Δ is not very large

limited by higher bands

Jaksch & Zoller (2003)
Aidelsburger, *et al* /Miyake, *et al* (2013)

Fang, *et al* (2012) – photons
Mukherjee, *et al* (2015) - photons

synthetic dimensions



internal states
[big gaps, no higher "bands"]

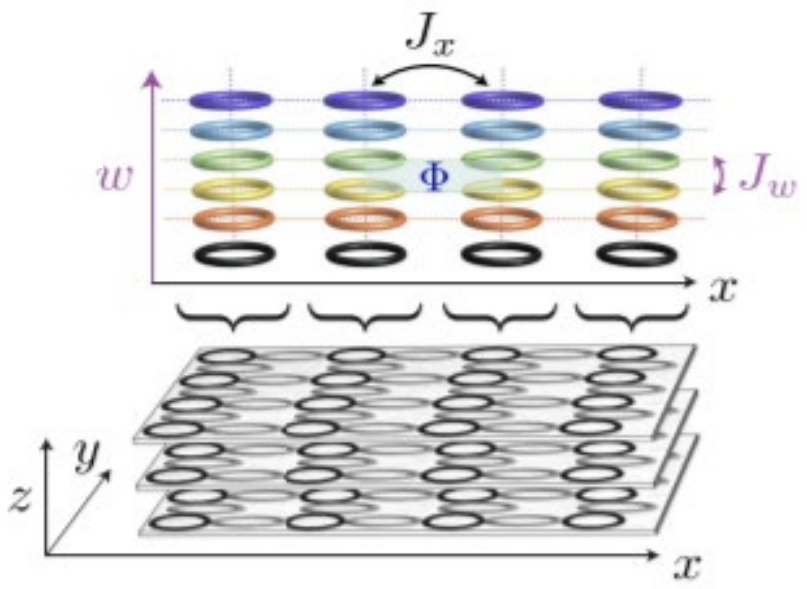
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$$J_{\text{eff}} = J \times J_{-1}(\delta/\Delta)e^{-i\varphi}$$

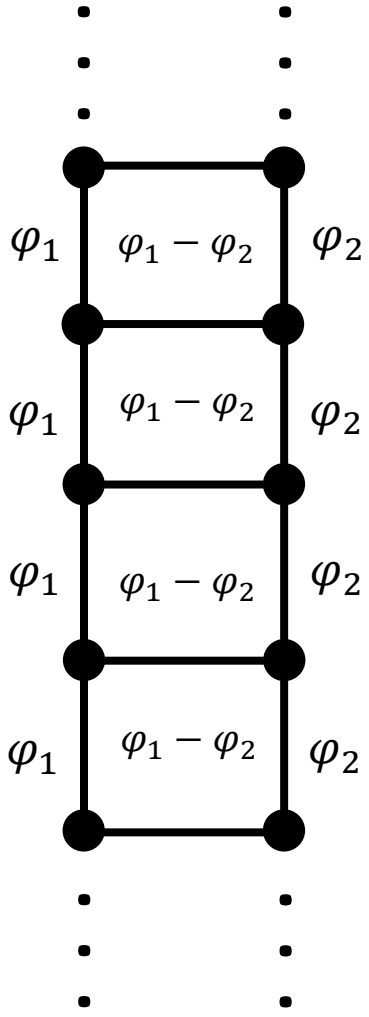
the synthetic dimensions zoo

photonics

modulated resonators – dynamics in a **synthetic frequency dimension**



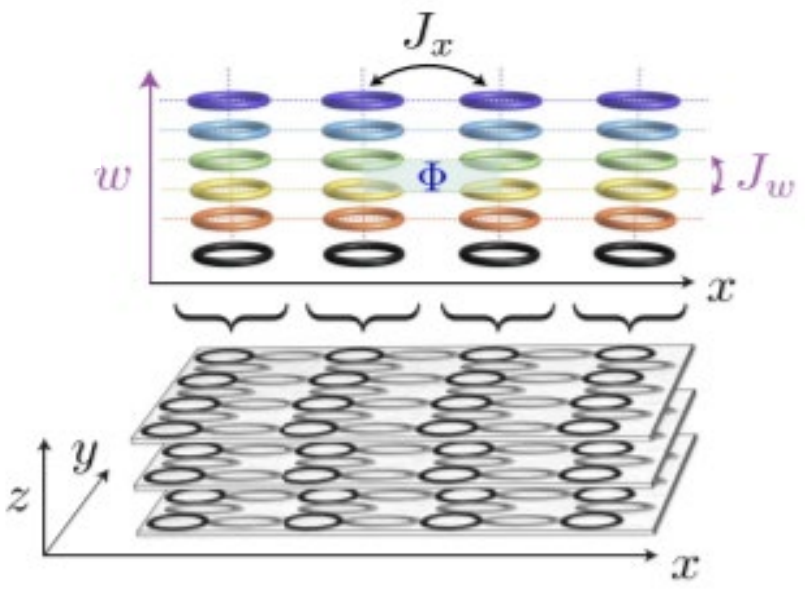
Ozawa, *et al*. (2016)



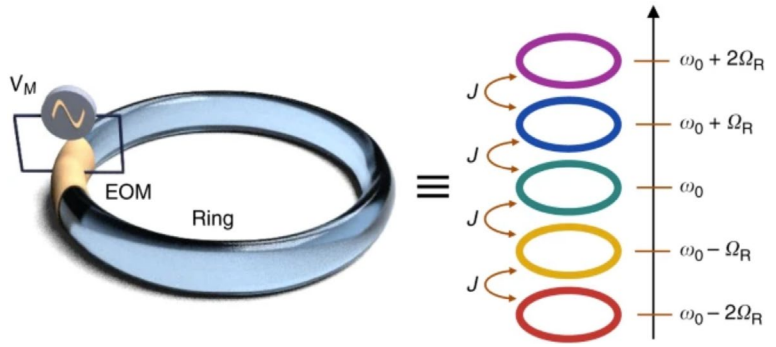
the synthetic dimensions zoo

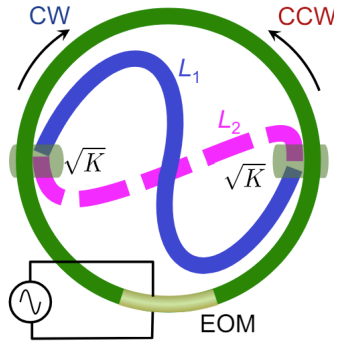
photonics

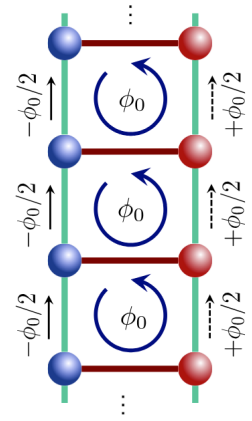
modulated resonators – dynamics in a **synthetic frequency dimension**



Ozawa, *et al*. (2016)







Fan/Dutt and other groups

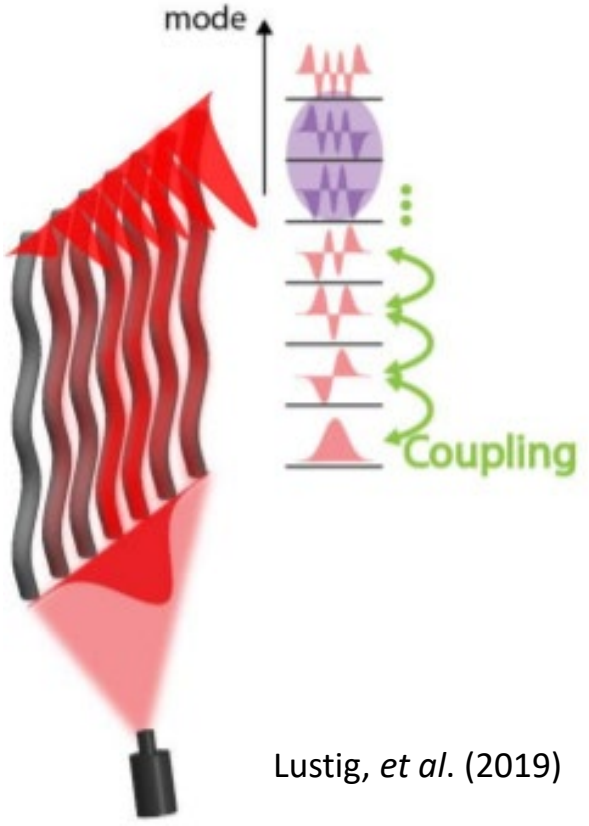
gauge fields, long-range hopping, non-reciprocity, and more

the synthetic dimensions zoo

photonics

modulated resonators – dynamics in a **synthetic frequency dimension**

modulated multi-mode waveguides – dynamics in a **synthetic "modal" dimension**



Lustig, *et al*. (2019)

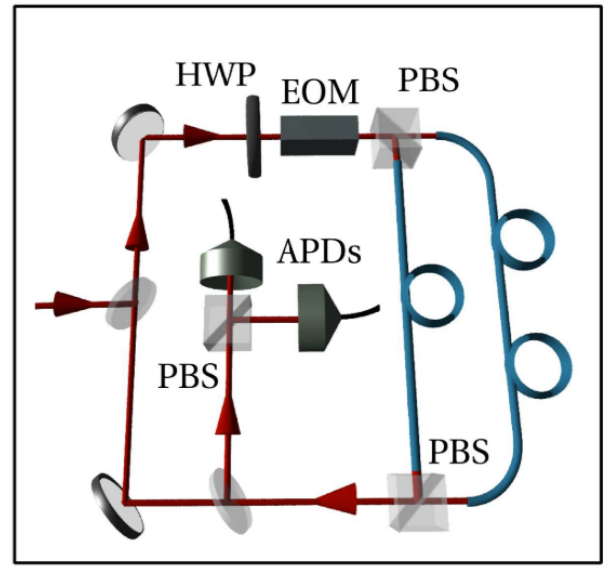
the synthetic dimensions zoo

photonics

modulated resonators – dynamics in a **synthetic frequency dimension**

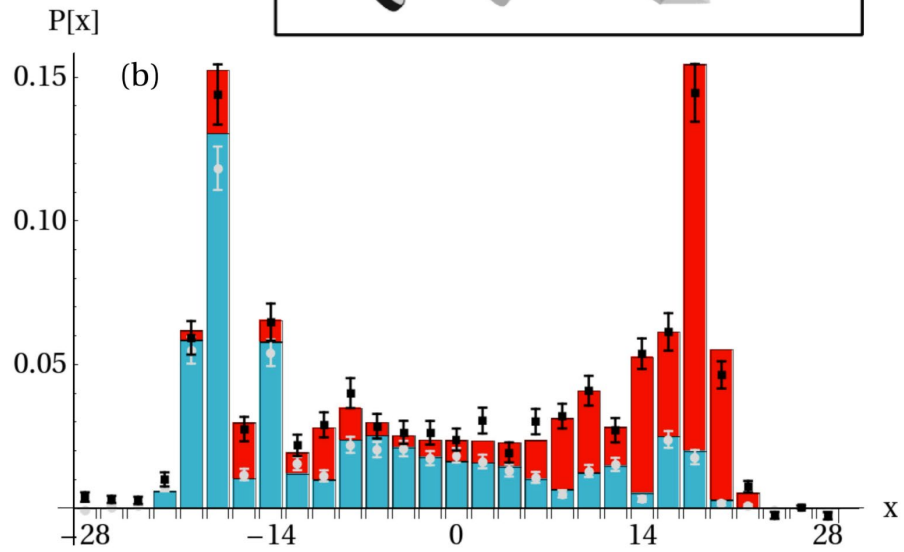
modulated multi-mode waveguides – dynamics in a **synthetic "modal" dimension**

time-multiplexed photonics – dynamics in a **synthetic "time-bin" dimension**



Silberhorn group, PRL (2011)

Wimmer
Hafezi/Waks
Marandi
Szameit/Segev
+ many others!



the synthetic dimensions zoo

photonics

modulated resonators – dynamics in a **synthetic frequency dimension**

modulated multi-mode waveguides – dynamics in a **synthetic "modal" dimension**

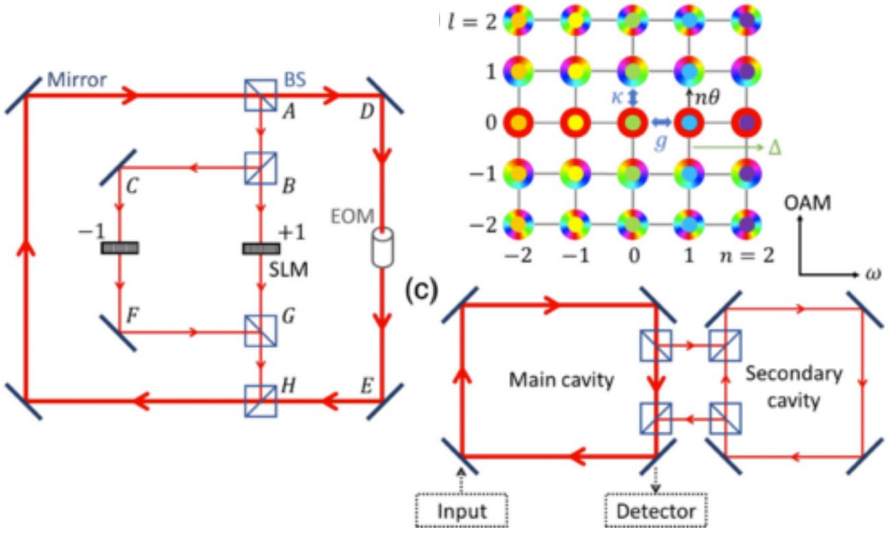
time-multiplexed photonics – dynamics in a **synthetic "time-bin" dimension**

orbital angular momentum

+ more degrees of freedom

Yuan, *et al*. PRL (2019)

Marrucci group (q-plates)
and more

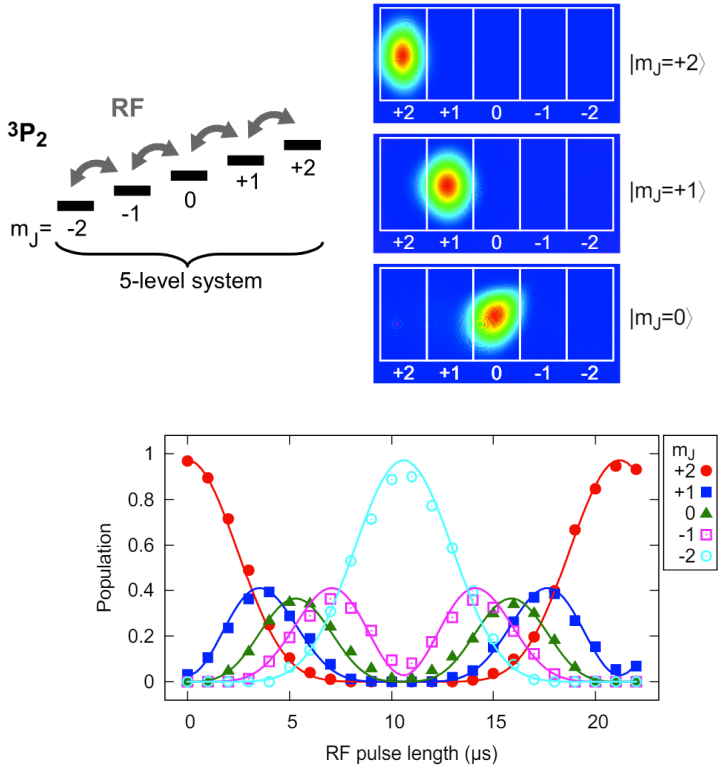


the synthetic dimensions zoo

photonics (frequency, modes, time-bin, OAM, more)

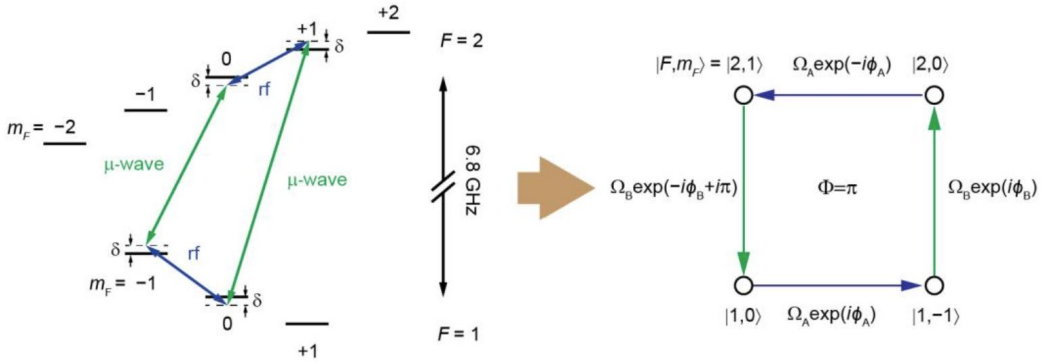
atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states

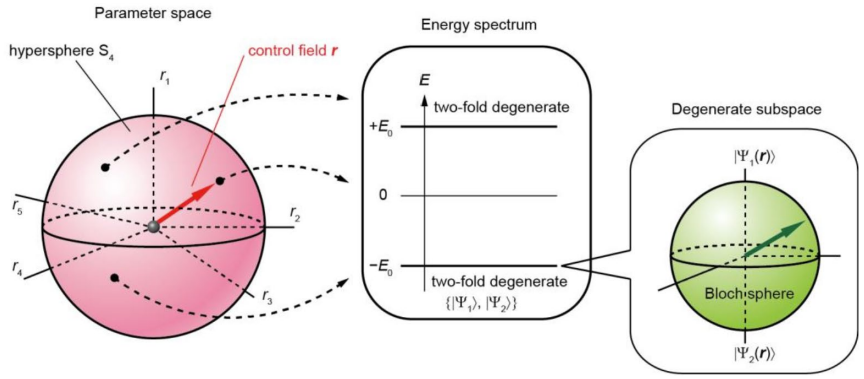


J. Phys B **55** 234004 (2022)

Birkl group (Darmstadt)



- small state space
- no gauge field (yet)



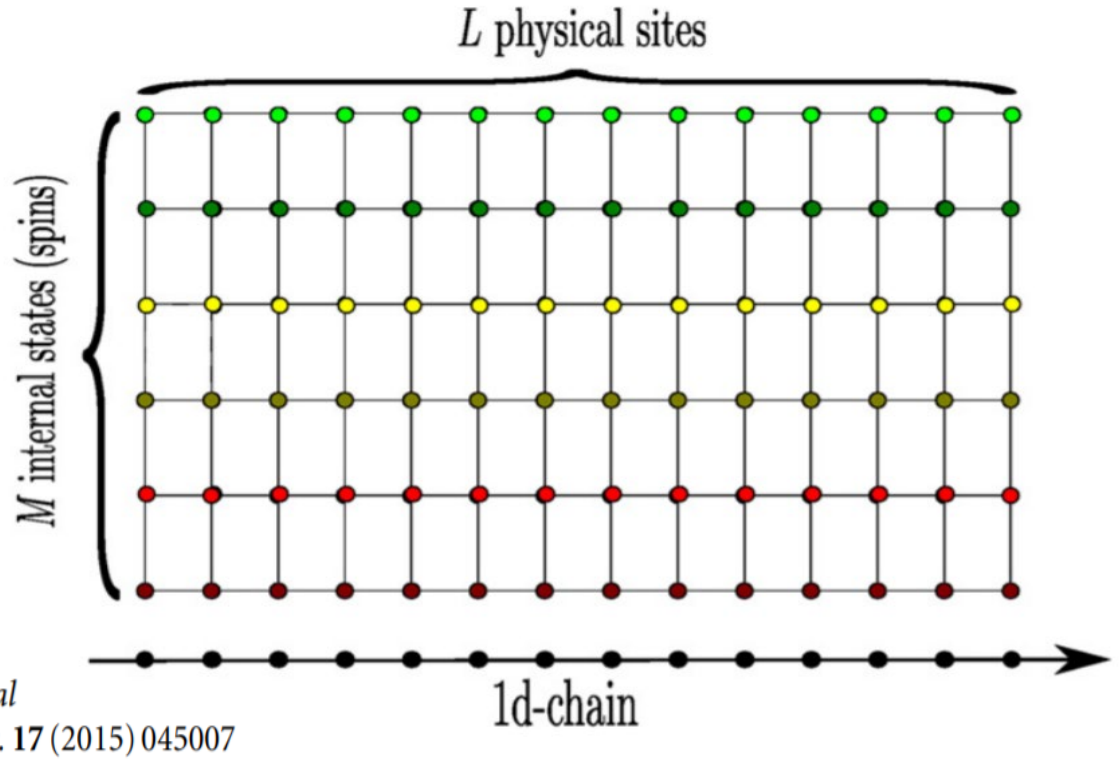
Sugawa, *et al*. Science **360**, 1429 (2018)

the synthetic dimensions zoo

photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states



O Boada *et al*
New J. Phys. **17** (2015) 045007

separable for rf- or μwave-based (long wavelength, equal phase)

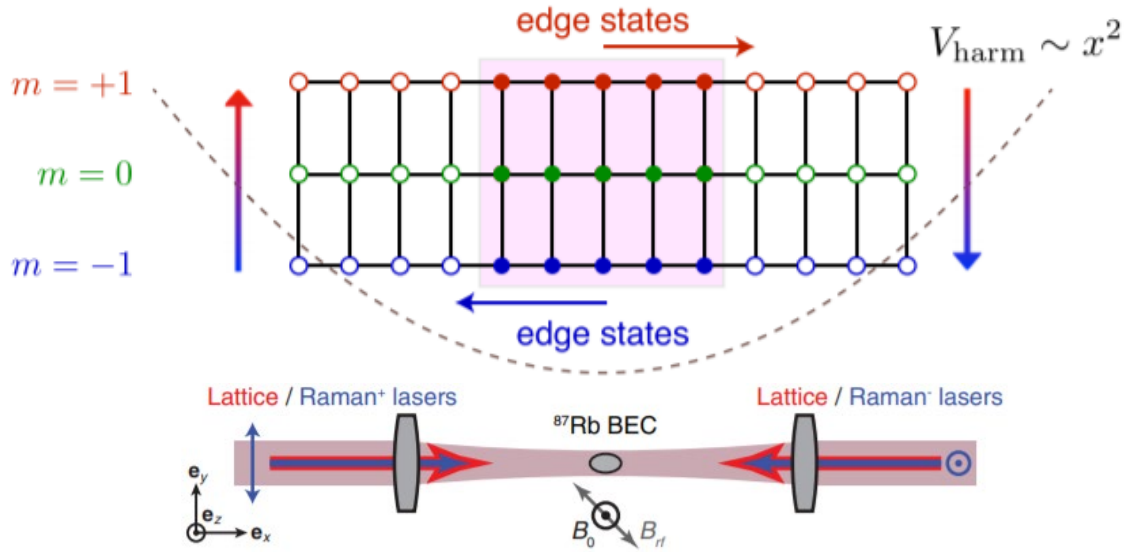
the synthetic dimensions zoo

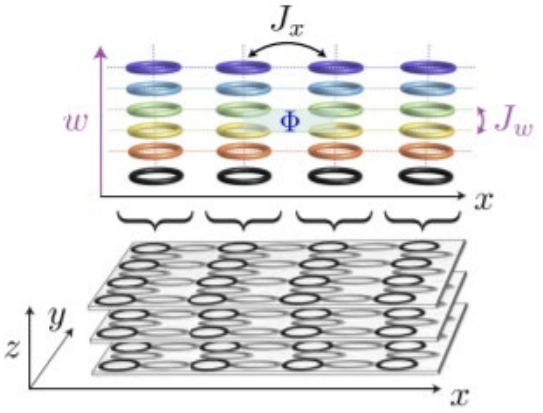
photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states

Synthetic Gauge Fields in Synthetic Dimensions
Celi, Massignan, Ruseckas, Goldman, Spielman, Juzeliūnas, and Lewenstein
Phys. Rev. Lett. **112**, 043001 (2014)





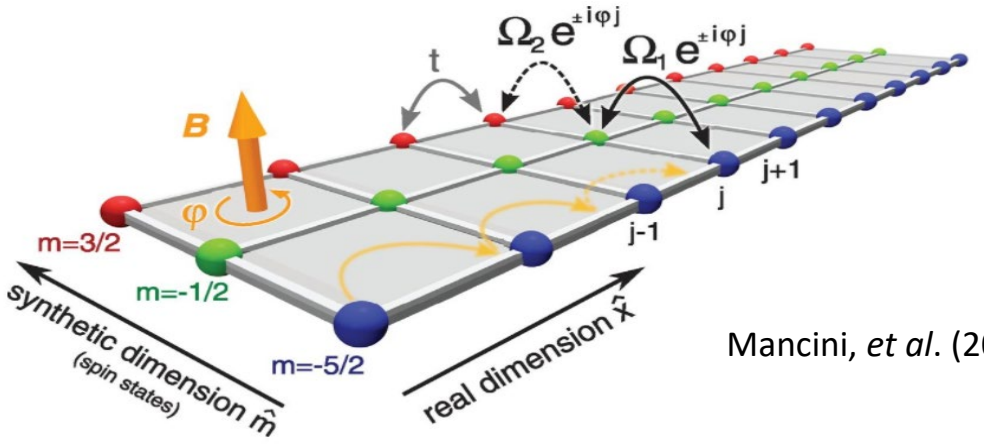
Ozawa, *et al*. (2016)

the synthetic dimensions zoo

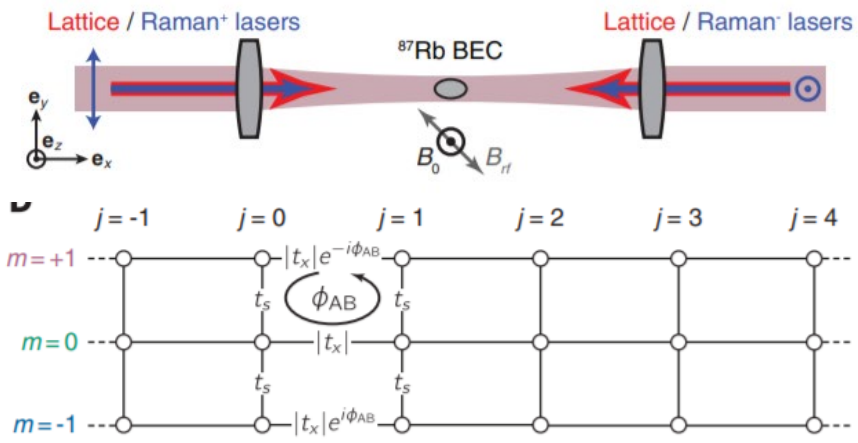
photonics (frequency, modes, time-bin, OAM, more)

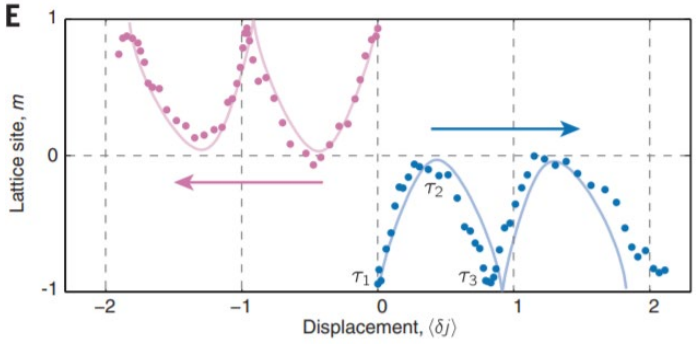
atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states



Mancini, *et al*. (2015)





Stuhl, *et al*. (2015)

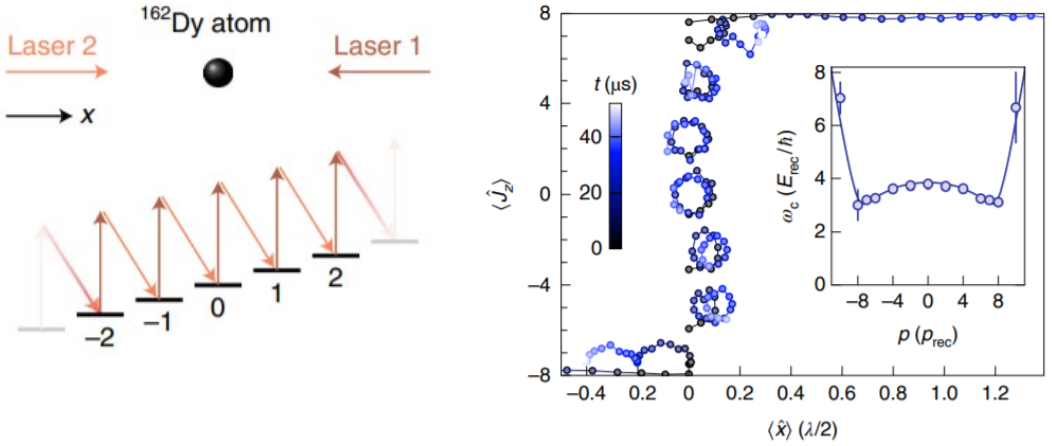
the synthetic dimensions zoo

photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems (wow, so many!)

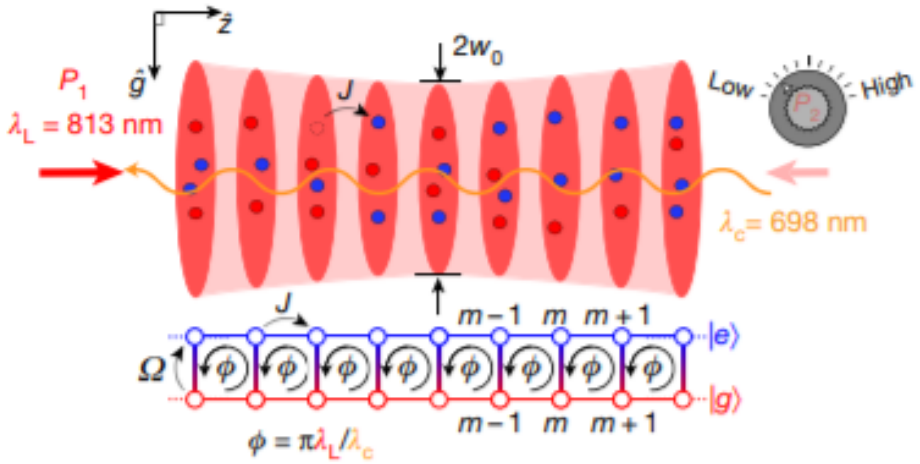
atomic hyperfine / internal states

magnetic atoms – large spin, many m_F states



Chalopin, *et al.* (Nat. Phys., 2019)

Kolkowitz, *et al*. Nature (2017)
Livi, *et al*. PRL (2016)



atomic “clock” states

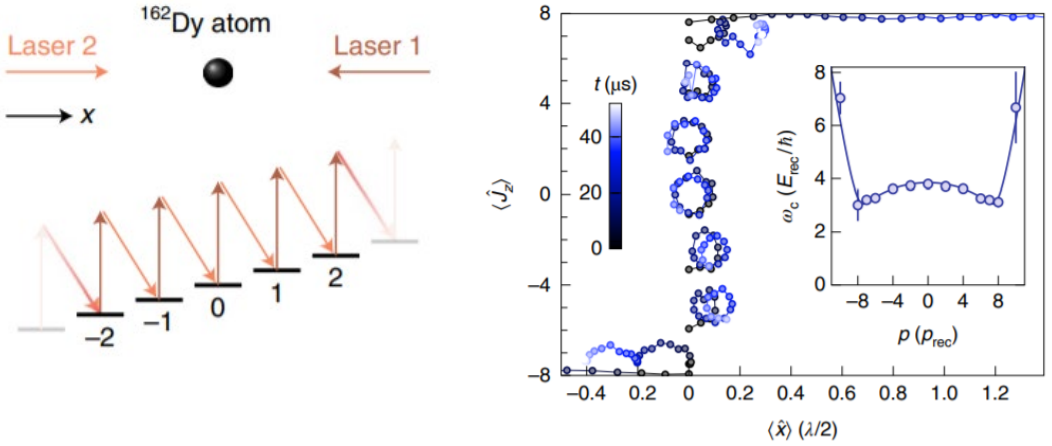
the synthetic dimensions zoo

photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states

magnetic atoms – large spin, many m_F states

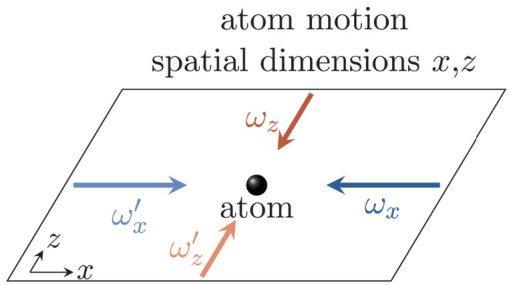


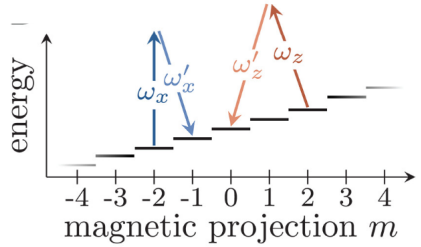
Chalopin, *et al.* (Nat. Phys., 2019)

Realization of an atomic quantum Hall system in four dimensions

JEAN-BAPTISTE BOUHIRON, AURÉLIEN FABRE, QI LIU, QUENTIN REDON, NEHAL MITTAL, TANISH SATOOR, RAPHAEL LOPES, AND SYLVAIN NASCIMBENE Authors Info & Affiliations

SCIENCE · 11 Apr 2024 · Vol 384, Issue 6692 · pp. 223-227 · DOI: 10.1126/science.adf8459





	Δn	Δm	$M\Delta\mathbf{v}$
x	1	1	$-2k\hat{\mathbf{x}}$
z	1	-2	$-2k\hat{\mathbf{z}}$

the synthetic dimensions zoo

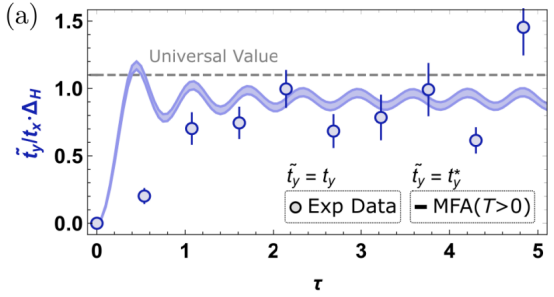
photonics (frequency, modes, time-bin, OAM, more)

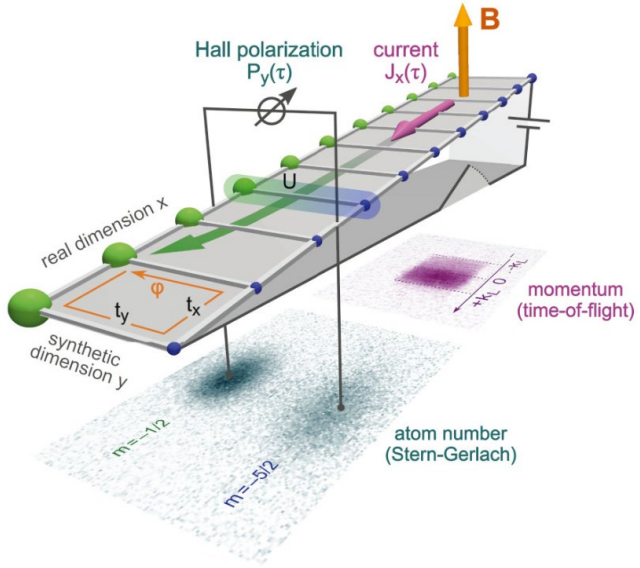
atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states

strong Hubbard-regime interactions (two spin states)

T.-W. Zhou *et al.*
Science **381**, 427 (2023)

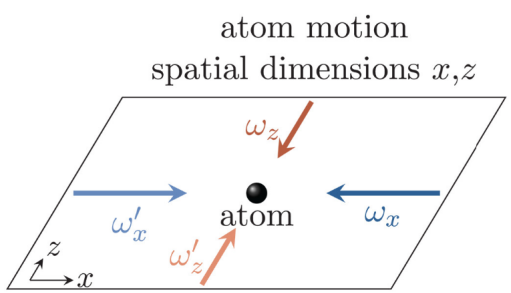


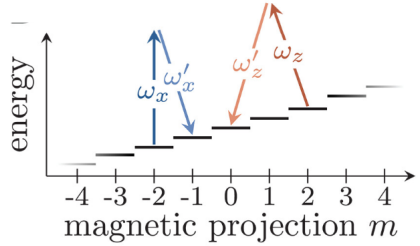


Realization of an atomic quantum Hall system in four dimensions

JEAN-BAPTISTE BOUHIRON, AURÉLIEN FABRE, QI LIU, QUENTIN REDON, NEHAL MITTAL, TANISH SATOOR, RAPHAEL LOPES, AND SYLVAIN NASCIMBENE Authors Info & Affiliations

SCIENCE · 11 Apr 2024 · Vol 384, Issue 6692 · pp. 223-227 · DOI: 10.1126/science.adf8459





	Δn	Δm	$M\Delta\mathbf{v}$
x	1	1	$-2k\hat{\mathbf{x}}$
z	1	-2	$-2k\hat{\mathbf{z}}$

the synthetic dimensions zoo

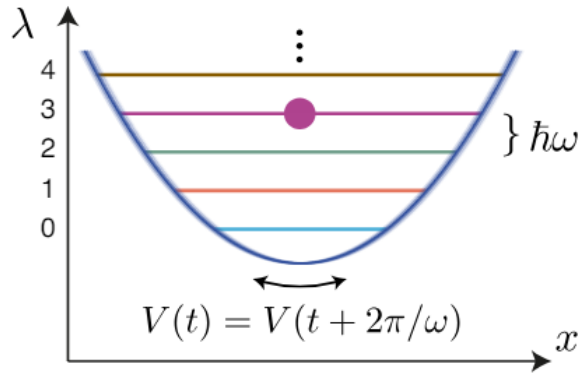
photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems (wow, so many!)

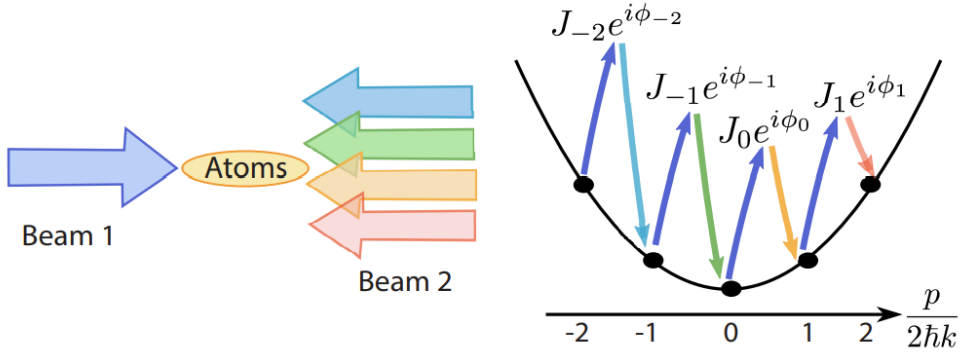
atomic hyperfine / internal states

atomic motional states

harmonic oscillator states



momentum states (Bragg transitions)



Ozawa and Price
Nat. Rev. Phys. (2019)

Proposal: Price, Ozawa, and Goldman, PRA (2017)
Experiment: Oliver, *et al*. Phys. Rev. Research (2023)

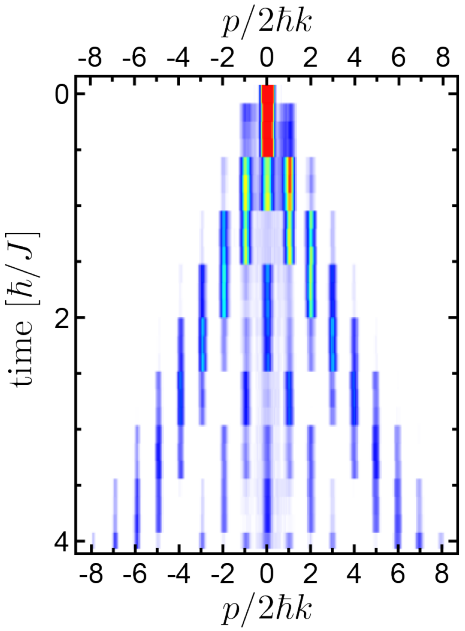
the synthetic dimensions zoo

photonics (frequency, modes, time-bin, OAM, more)

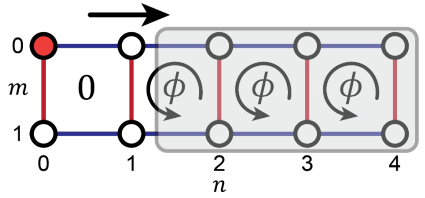
atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states

atomic motional states

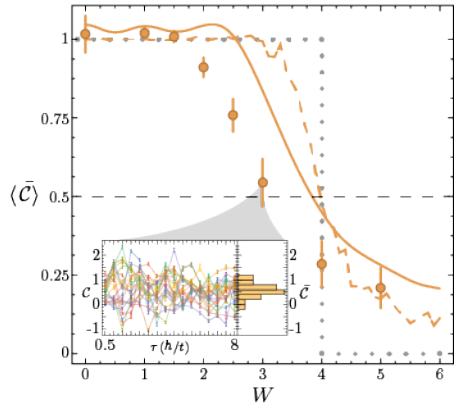


Tunable gauge fields



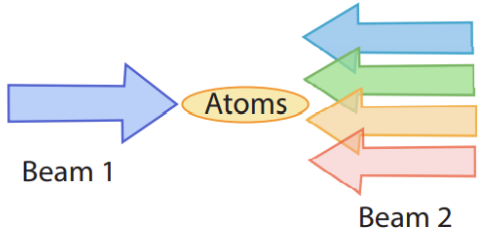
An, Meier, and BG, *Sci. Adv.* (2017)

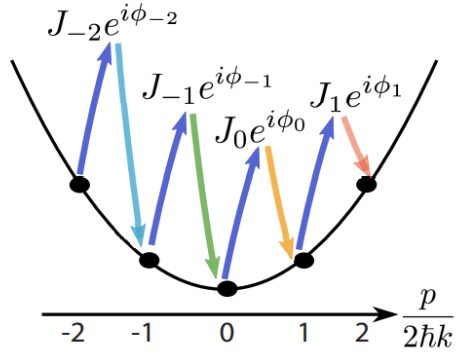
Disorder-induced topology



E.J. Meier *et al.*, *Science* (2018)

momentum states (Bragg transitions)





Ozawa and Price
Nat. Rev. Phys. (2019)

the synthetic dimensions zoo

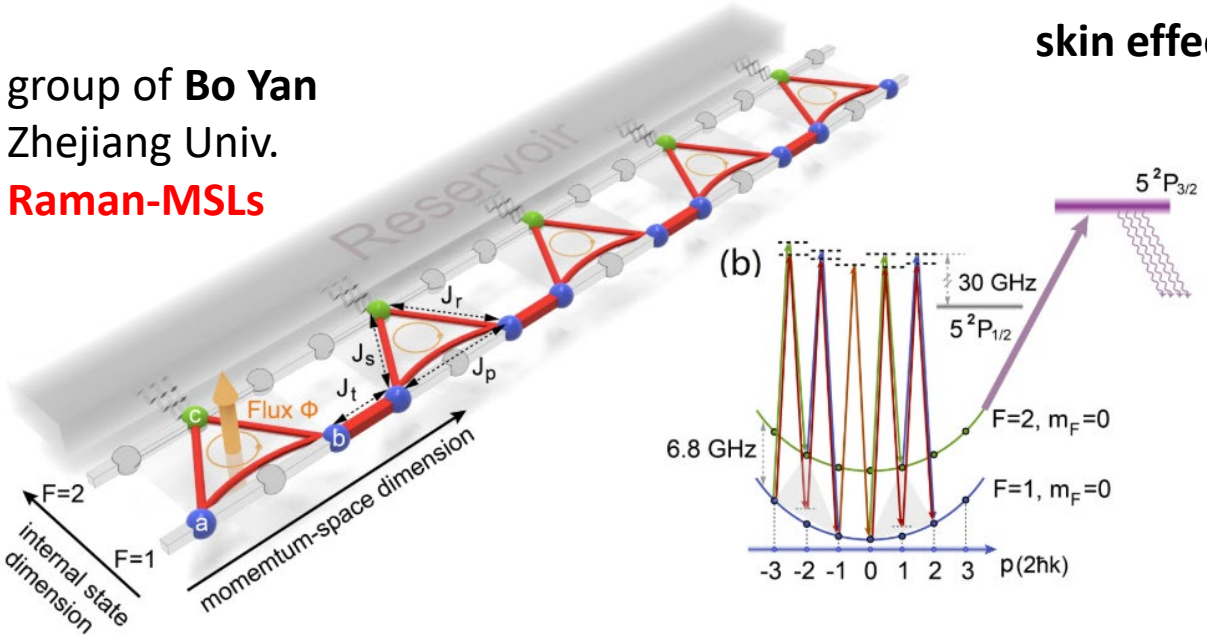
photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states

atomic motional states

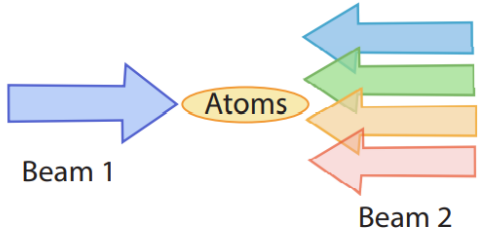
group of **Bo Yan**
Zhejiang Univ.
Raman-MSLs

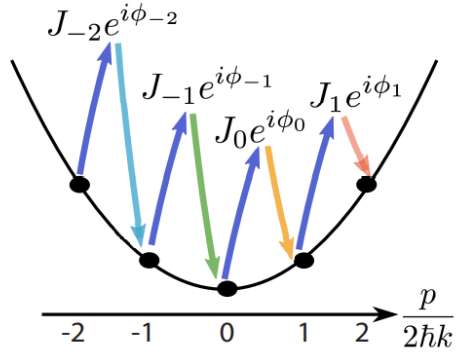


Qian Liang, *et al*. PRL (2022)

non-Hermitian skin effect

momentum states (Bragg transitions)





Ozawa and Price
Nat. Rev. Phys. (2019)

the synthetic dimensions zoo

photonics (frequency, modes, time-bin, OAM, more)

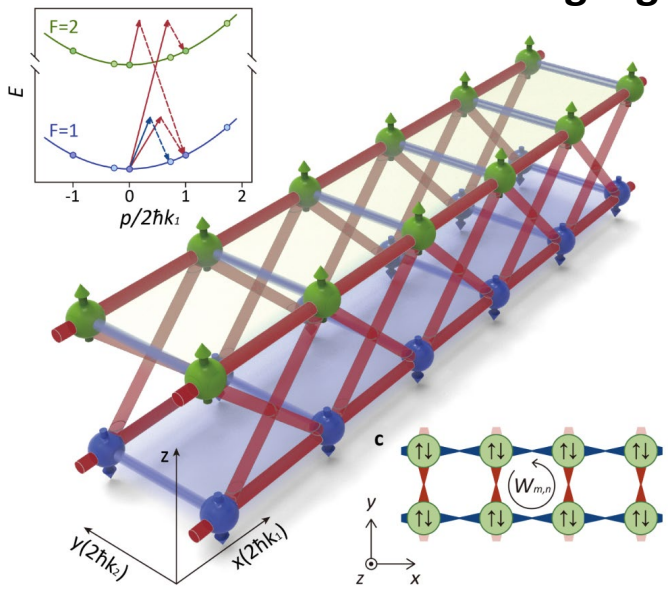
atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states

atomic motional states

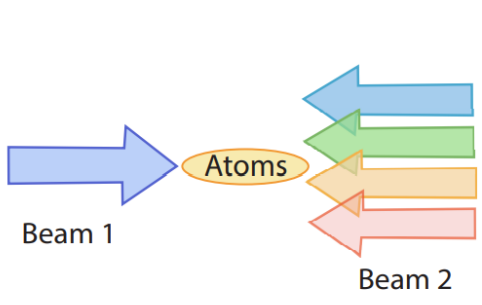
group of **Bo Yan**
Zhejiang Univ.
Raman-MSLs

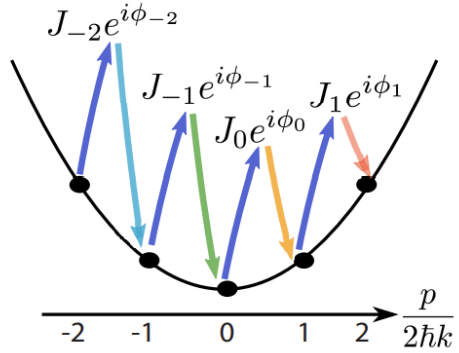
non-Abelian gauge fields



Qian Liang, *et al*. arXiv:2401.03612

momentum states (Bragg transitions)





Ozawa and Price
Nat. Rev. Phys. (2019)

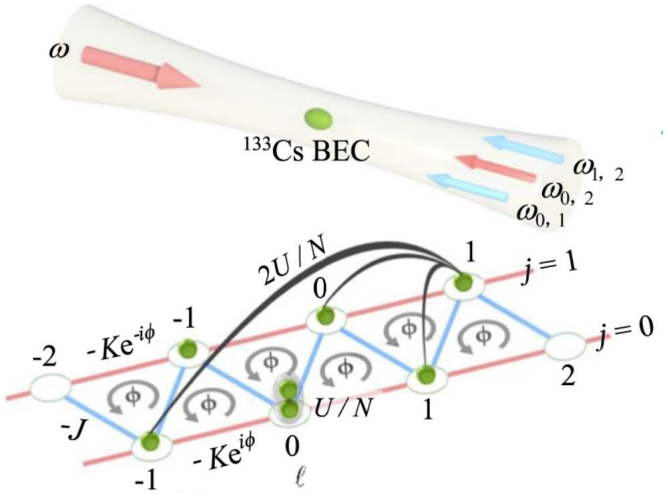
the synthetic dimensions zoo

photonics (frequency, modes, time-bin, OAM, more)

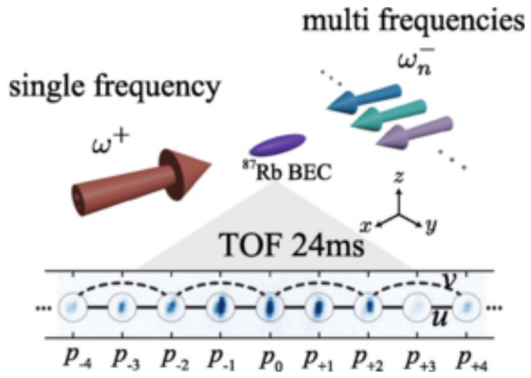
atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states

atomic motional states

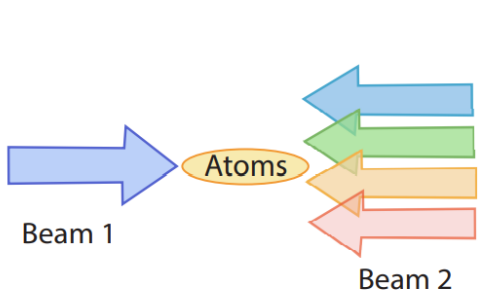


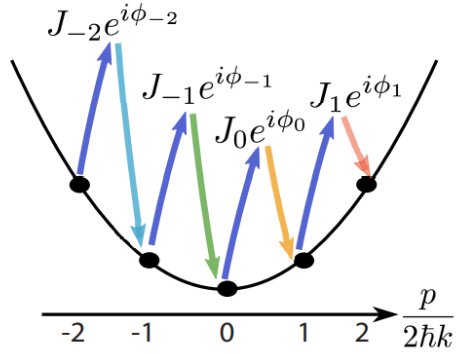
Yuqing Li, *et al*. Nat. Comm. (2023)



Zheng, *et al*. PRL (2024)

momentum states (Bragg transitions)





Ozawa and Price
Nat. Rev. Phys. (2019)

group of **Suotang Jia, Shanxi Univ.**
group of **Jian-Wei Pan Jia, USTC**

the synthetic dimensions zoo

photonics (frequency, modes, time-bin, OAM, more)

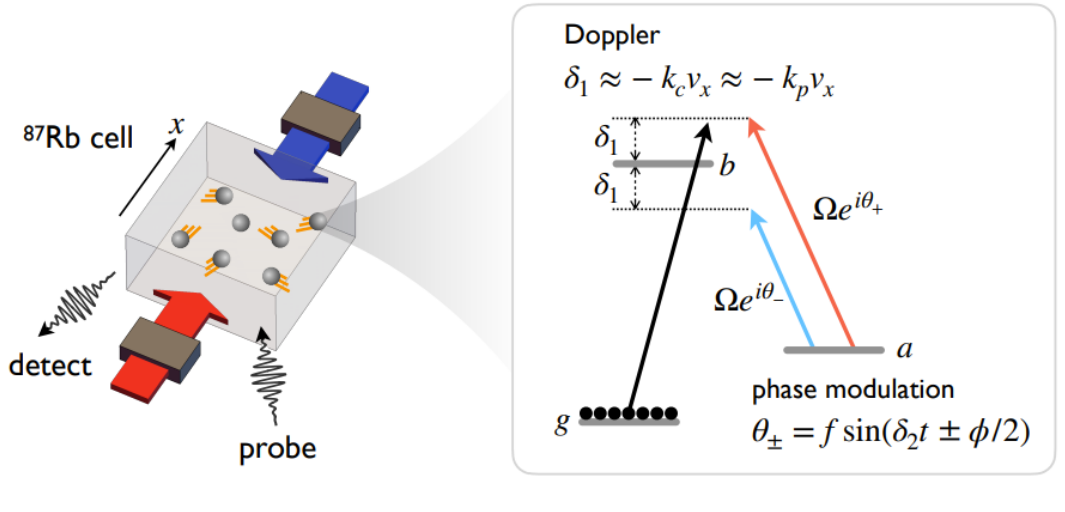
atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states

atomic motional states

superradiance lattices

D.-W. Wang & co-workers



the synthetic dimensions zoo

photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems (wow, so many!)

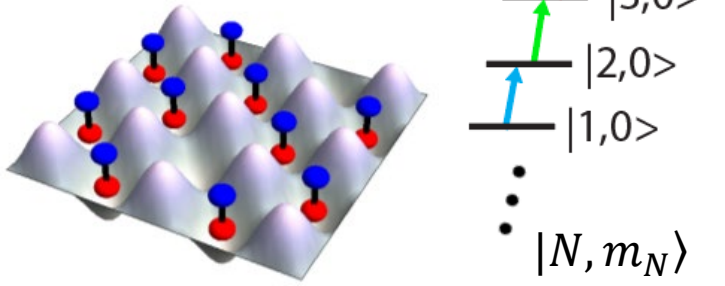
atomic hyperfine / internal states
atomic motional states
emerging platforms: **molecules, Rydberg atoms, and trapped ions**

the synthetic dimensions zoo

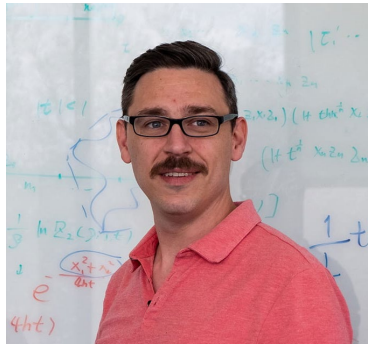
photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states
atomic motional states
emerging platforms: **molecules**, Rydberg atoms, and trapped ions

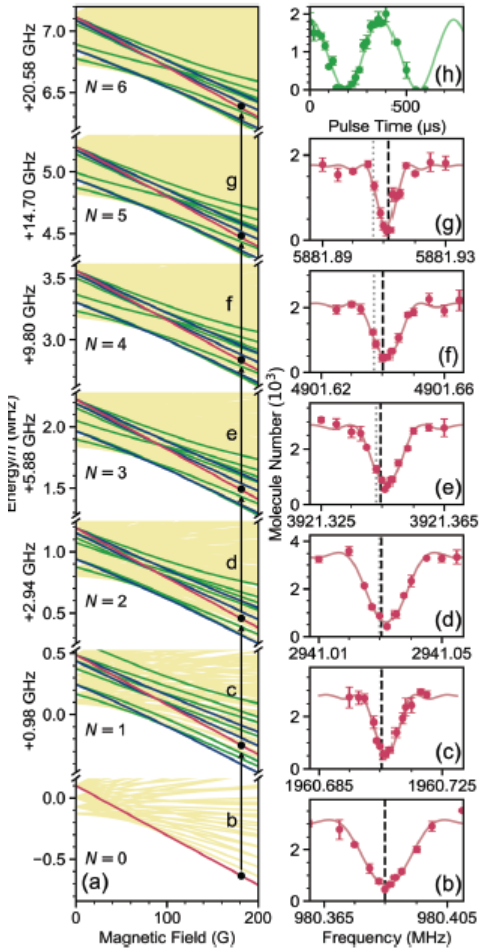


Moses, *et al.* (2019)



Sundar, BG, and Hazzard, Scientific Reports (2018)
Sundar, Thibodeau, Wang, BG, and Hazzard, PRA (2019)

Blackmore, *et al*. PCCP (2021)



the synthetic dimensions zoo

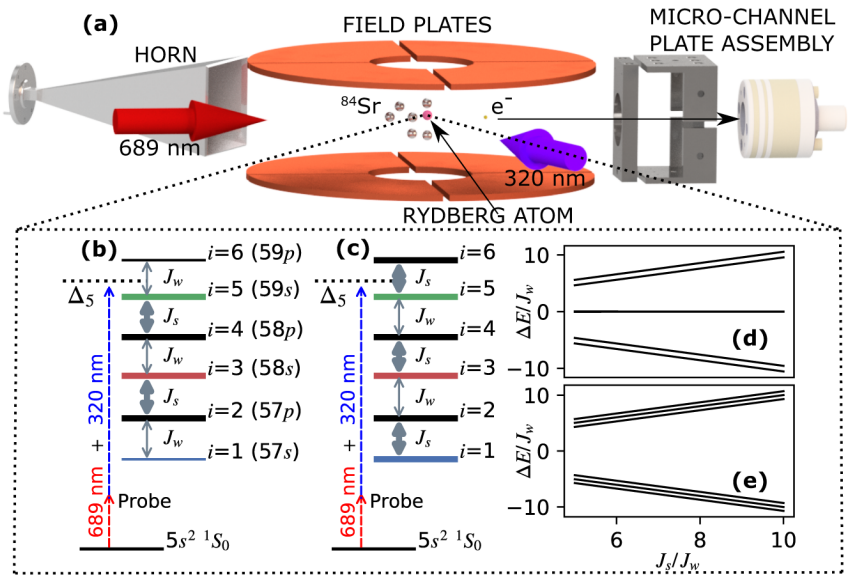
photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems (wow, so many!)

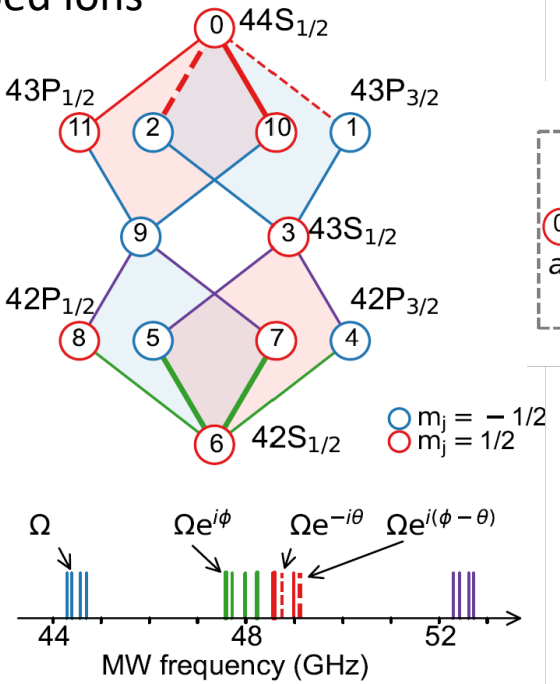
atomic hyperfine / internal states

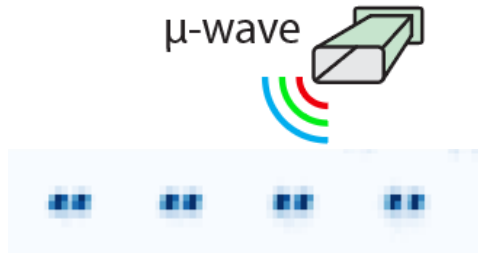
atomic motional states

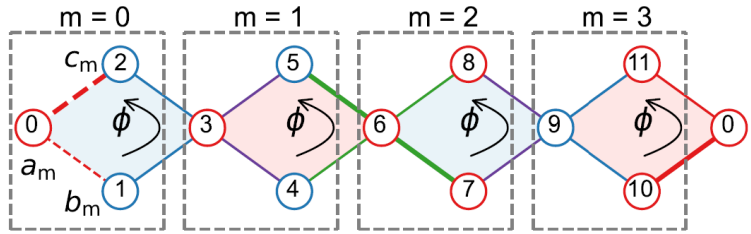
emerging platforms: molecules, **Rydberg atoms**, and trapped ions



S.K. Kanungo, *et al. Nat Commun* **13**, 972 (2022)
Lu. *et al*. PRA **109**, 032801 (2024)







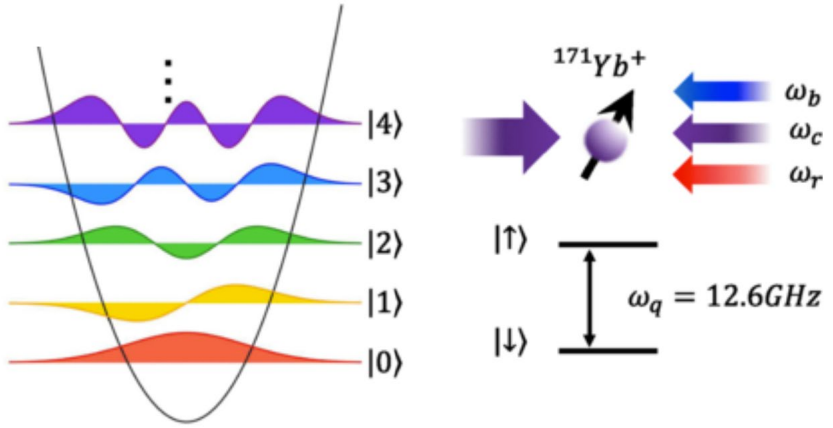
T. Chen*, C. Huang*, *et al. Nat Comm.*(2023)
T. Chen*, C. Huang*, *et al.* arXiv:2404.00737
T. Chen, *et al.* arXiv:2404.00740
(+ several more in preparation)

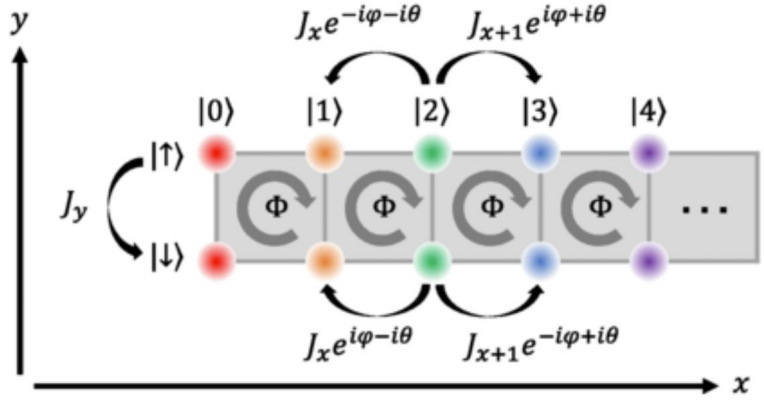
the synthetic dimensions zoo

photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems (wow, so many!)

atomic hyperfine / internal states
atomic motional states
emerging platforms: molecules, Rydberg atoms, and **trapped ions**





Realizing Synthetic Dimensions and Artificial Magnetic Flux in a Trapped-Ion Quantum Simulator

Y. Wang, Y.-K. Wu, Y. Jiang, M.-L. Cai, B.-W. Li, Q.-X. Mei, B.-X. Qi, Z.-C. Zhou, and L.-M. Duan
Phys. Rev. Lett. **132**, 130601 – Published 25 March 2024

the synthetic dimensions zoo – common themes / differences

photonics (frequency, modes, time-bin, OAM, more)

atomic & molecular systems

atomic hyperfine / internal states
atomic motional states
emerging platforms: molecules, Rydberg atoms, and trapped ions

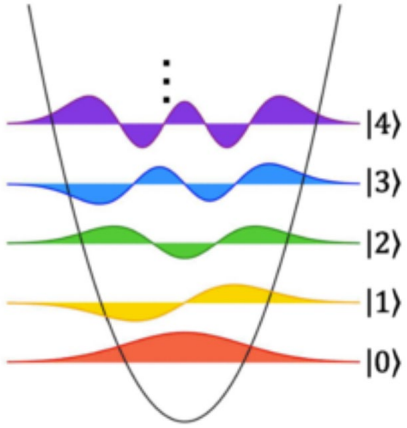
What are some commonalities and differences between these platforms?

Are there unifying ways to think about these systems?

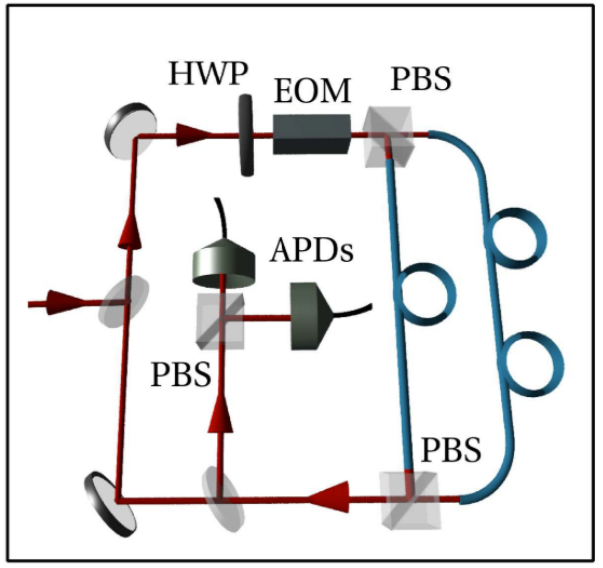
the synthetic dimensions zoo – common themes / differences

are the hopping terms “activated”?

if so, free control of hopping phases, site energies, and more



Duan group, PRL (2024)



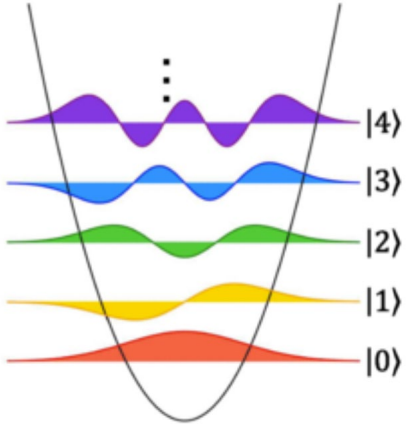
Silberhorn group, PRL (2011)

the synthetic dimensions zoo – common themes / differences

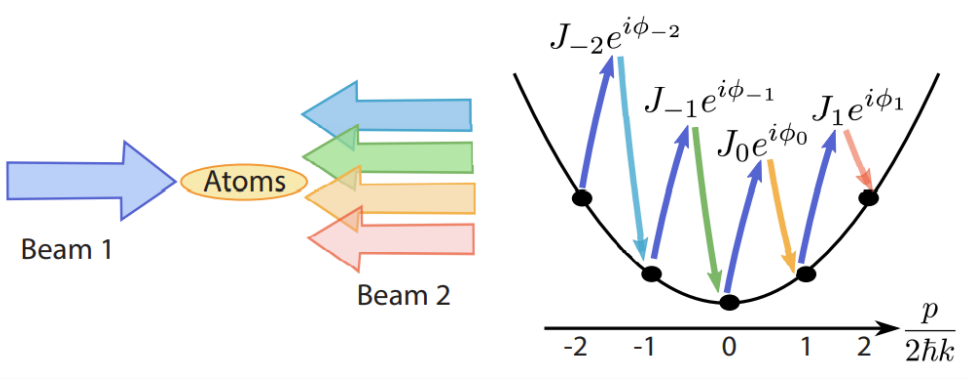
are the hopping terms “activated”?

is the system harmonic? or do transitions have unique resonances?

if unique, further capabilities for topological and disordered lattices



Duan group, PRL (2024)



Ozawa and Price, Nat. Rev. Phys. (2019)

the synthetic dimensions zoo – common themes / differences

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is the system harmonic? or do transitions have unique resonances?

what does the system look like in real space?

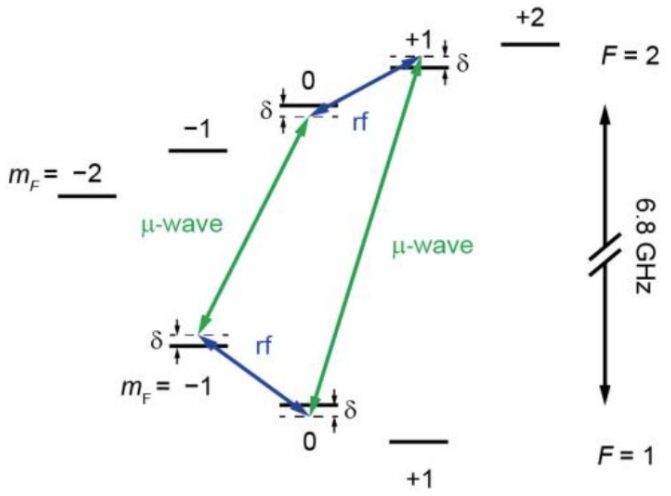
the synthetic dimensions zoo – common themes / differences

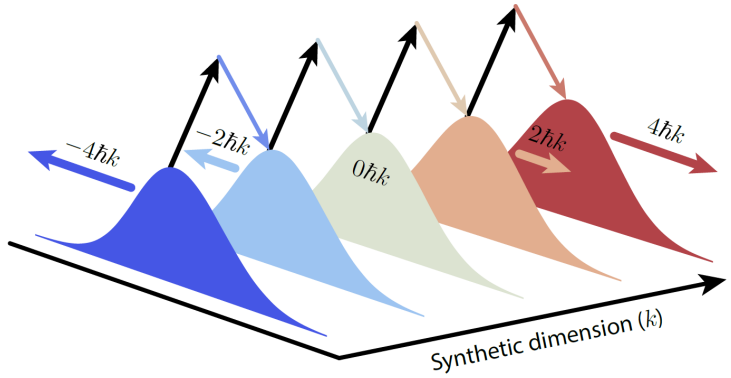
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- fully synthetic (single spatial mode)?
- mixed (real + synthetic) w/ real-space hopping?
- mixed (real + synthetic) w/ no hopping but long-ranged interactions?





for bosons, can be described as a type of “spinor gas” or multi-component condensate

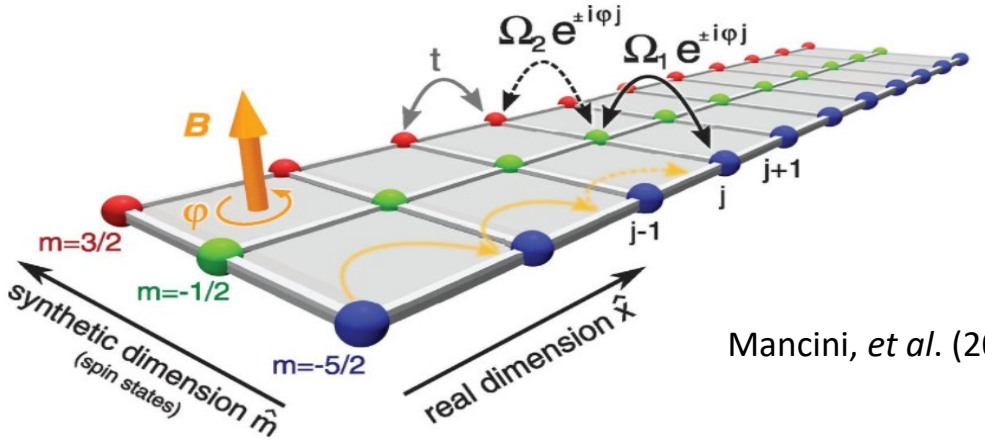
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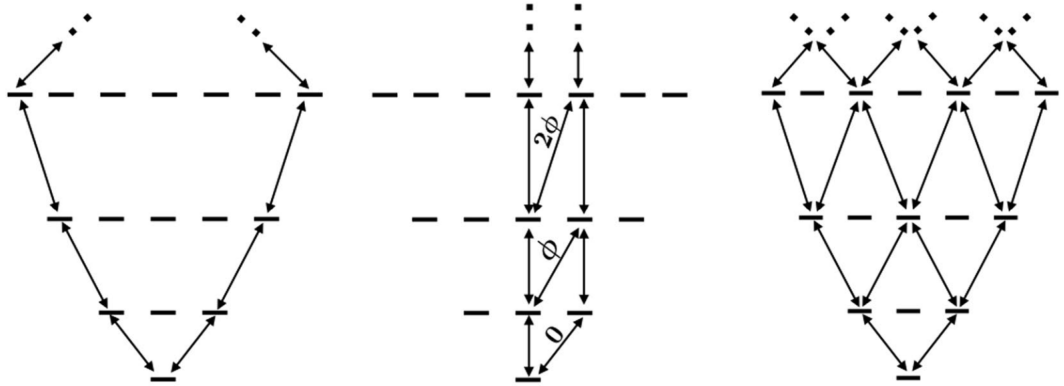
Mancini, *et al*. (2015)

- $(d_{\text{spatial}} + d_{\text{internal}})$-dimensional lattice
- bosons and fermions
- can be realized in the Hubbard regime
- non-separable (gauge fields, etc.) for spatially-varying Ω
- interactions in the synthetic dimension…

the synthetic dimensions zoo – common themes / differences

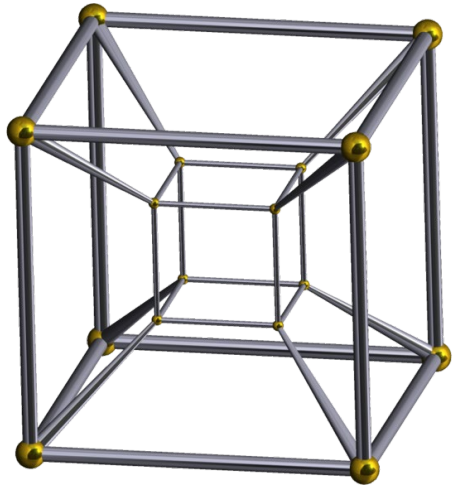
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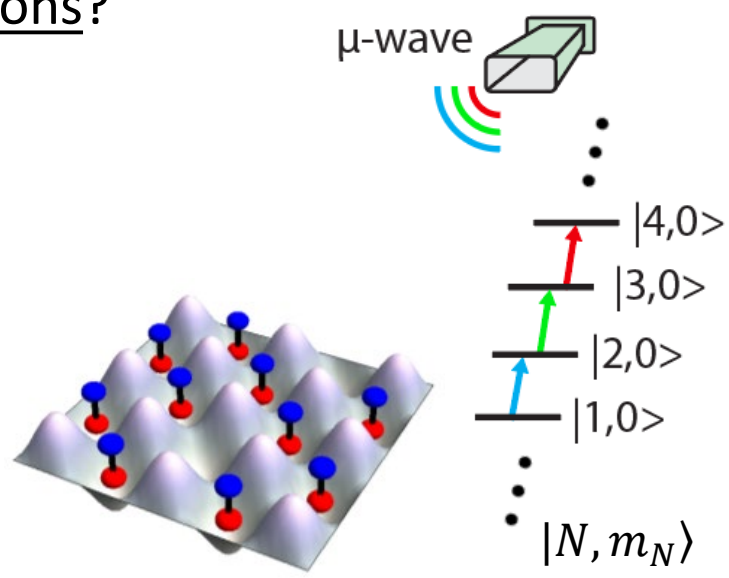


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4D hypercube
Robert Webb,
wikipedia

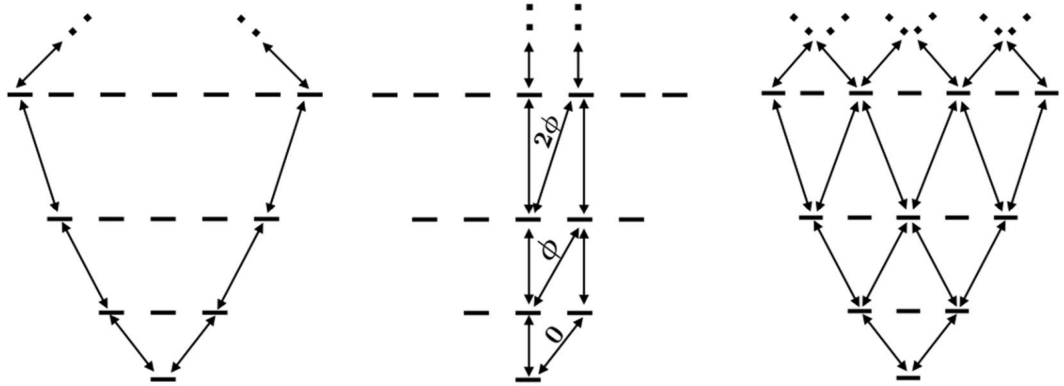


Moses, *et al.* (2019)

the synthetic dimensions zoo – common themes / differences

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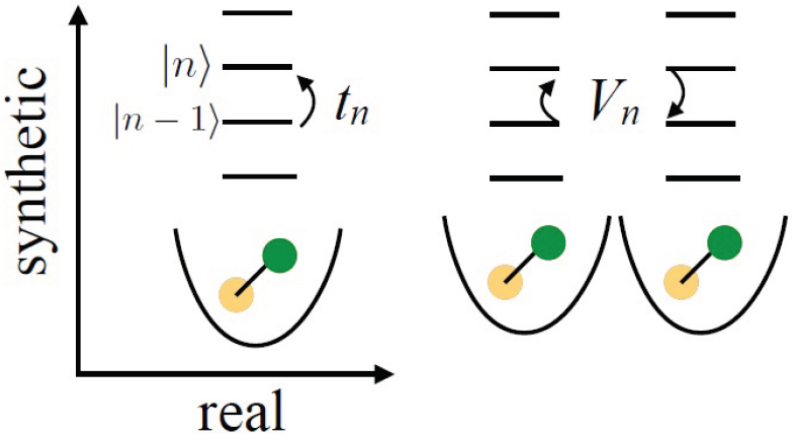
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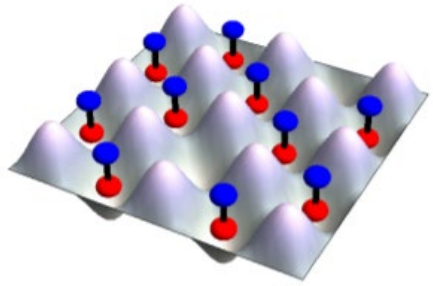
<u>it’s a spin system</u>

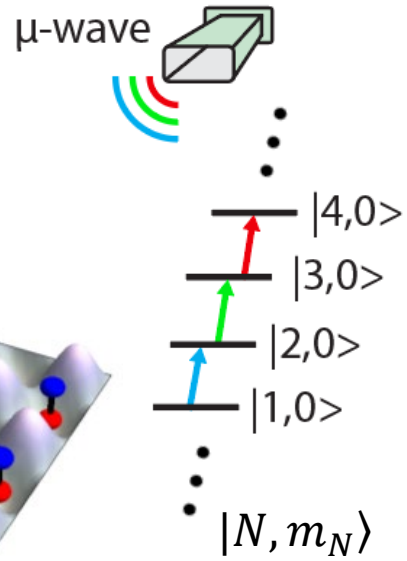
lattice TB models in exotic/restricted regimes

opportunities for exotic spin models



Sundar, BG, and Hazzard, Scientific Reports (2018)
Sundar, Thibodeau, Wang, BG, and Hazzard, PRA (2019)



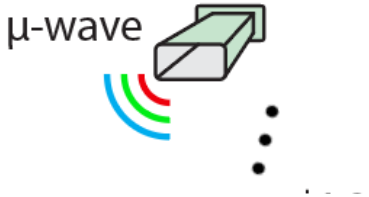


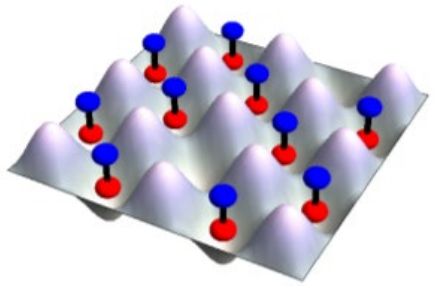
Moses, *et al.* (2019)

why synthetic dimensions? what do we get?

why?

experimentally, it's trivial and robust





Moses, *et al.* (2019)

why synthetic dimensions? what do we get?

why?

experimentally, it's trivial and robust

perfect for realizing exotic Hamiltonians

- gauge fields
- disorder
- dissipation
- "novel probing"

opportunities to realize **exotic interactions** (more tomorrow)

- pair-hopping
- density-dependent gauge fields
- controlled multi-particle loss

summary of Synthetic Dimensions 1

internal states are a readily available degree of freedom for atoms and molecules

some real-space tasks become simple if performed in synthetic dimensions

the roadmap ahead may be unclear, but there are real opportunities for analog simulation and even for quantum computing (qudit-based approaches)

tomorrow (9 am lecture):

- interactions in the synthetic dimension
- recent advances in correlated dynamics/phases in synthetic dimensions

Synthetic Dimensions II: interactions in synthetic lattices

Bryce Gadway

Penn State University (& University of Illinois)

2024 Croucher Summer Course on Ultracold Atomic Physics

May 22, 2024



please ask questions!

Yesterday: Synthetic Dimensions I

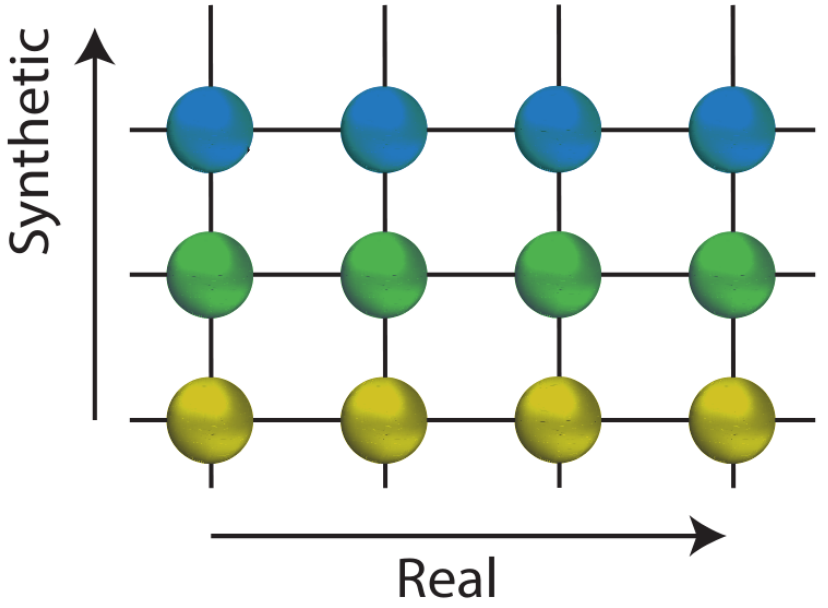
- systems, states, and capabilities
- some unifying themes & considerations

Today: Synthetic Dimensions II

- interactions in synthetic dimensions
- recent results on correlated dynamics in synthetic dynamics

outline for Synthetic Dimensions II

- interactions along a synthetic dimension – what's different from real space?
- atoms, molecules, Rydberg atoms, . . .
- the current state of affairs
- challenges and prospects
- summary



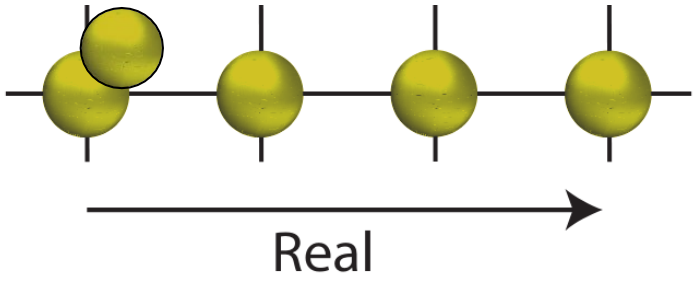
Ozawa & Price, Nat. Rev. Phys. (2019)

interactions along a **real** dimension

- **typically local or short-ranged in space**

  contact interactions

$$U_{ij} \propto \delta_{ij}$$



Ozawa & Price, Nat. Rev. Phys. (2019)

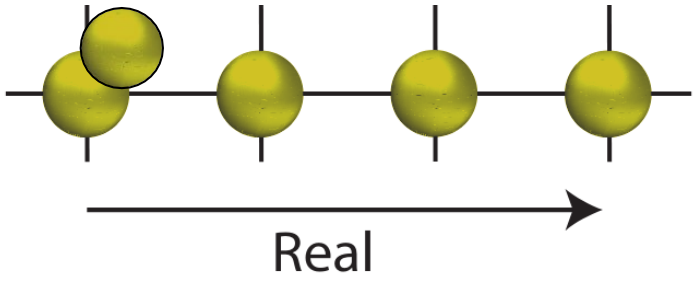
interactions along a **real** dimension

- **typically local or short-ranged in space**

  contact interactions

  dipolar interactions

$$U_{ij} \propto |i-j|^{-3}$$



Ozawa & Price, Nat. Rev. Phys. (2019)

interactions along a **real** dimension

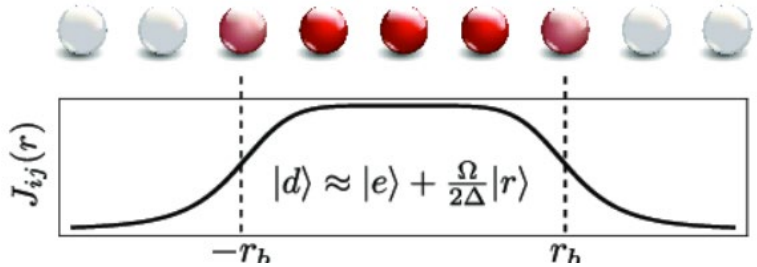
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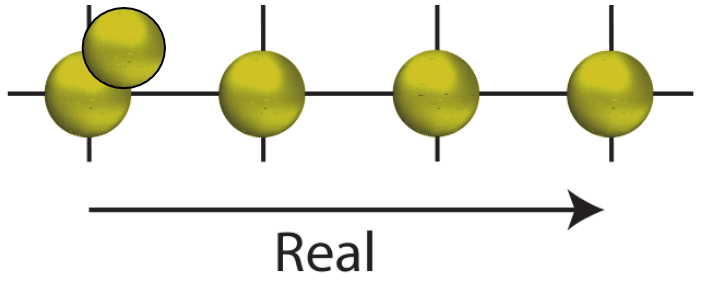
  dipolar interactions

  soft-core

Pupillo, *et al*. PRL (2010)



$U_{ij} \propto$



Ozawa & Price, Nat. Rev. Phys. (2019)

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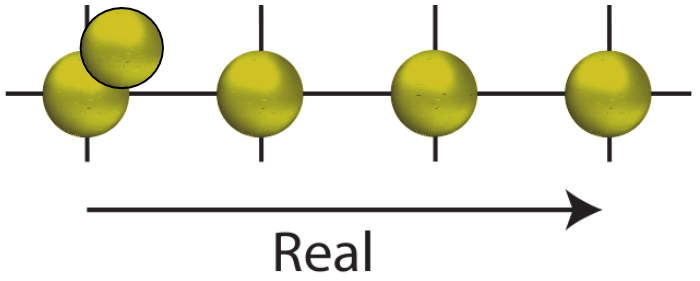
  contact interactions

  dipolar interactions

  soft-core

  Coulombic

$$U_{ij} \propto |i-j|^{-1}$$



Ozawa & Price, Nat. Rev. Phys. (2019)

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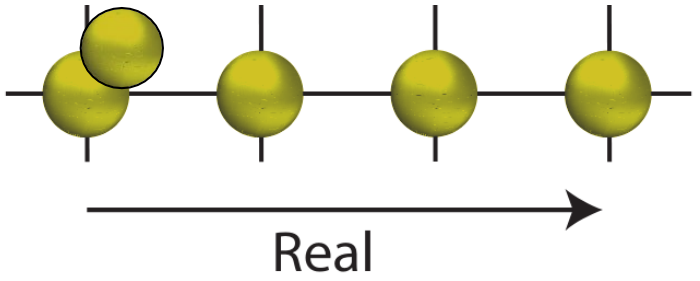
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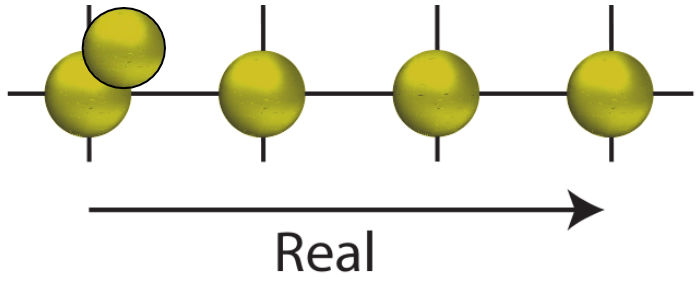


Ozawa & Price, Nat. Rev. Phys. (2019)

interactions along a **real** dimension

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- **typically diagonal in Fock space**

$$\hat{H}_{int} = \sum_{i,j} U_{ij}\, \hat{c}_j^{\dagger} \hat{c}_i^{\dagger} \hat{c}_j \hat{c}_i$$



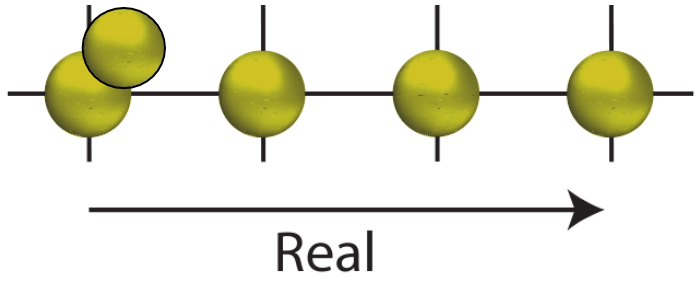
Ozawa & Price, Nat. Rev. Phys. (2019)

interactions along a **real** dimension

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- **typically diagonal in Fock space**
- **obeys discrete translation symmetry**

$$U_{i,j} = U_{i+nT,j+nT}$$

$$U_{1,1} = U_{4,4}$$



Ozawa & Price, Nat. Rev. Phys. (2019)

interactions along a **synthetic** dimension

- **typically local or short-ranged in space**

  → **not true in general!**

- **typically diagonal in Fock space**

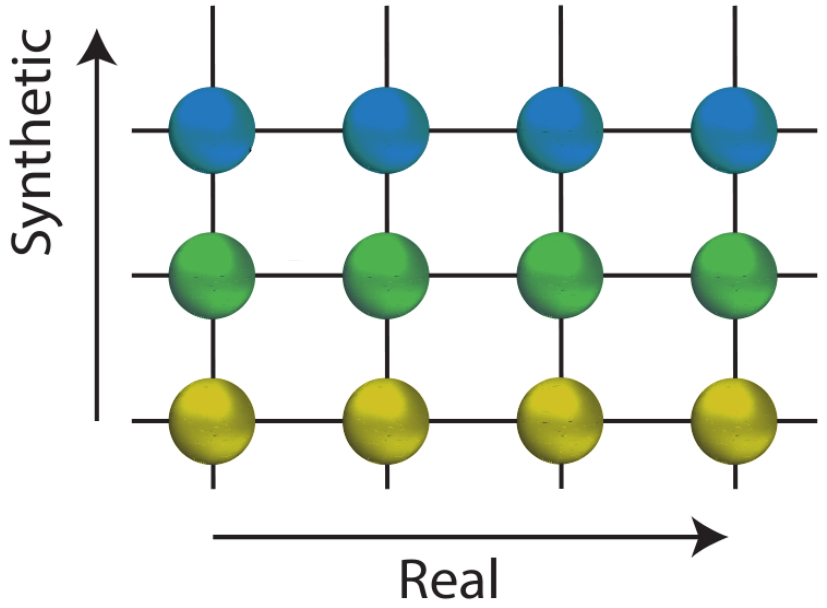
  → **yes, but also "spin-changing" (correlated pair hopping)!**

- **obeys discrete translation symmetry**

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The structure of interactions in synthetic space:

- complicates the mapping to certain canonical model
- presents the opportunity to explore some exotic phenomena



Ozawa & Price, Nat. Rev. Phys. (2019)

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Synthetic gauge fields in synthetic dimensions: interactions and chiral edge modes

Simone Barbarino[6,1], Luca Taddia[2,3], Davide Rossini[1], Leonardo Mazza[4,1] and Rosario Fazio[5,1]

Published 11 March 2016 • © 2016 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

New Journal of Physics, Volume 18, March 2016

Many-body localization due to random interactions

Piotr Sierant, Dominique Delande, and Jakub Zakrzewski
Phys. Rev. A **95**, 021601(R) – Published 9 February 2017



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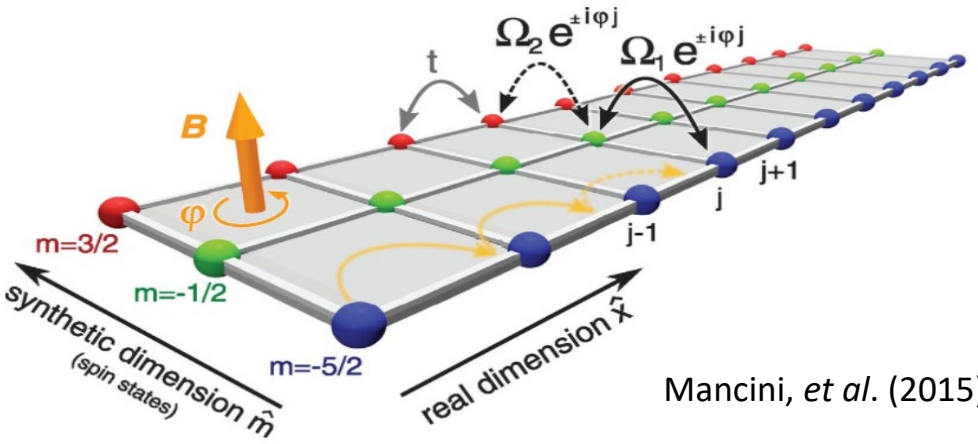
Ozawa & Price, Nat. Rev. Phys. (2019)

The structure of interactions in synthetic space:

- complicates the mapping to certain canonical model
- presents the opportunity to explore some exotic phenomena
- for non-diagonal (state-non-conserving) – be careful for the RWA…

“synthetic-space” interactions for **neutral atom spin states**

- **Interactions depend on intra-state and inter-state scattering lengths**



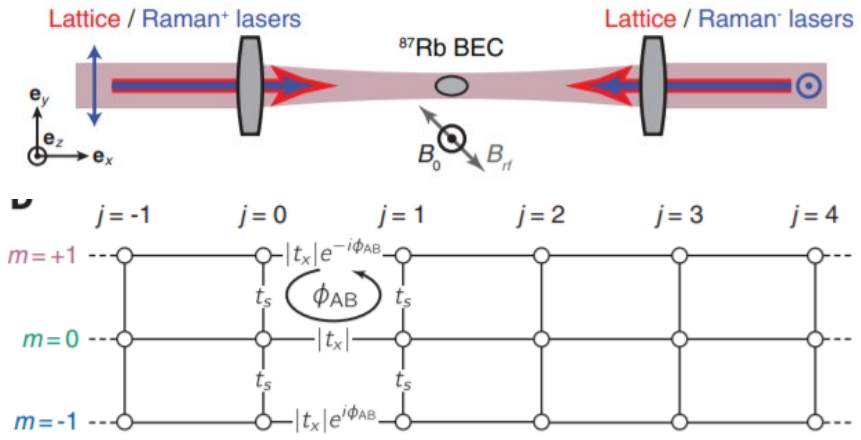
Mancini, *et al*. (2015)

alkaline-earth fermions – SU(N) symmetric atoms

- $U_{i,j}$ the same for all i , j (all-to-all, non-local)

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Stuhl, *et al*. (2015)

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rubidium 87 internal states
- almost completely all-to-all in internal space

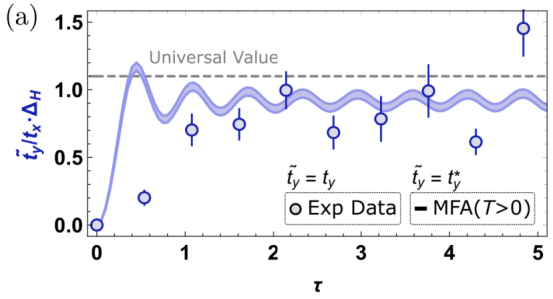
not generic, but the first systems realized in experiment

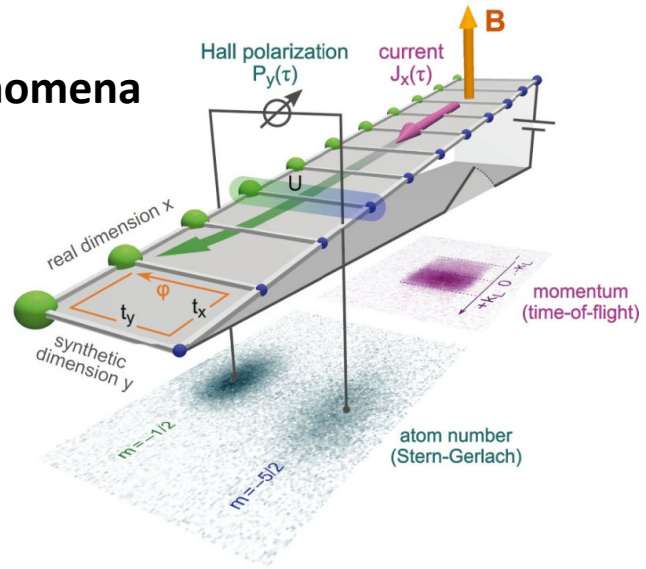
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Strongly correlated phenomena

T.-W. Zhou *et al*.
Science **381**, 427 (2023)





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- almost completely all-to-all in internal space

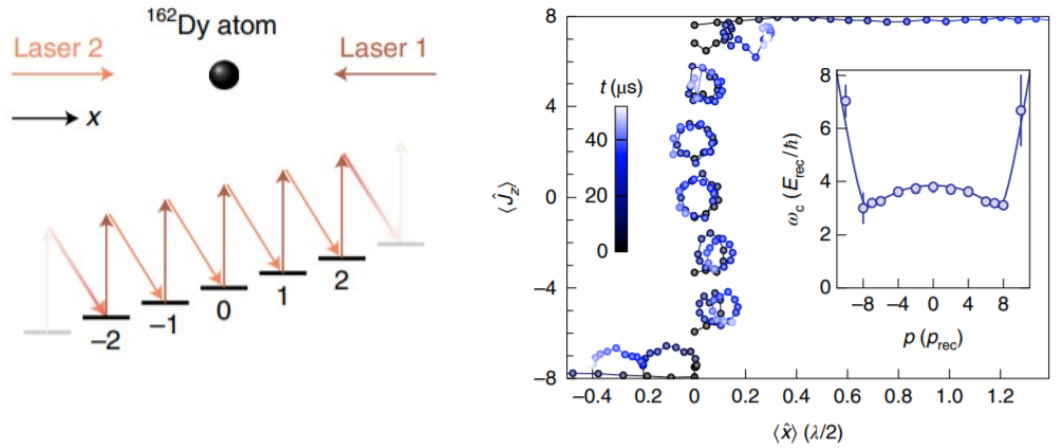
not generic, but the first systems realized in experiment

does not preclude the study of strongly correlated physics

see talk from Thierry!

“synthetic-space” interactions for **neutral atom spin states**

- **Interactions depend on intra-state and inter-state scattering lengths**



Chalopin, *et al.* (Nat. Phys., 2019)

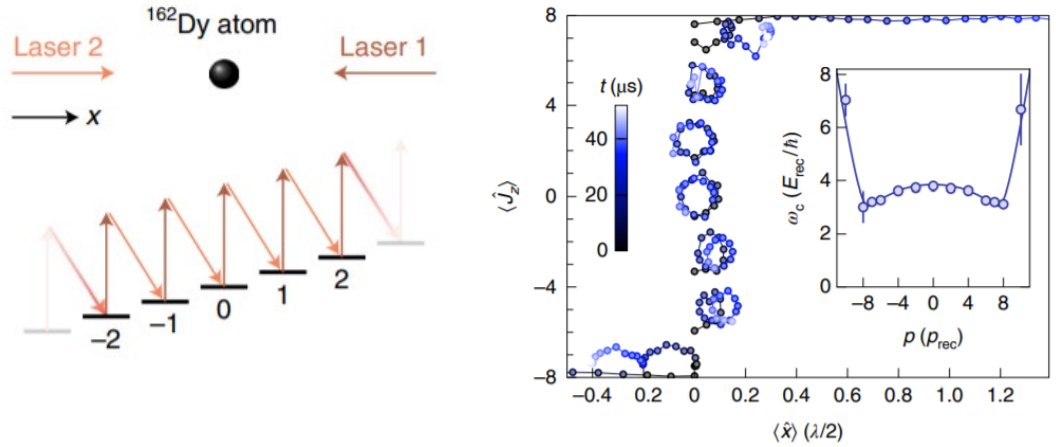
alkaline-earth fermions – SU(N) symmetric atoms
- $U_{i,j}$ the same for all i , j (all-to-all, non-local)

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slightly more generic: **dysprosium**

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1,1	1,2	1,3	.	.	.										
2,1	2,2														
3,1		.													
4,1			.												
.				.											
.															
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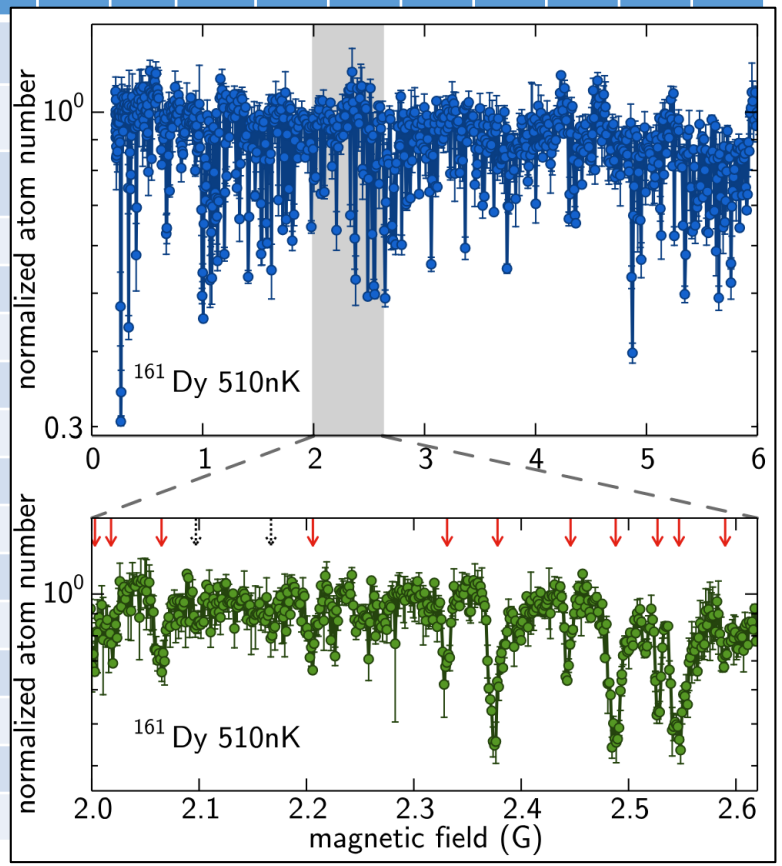
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← **“diagonal” terms – 153 unique values**

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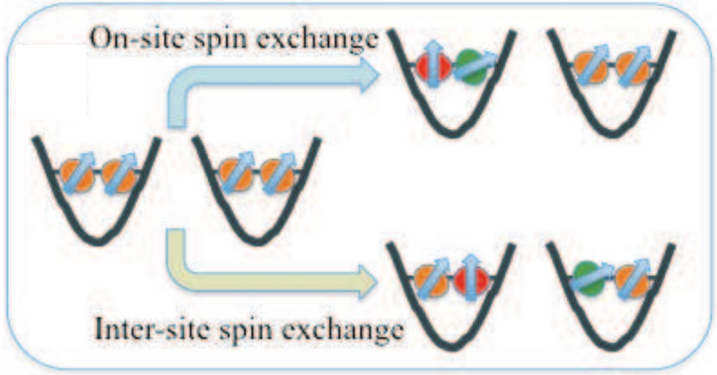
← **strong B-field dependencies**

Baumann, *et al*. PRA (2014)

“synthetic-space” interactions for **neutral atom spin states**

- **Interactions depend on intra-state and inter-state scattering lengths**

de Paz, *et al*. PRL (2014)



can also have strong state-changing collisions (non-diagonal terms)

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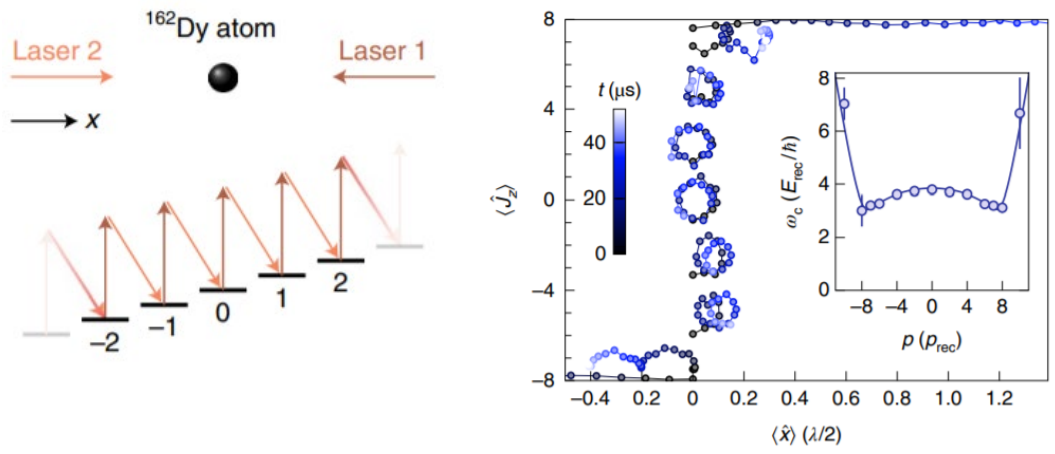
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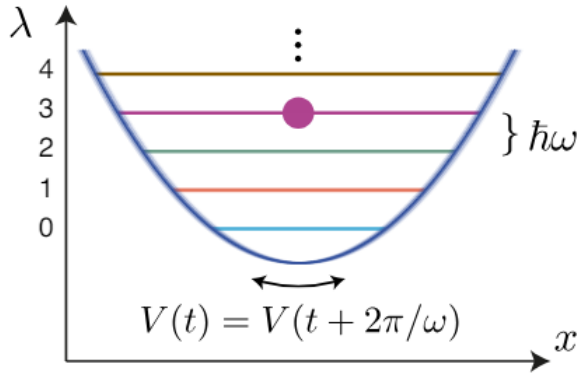
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“synthetic-space” interactions for **neutral atom motional states**

harmonic oscillator states

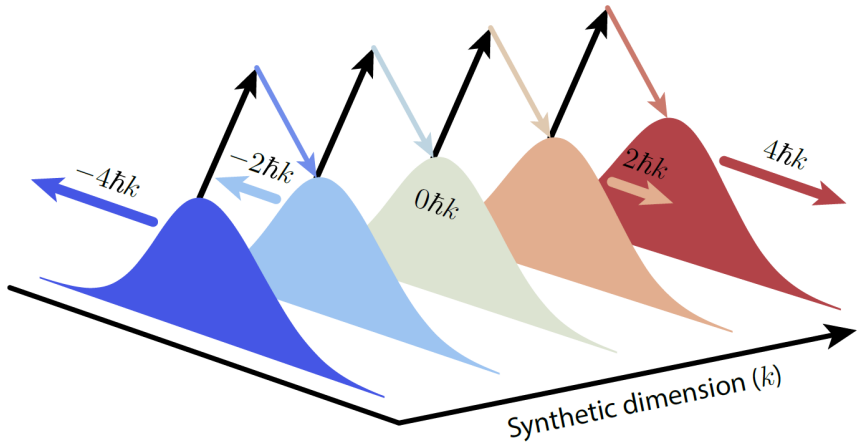


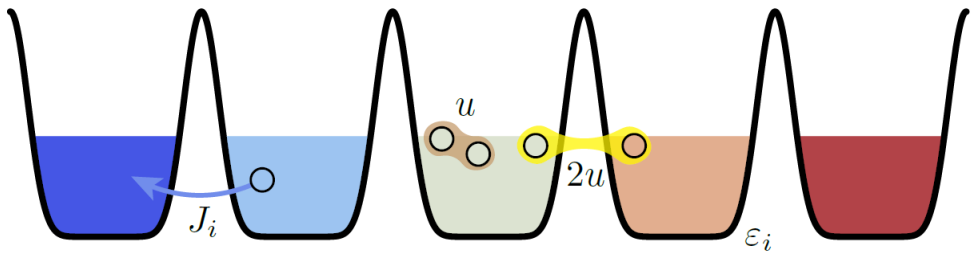
Proposal: Price, Ozawa, and Goldman, PRA (2017)
Experiment: Oliver, *et al*. Phys. Rev. Research (2023)

- unlike internal states, regular/structured interactions
- λ scaling due to changing size of orbitals
  [i.e., not translationally invariant]
- state-changing (non-diagonal) collisions allowed,
  and resonant

“synthetic-space” interactions for **neutral atom motional states**

momentum states

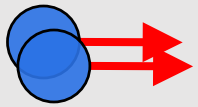




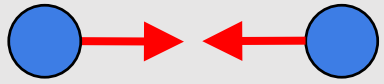
$$H_{int} = -(u/2)\sum_i n_i^2$$

effectively attractive & local in momentum space

- completely translation invariant
- can typically ignore (non-resonant) state-changing collisions
- short-ranged in real space, interactions should be long-ranged in momentum space



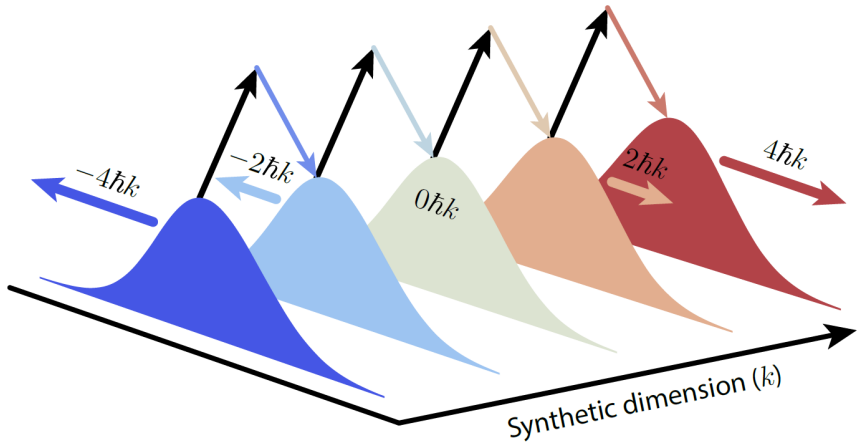
$i = j$: repulsive interaction u

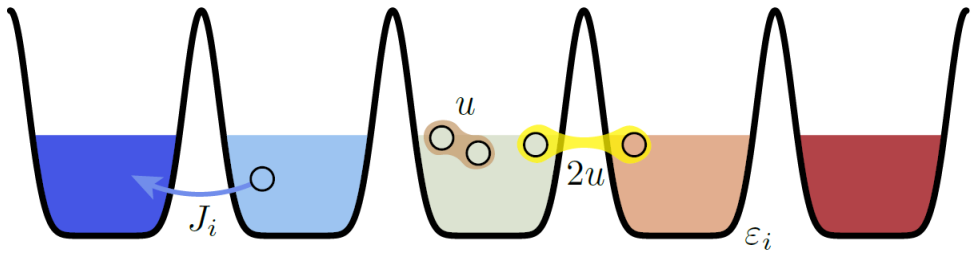


$i \neq j$: 2× repulsive interaction, $2u$
→ **additional “exchange energy”**

“synthetic-space” interactions for **neutral atom motional states**

momentum states



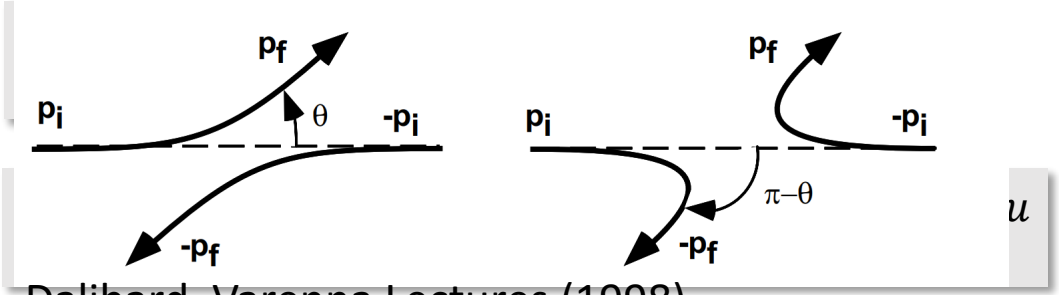


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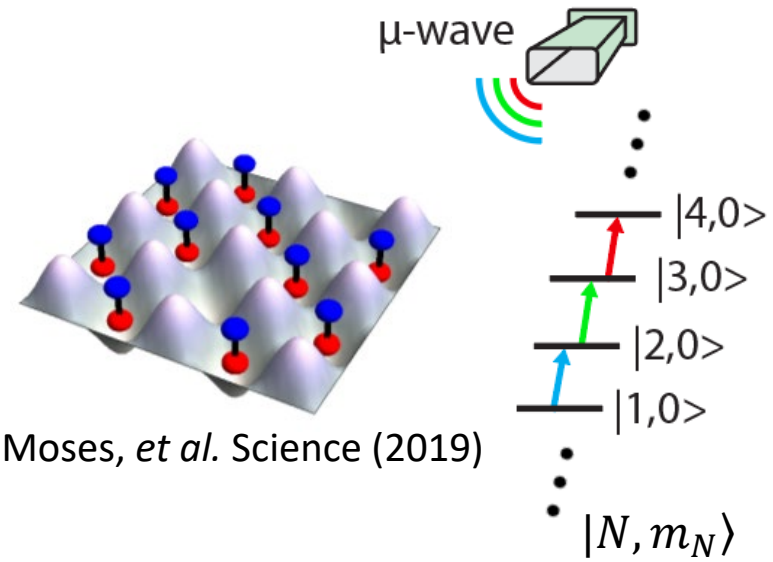
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JEAN DALIBARD

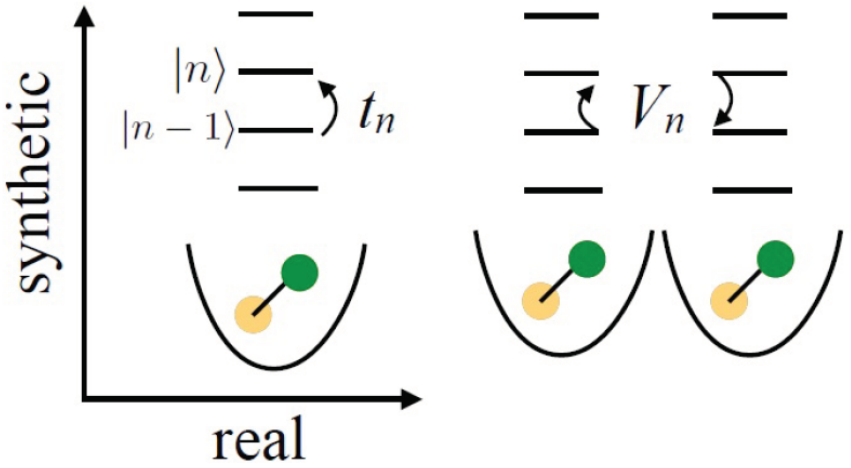


Dalibard, Varenna Lectures (1998)

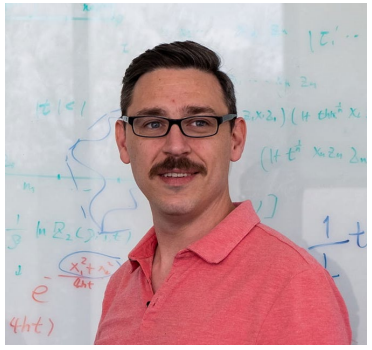
"synthetic-space" interactions for **molecules & Rydberg atoms**



Moses, *et al.* Science (2019)



Really a dipolar spin system (hard-core bosons, one occupancy per spatial trap)



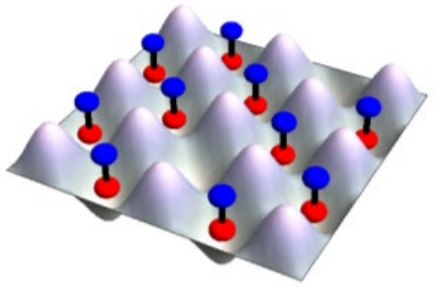
Kaden Hazzard

Sundar, BG, and Hazzard, Scientific Reports (2018)
Sundar, Thibodeau, Wang, BG, and Hazzard, PRA (2019)

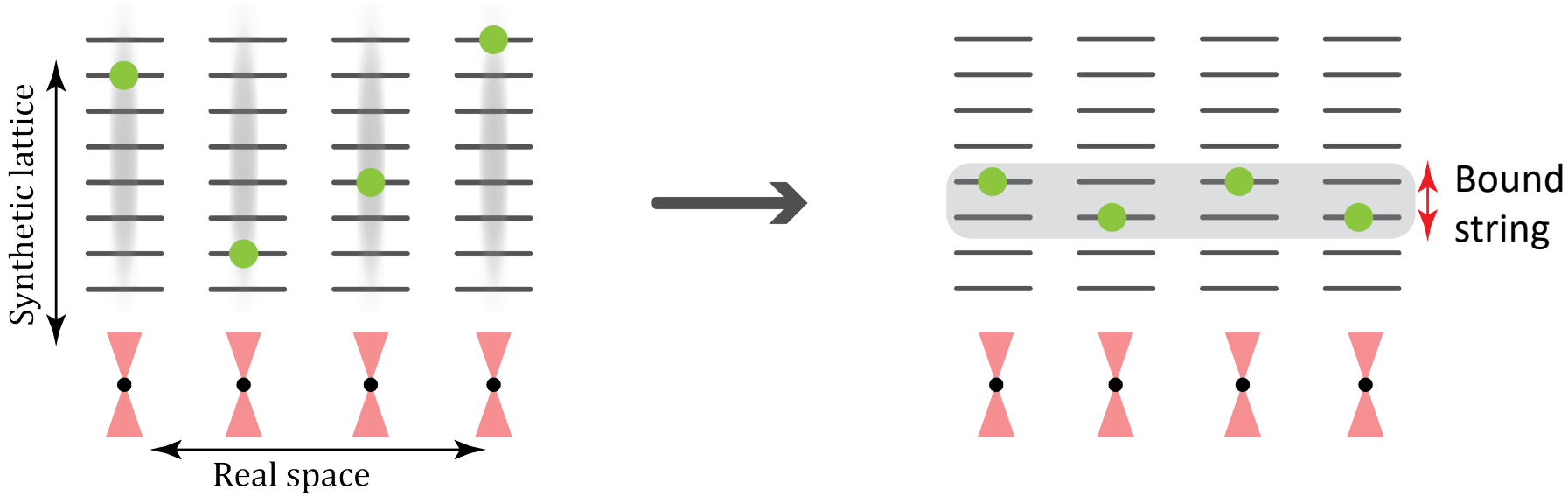
off-diagonal, but overall state-conserving

- $1/r^3$ in real space, so particles interact even if at distinct locations
- can typically ignore all but the resonant **flip-flop exchange terms**
- short-ranged in synthetic space, but not translationally invariant [structured but not "random"], can calculate

“synthetic-space” interactions for **molecules & Rydberg atoms**



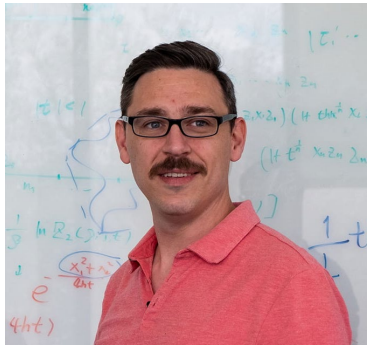
Moses, *et al.* Science (2019)



$0 < V_{dd} < 2t$

$V_{dd} > 2t$

some new many-body physics predicted for most generic “synthetic lattice”



Kaden Hazzard

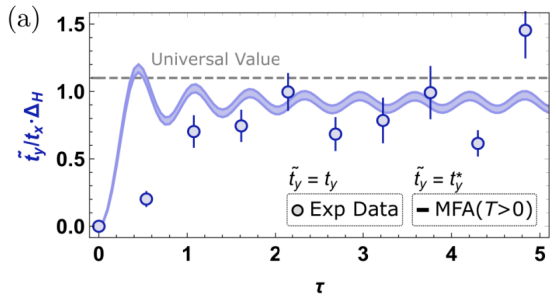
Sundar, BG, and Hazzard, Scientific Reports (2018)
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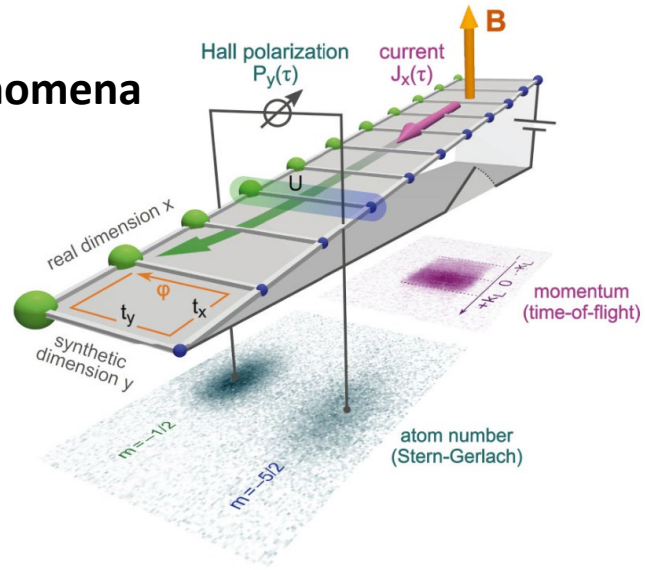
role of interactions in synthetic dimensions experiments

internal state synthetic dimensions: mostly single-particle physics, but significant recent progress

Strongly correlated phenomena

T.-W. Zhou *et al*.
Science **381**, 427 (2023)

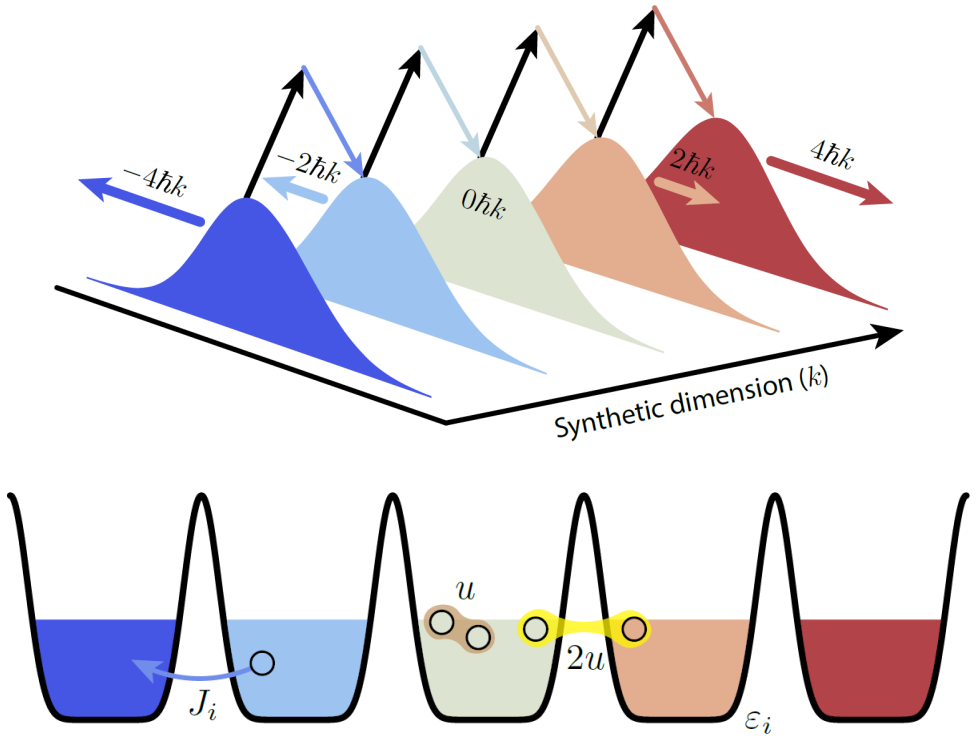


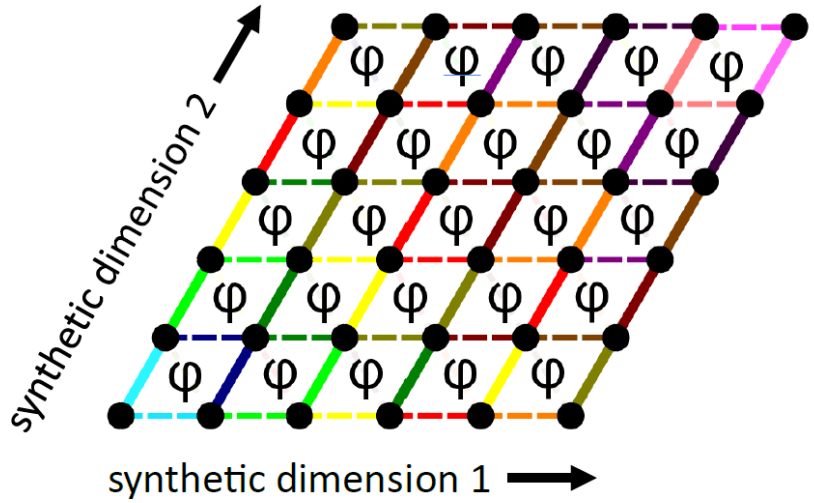


see talk from Thierry!

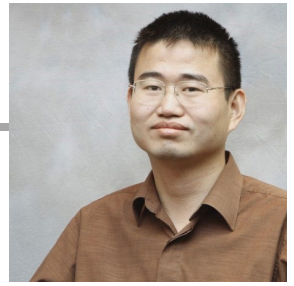
role of interactions in synthetic dimensions experiments

“momentum state lattices” – significant collective interactions (bulk gases)





collective nonlinearities in momentum state lattices



Chuanwei Zhang

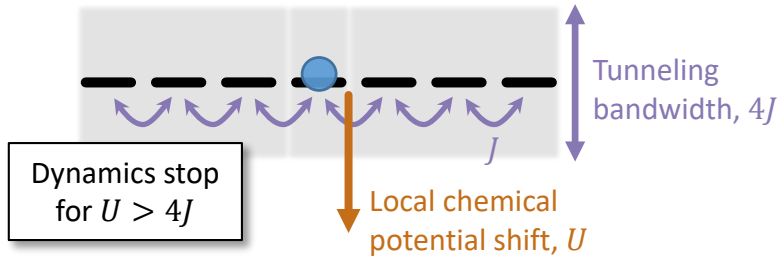


Kaden Hazzard



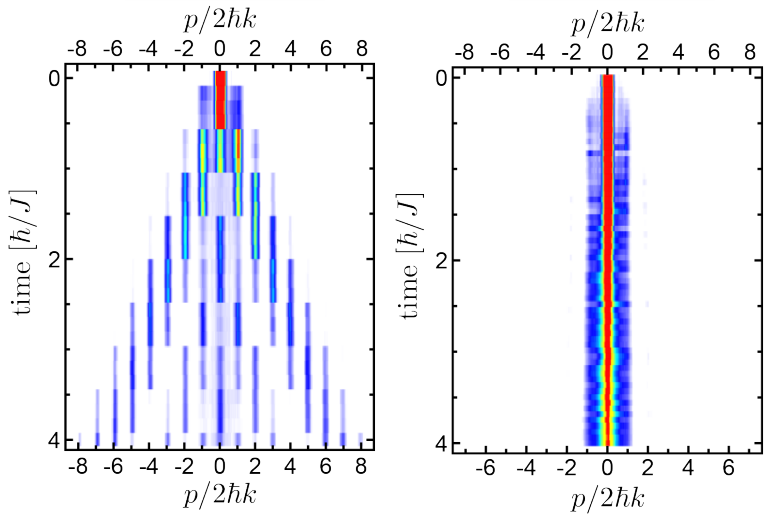
Fangzhao Alex An

Simplest system: single-band 1D lattice (flat)



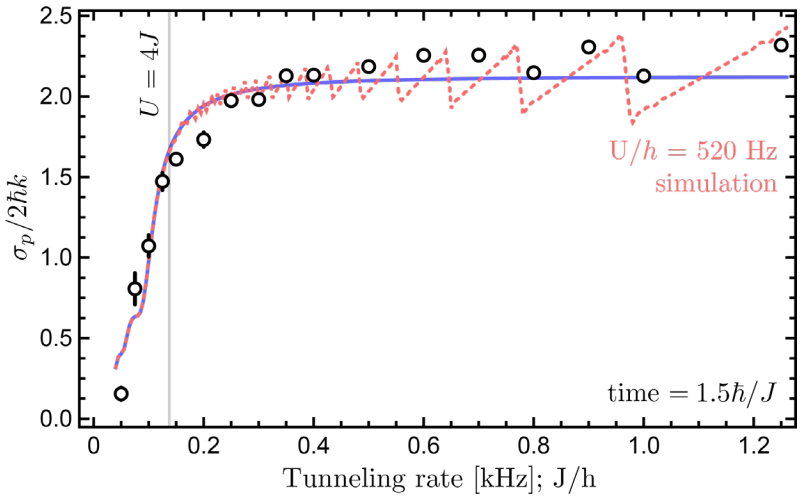
$U < J$ limit

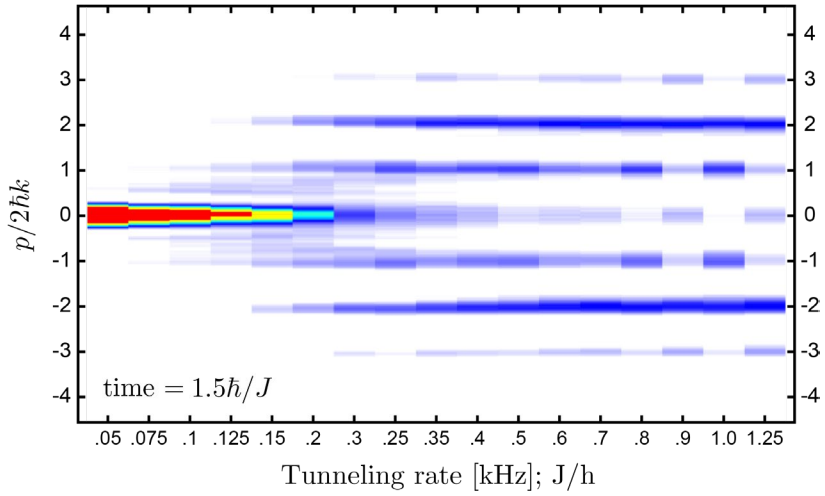
$U \gg J$ limit



Dispersive spreading

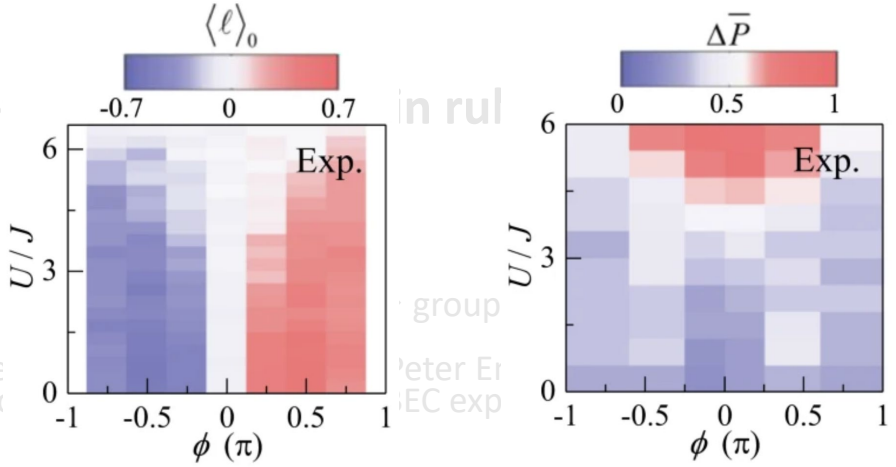
Self-trapping



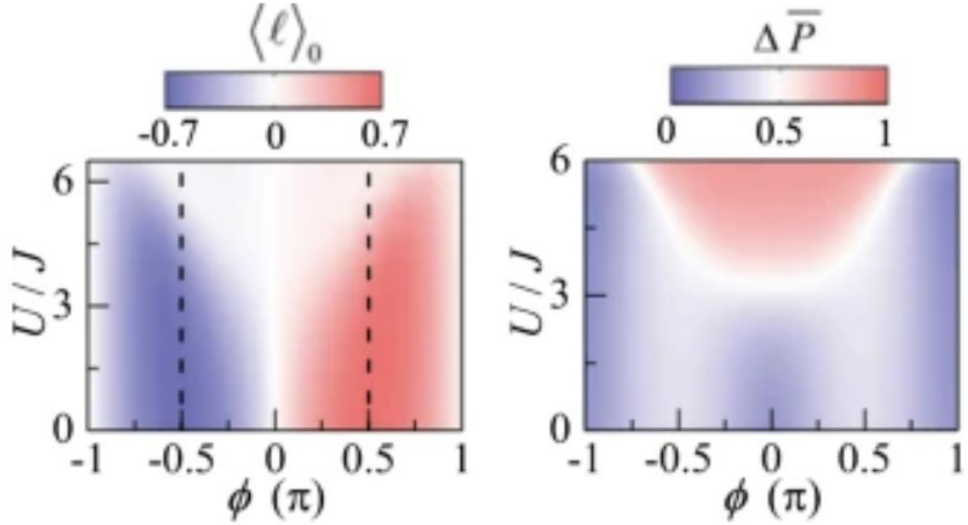


An, *et al*. PRL (2021)
[also, An, *et al*. PRL (2018)]

collective nonlinearities in momentum state lattices

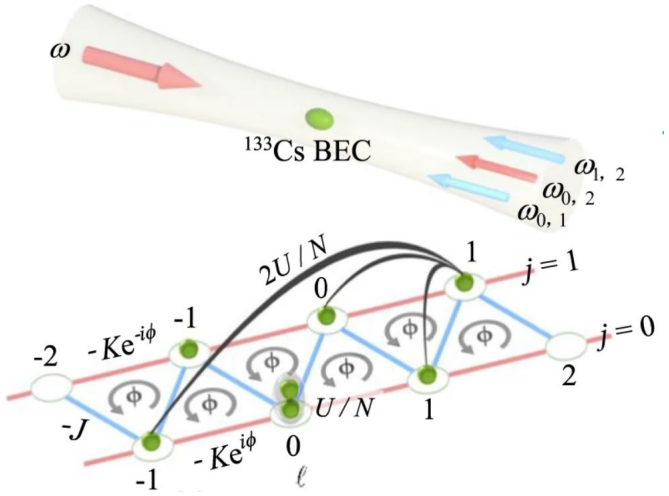


flux-dependent self-trapping



recently: Feshbach control w/ Cs

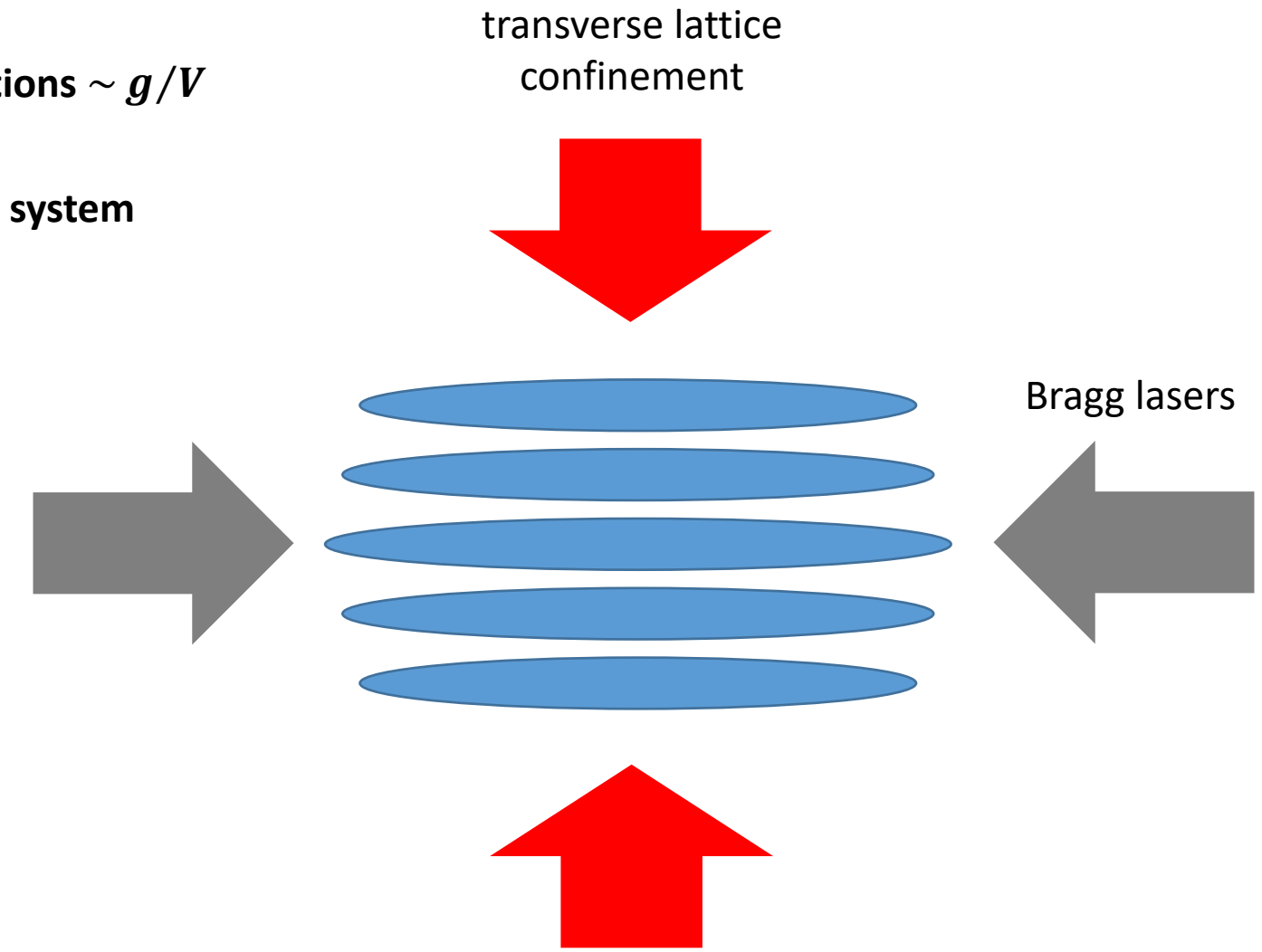
group of Suotang Jia @ Shanxi University



Yuqing Li, *et al*. Nat. Comm. (2023)

how to go beyond mean-field (classical) effects?

- **increase the scale of pair-wise interactions $\sim g/V$**
- **hybrid real-space & momentum-space system [maybe into the Hubbard regime?]**



how to go beyond mean-field (classical) effects?

- **just look more carefully**



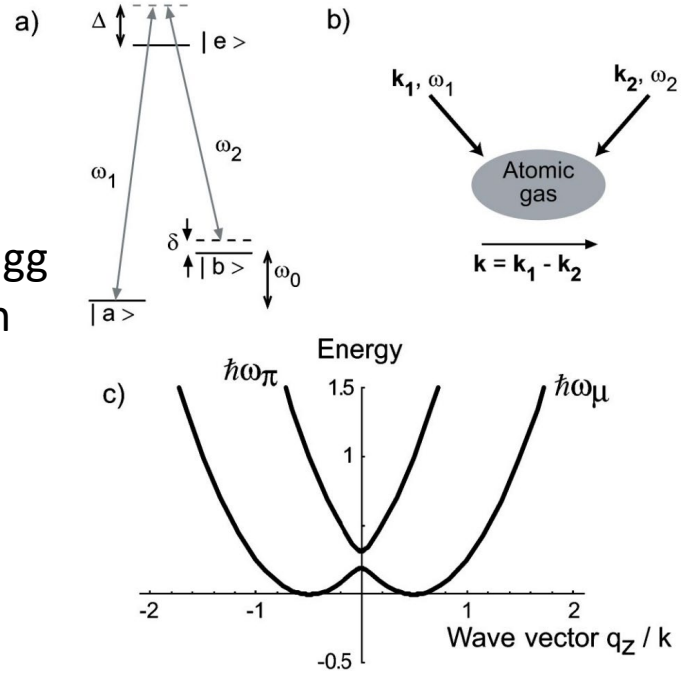
looking for beyond-mean-field effects

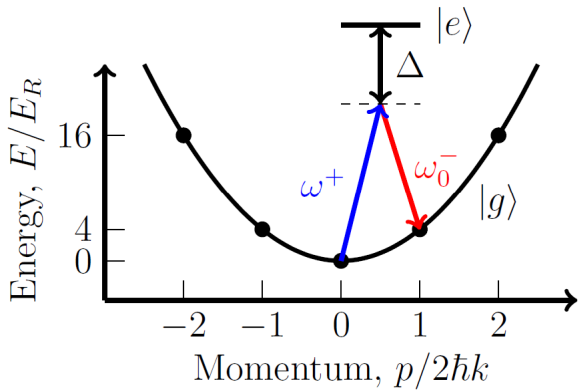
PHYSICAL REVIEW A **69**, 053605 (2004)

Generating macroscopic-quantum-superposition states in momentum and internal-state space from Bose-Einstein condensates with repulsive interactions

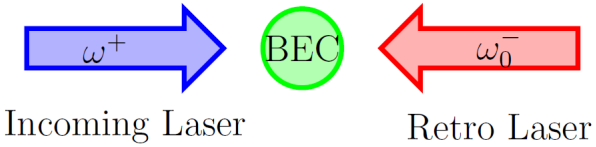
J. Higbie and D. M. Stamper-Kurn
Department of Physics, University of California, Berkeley, California 94720, USA
(Received 23 October 2003; published 7 May 2004)

Raman-Bragg transition





Bragg transition



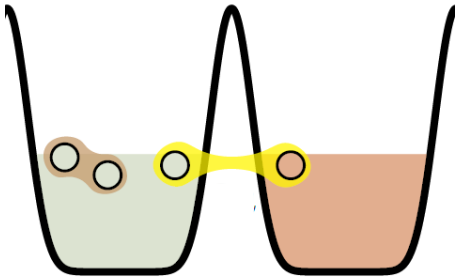
- prediction of interaction-based squeezing
- Raman-dressed spin-orbit coupling

looking for beyond-mean-field effects

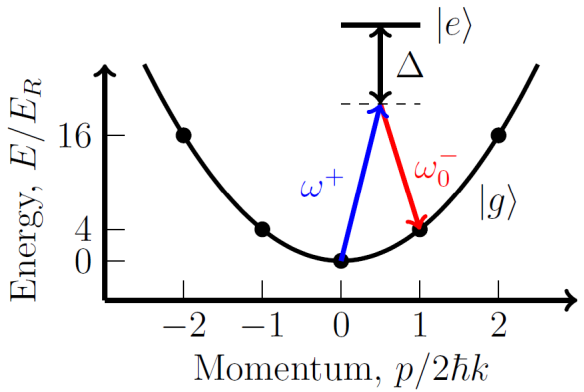
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mode 0 mode 1



Bragg transition

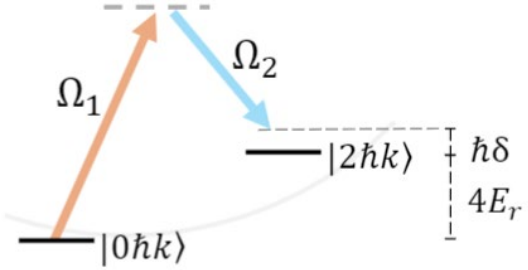


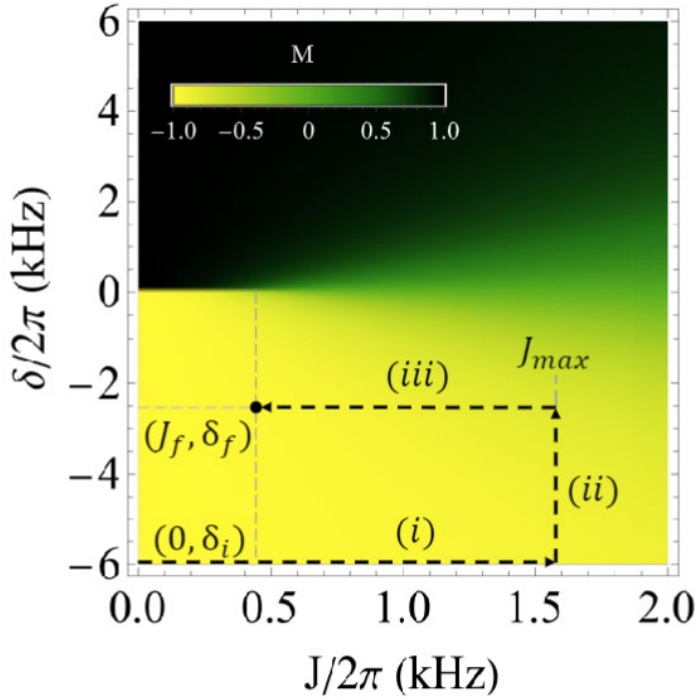
$$H_{\mathrm{sq}} = \kappa \hat{J}_z^2 \qquad \kappa = (u_{00} + u_{11})/2 - u_{01}$$

An, *et al*. Phys. Rev. Lett. (2018)

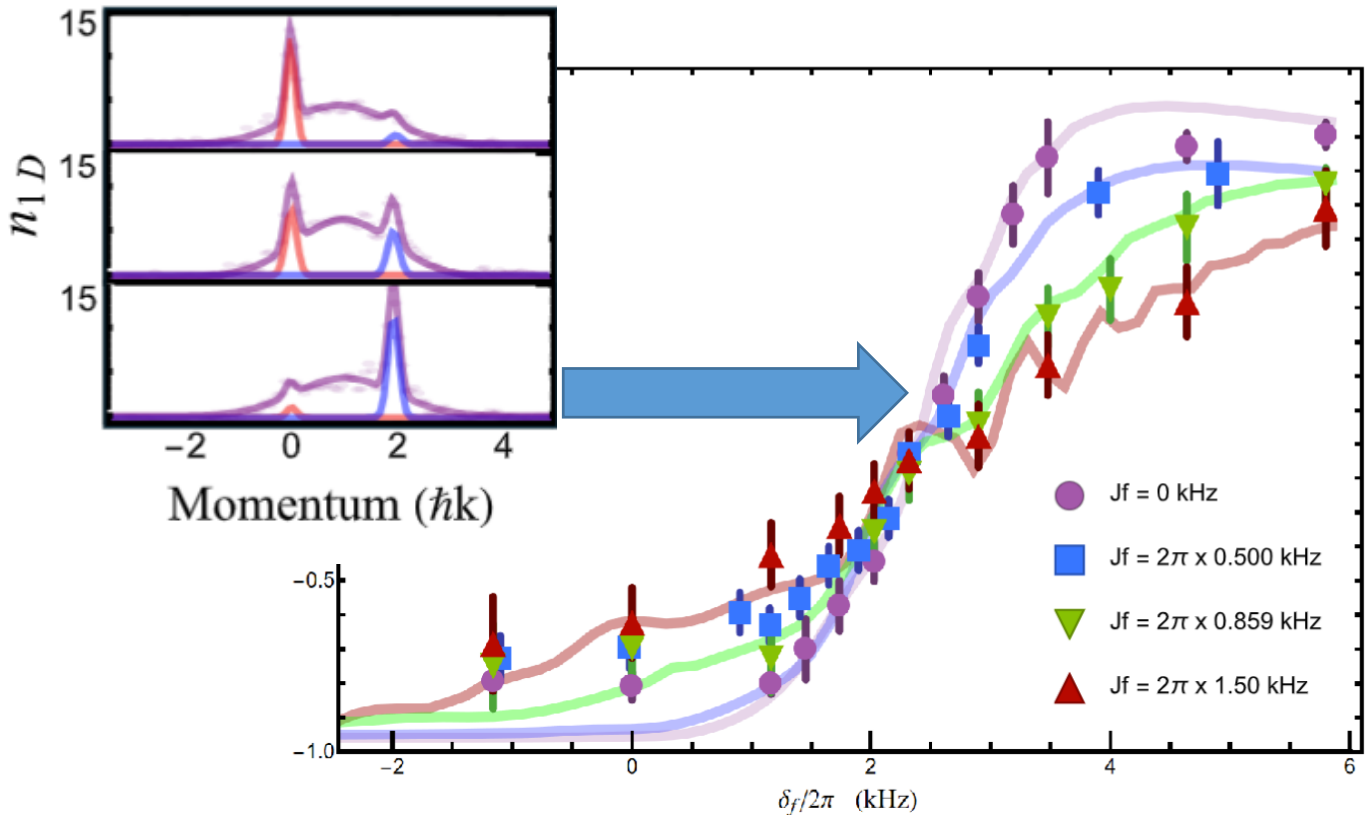
optimal squeezing time of $\boldsymbol{\tau_{sq} \approx 1}$ **ms** for typical scattering lengths and atom numbers

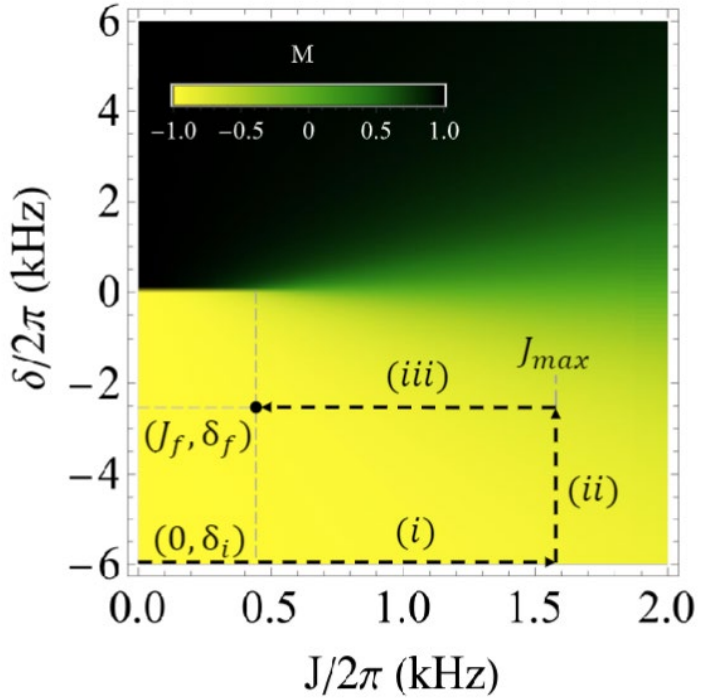
adiabatic state preparation – “pseudomagnetic” response



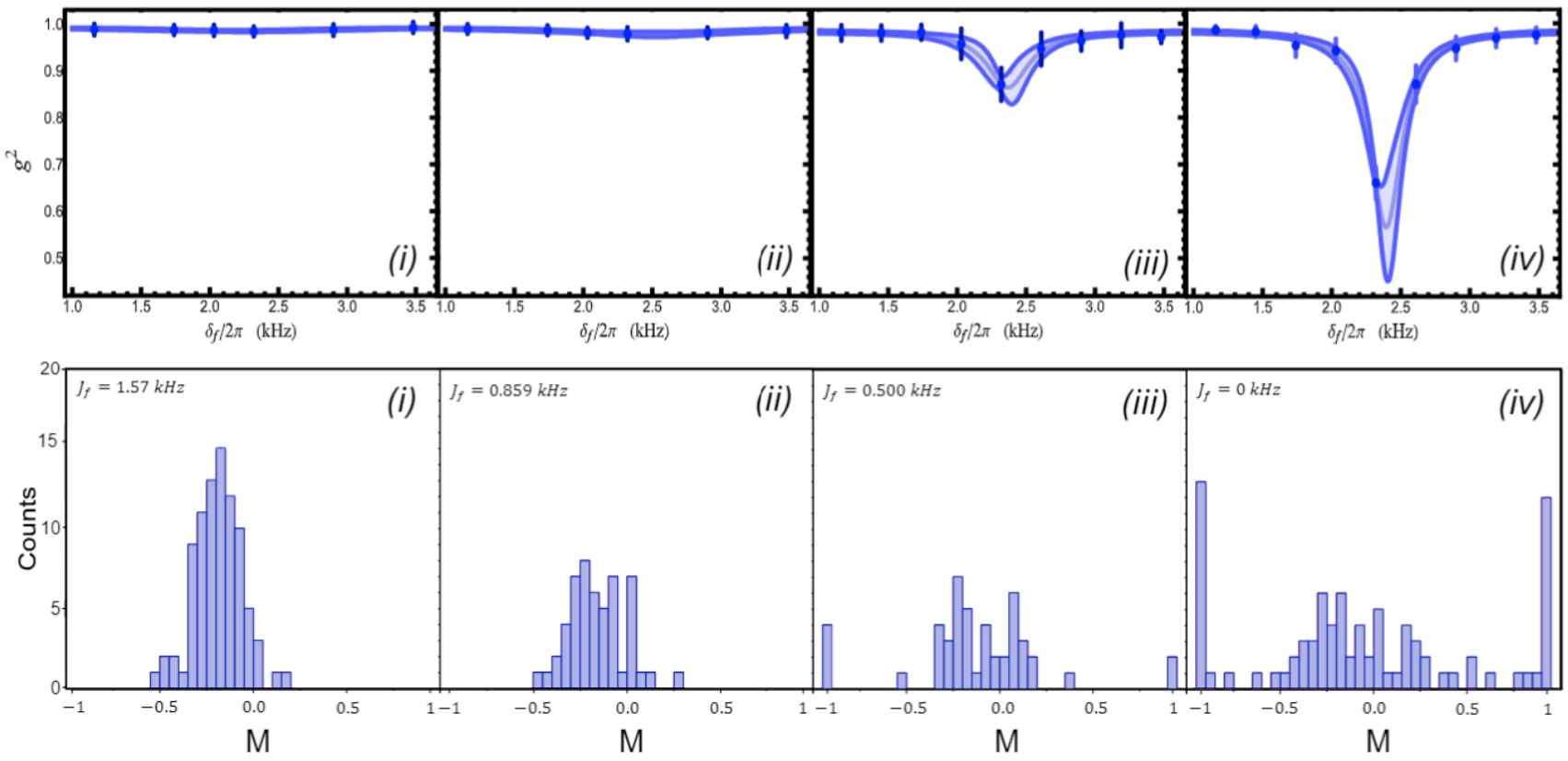


adiabatic state preparation – “pseudomagnetic” response



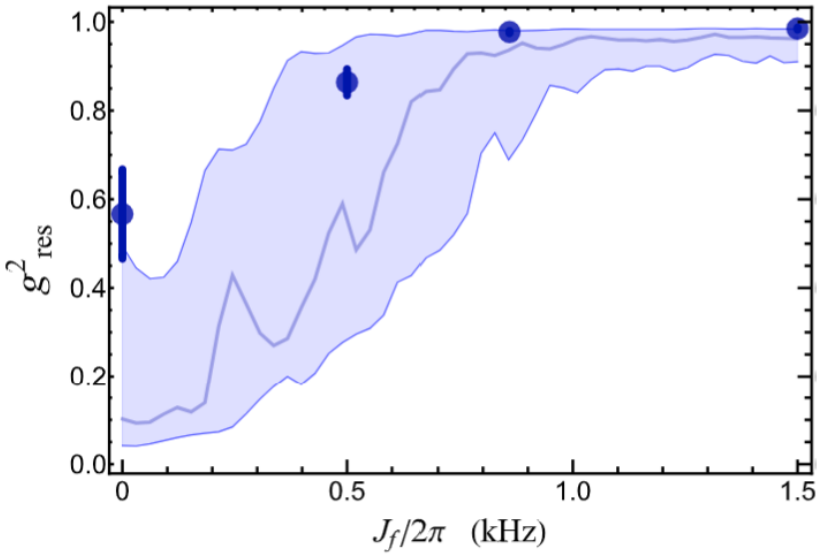


adiabatic state preparation – "pseudomagnetic" response



increasing U/J

adiabatic state preparation – "pseudomagnetic" response



comparison to naive expected response (single-mode ED simulations)

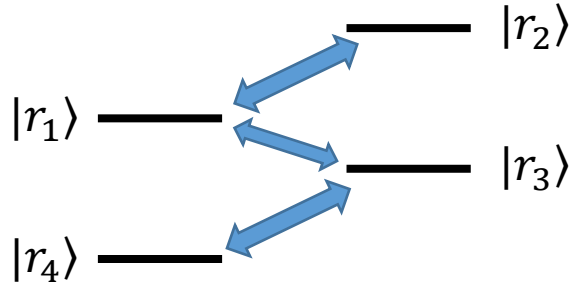
- **suggestive of momentum state "squeezing" (<u>further measurements needed</u> to prove this)**

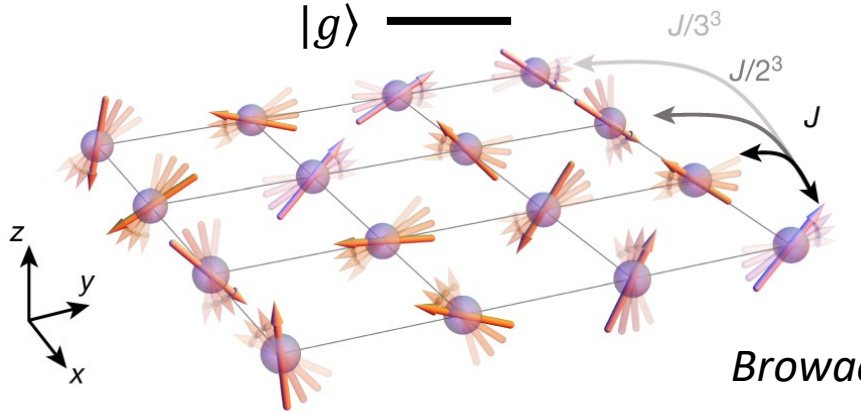
role of interactions in synthetic dimensions experiments

Rydberg state synthetic lattices

start with strong interactions – essentially a spin system

Heisenberg models
in **Rydberg-Rydberg** experiments





Browaeys group

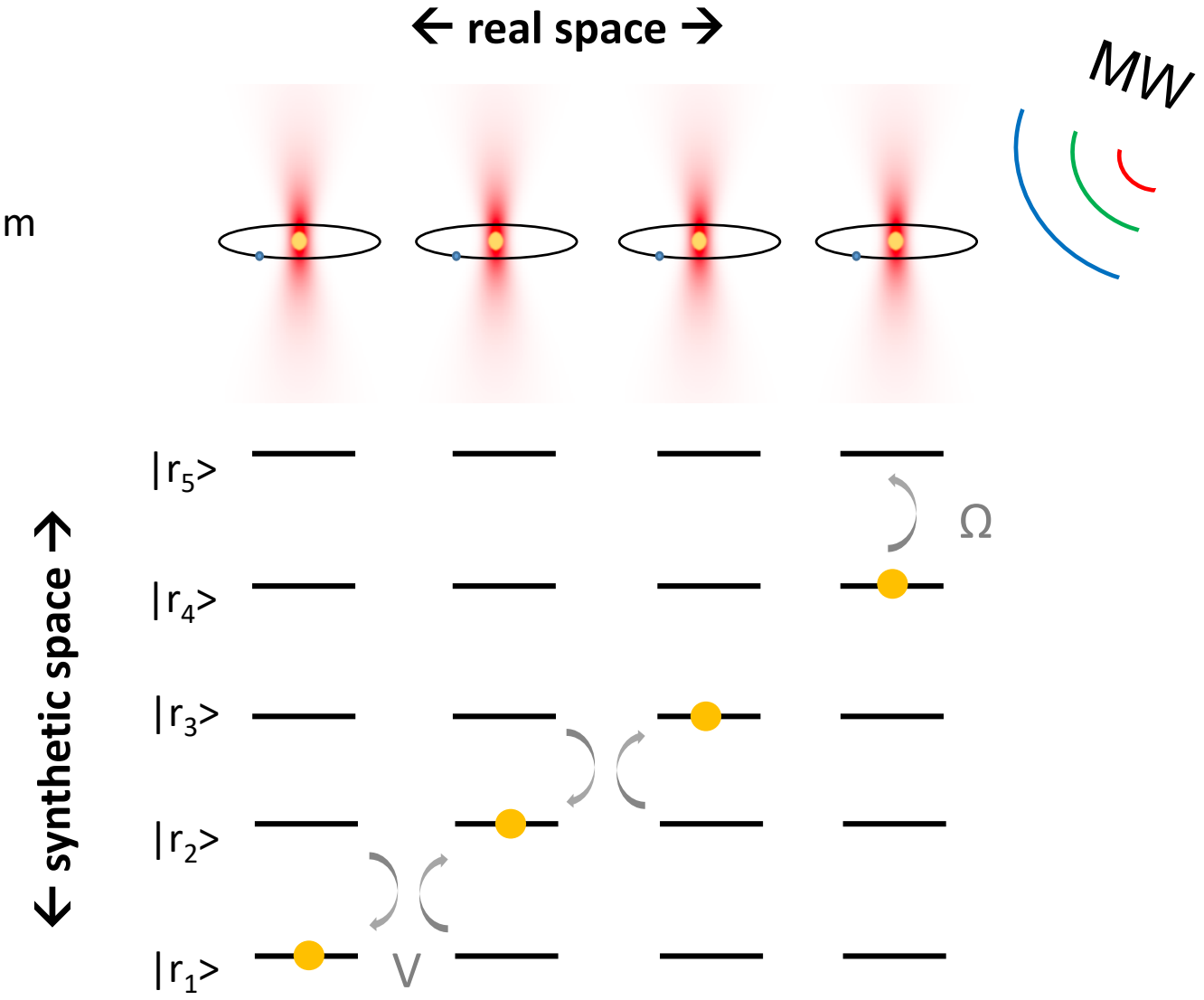
C. Chen, G. Bornet, M. Bintz *et al. Nature* **616**, 691–695 (2023).

role of interactions in synthetic dimensions experiments

Rydberg state synthetic lattices

start with strong interactions – essentially a spin system

the V and Ω terms both tunable and on MHz scale



role of interactions in synthetic dimensions experiments

Rydberg state synthetic lattices

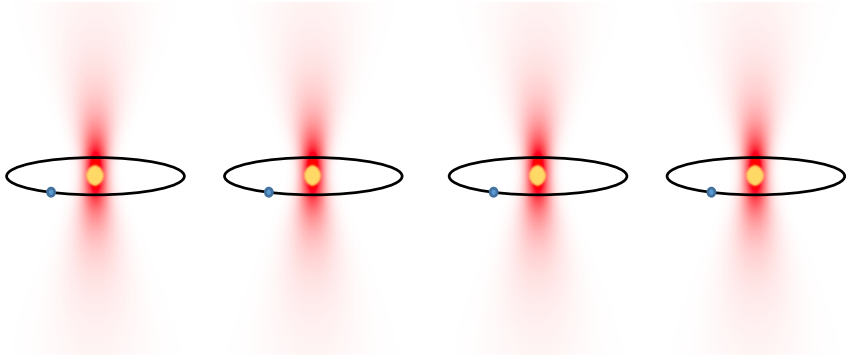
start with strong interactions – essentially a spin system

Ivan Velkovsky Dr. Tao Chen

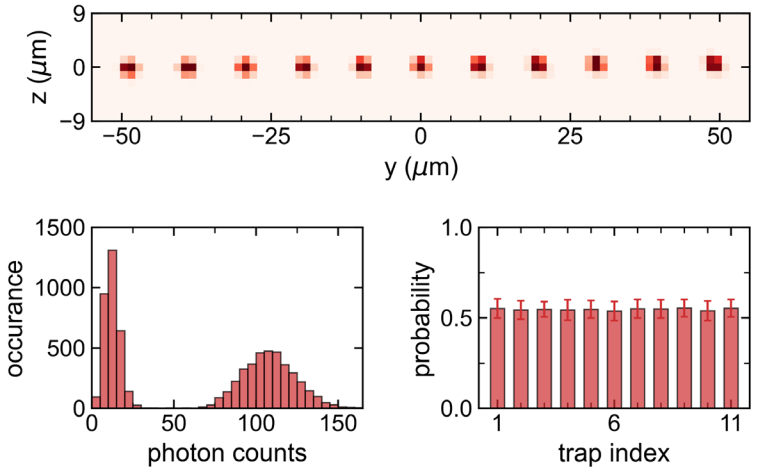


Chenxi Huang Jackson Ang'ong'a (alum, Tubingen)

← real space →



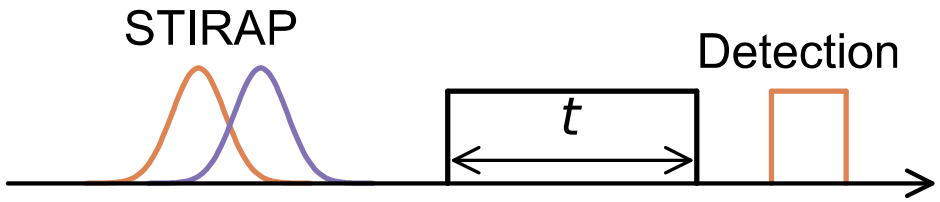
our experiments - ^{39}K atoms



role of interactions in synthetic dimensions experiments

Rydberg state synthetic lattices

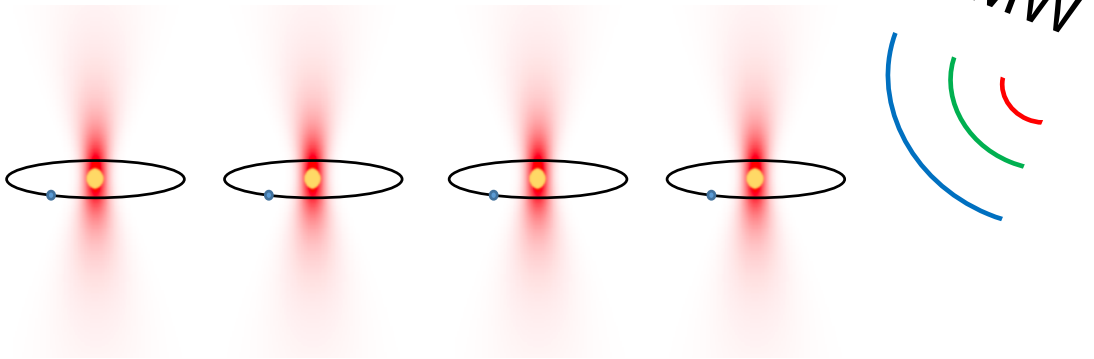
“science” portion of experiment



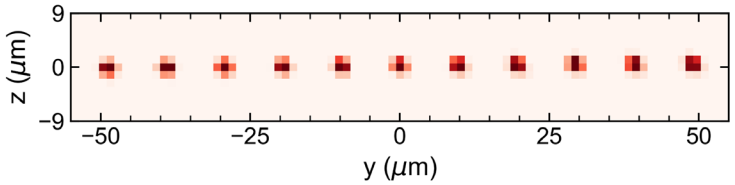
microwave-controlled dynamics in the synthetic space

STIRAP transfer (94% efficiency) to 42s $m_J = +1/2$ state

← real space →



our experiments - ^{39}K atoms



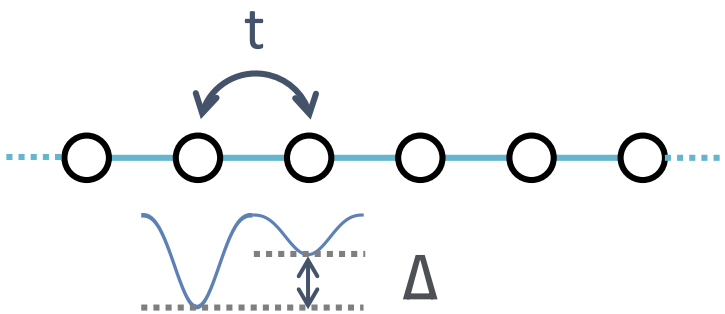
technical limit:
can’t remove cases where atoms are not excited to the Rydberg state

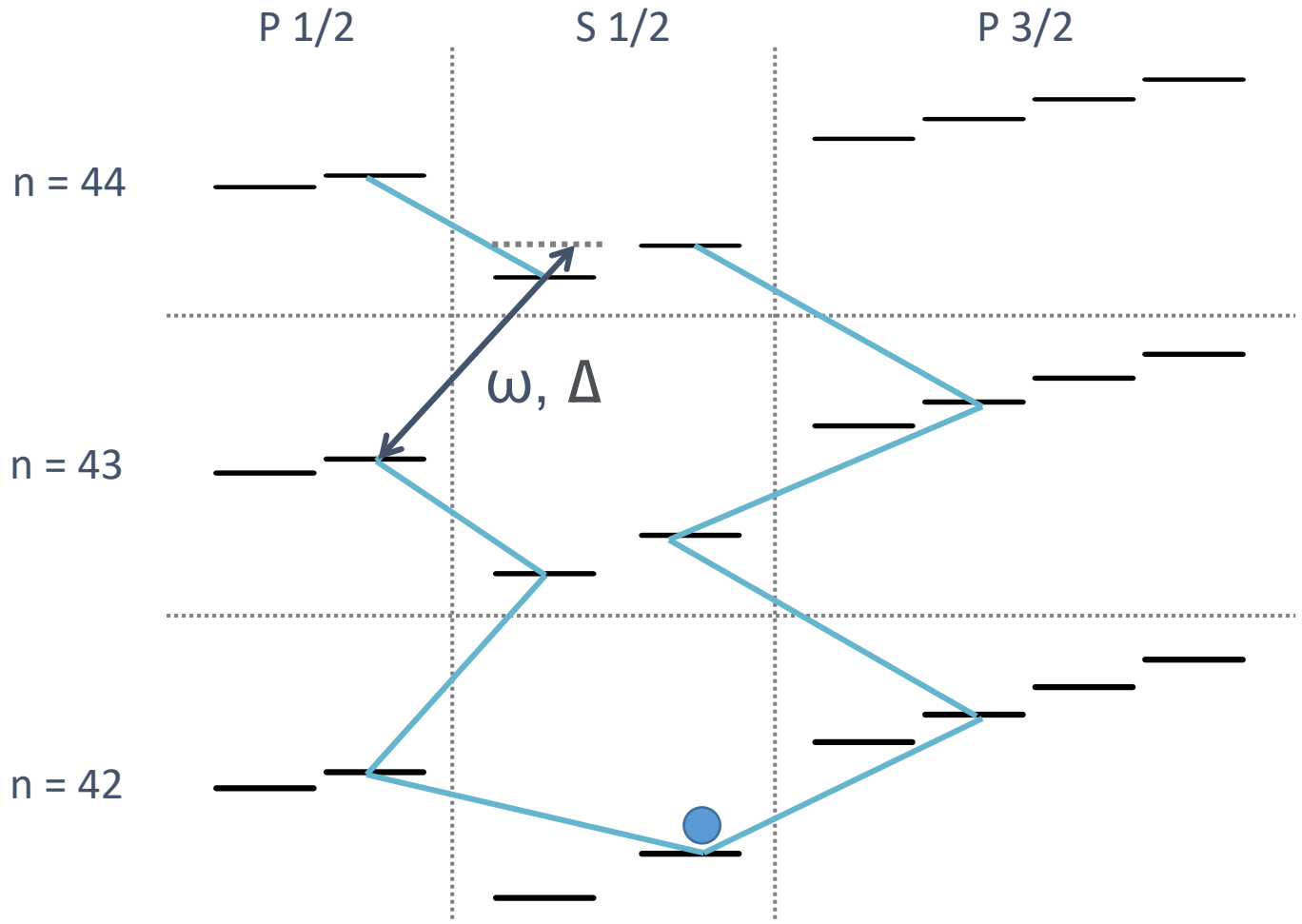
→ studies of few-body systems (1-6 atoms)

Rydberg synthetic lattices

see also work from Killian group:

S.K. Kanungo, *et al.* *Nat Commun* **13**, 972 (2022) ; Lu. *et al*. PRA **109**, 032801 (2024)

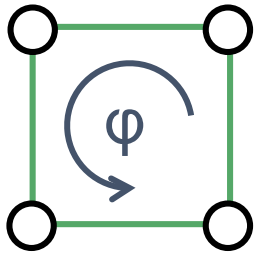


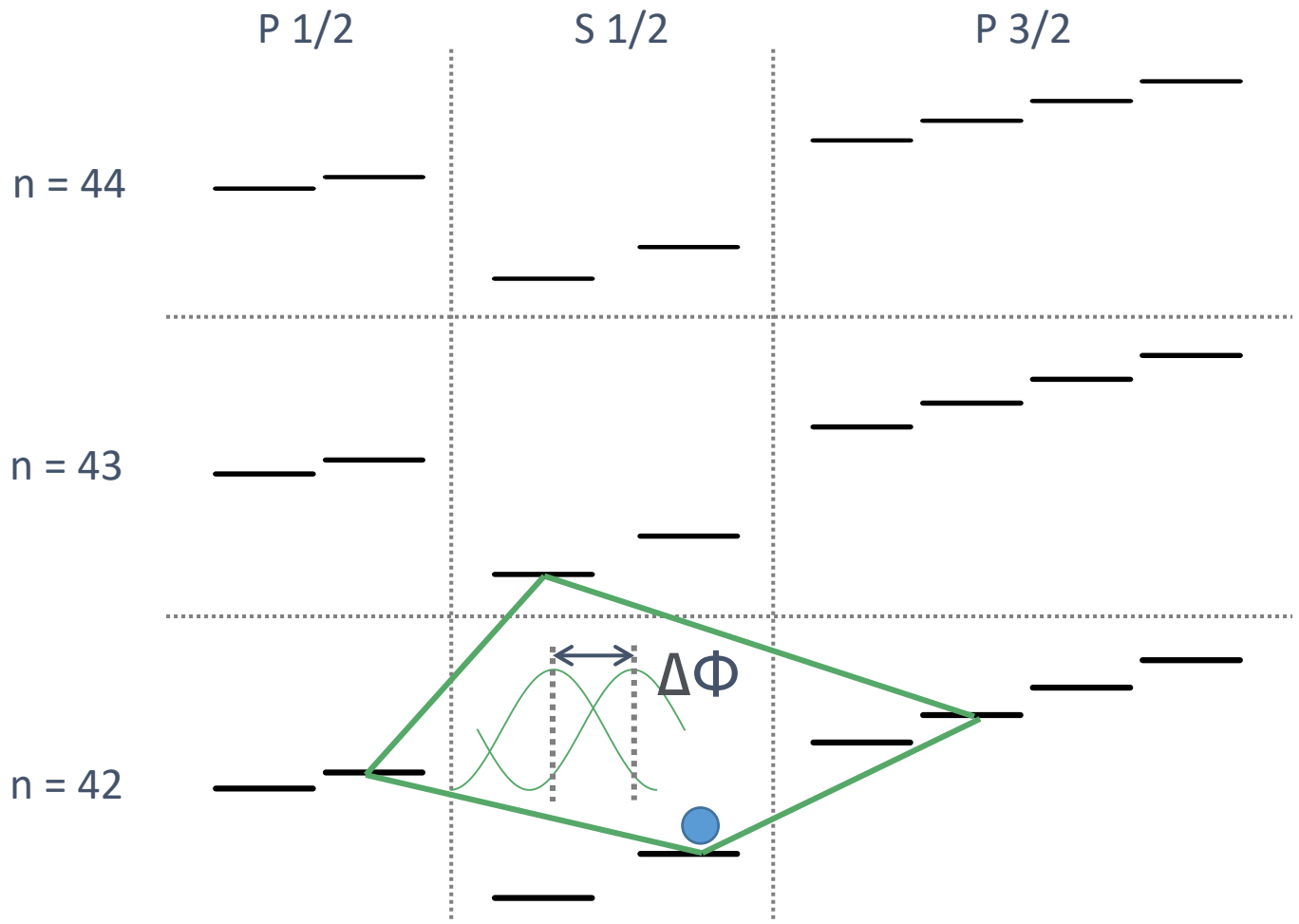


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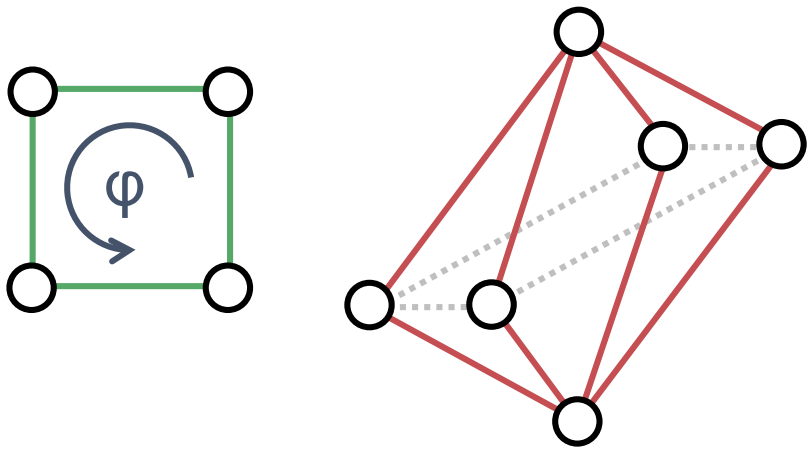


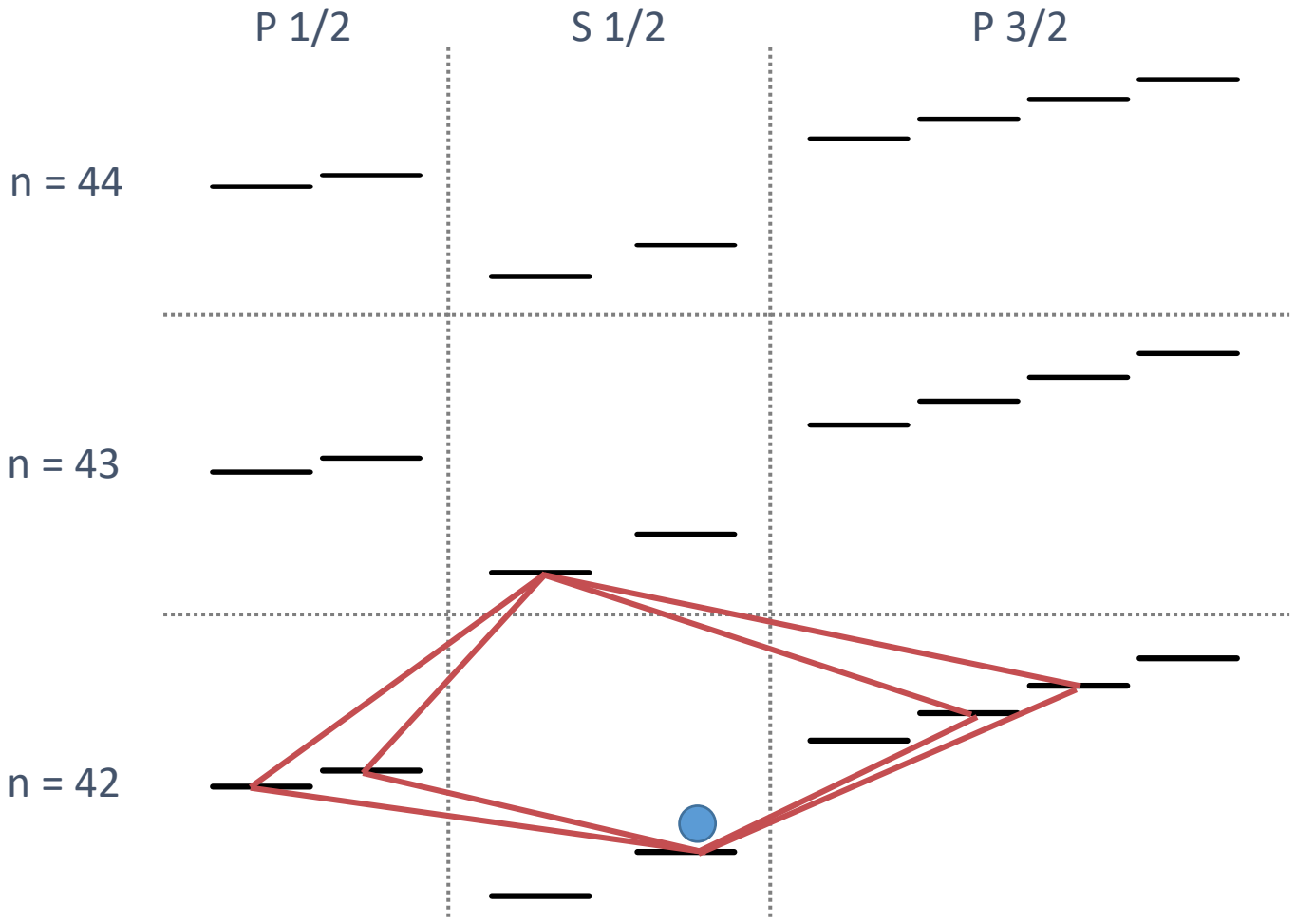


Rydberg synthetic lattices

see also work from Killian group:

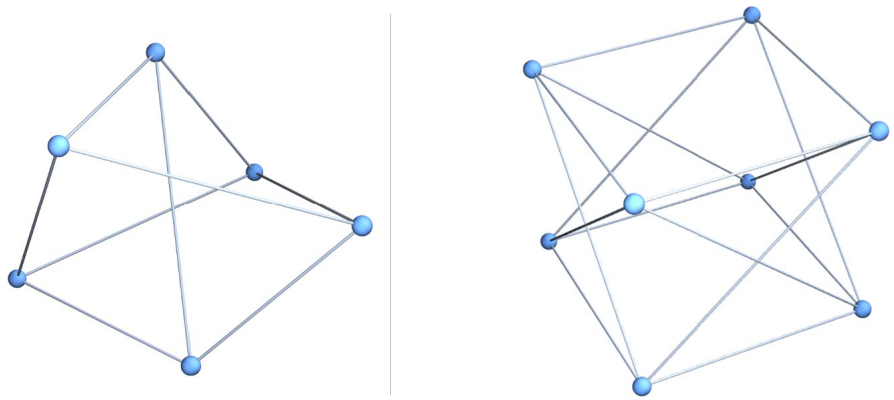
S.K. Kanungo, *et al.* *Nat Commun* **13**, 972 (2022) ; Lu. *et al*. PRA **109**, 032801 (2024)

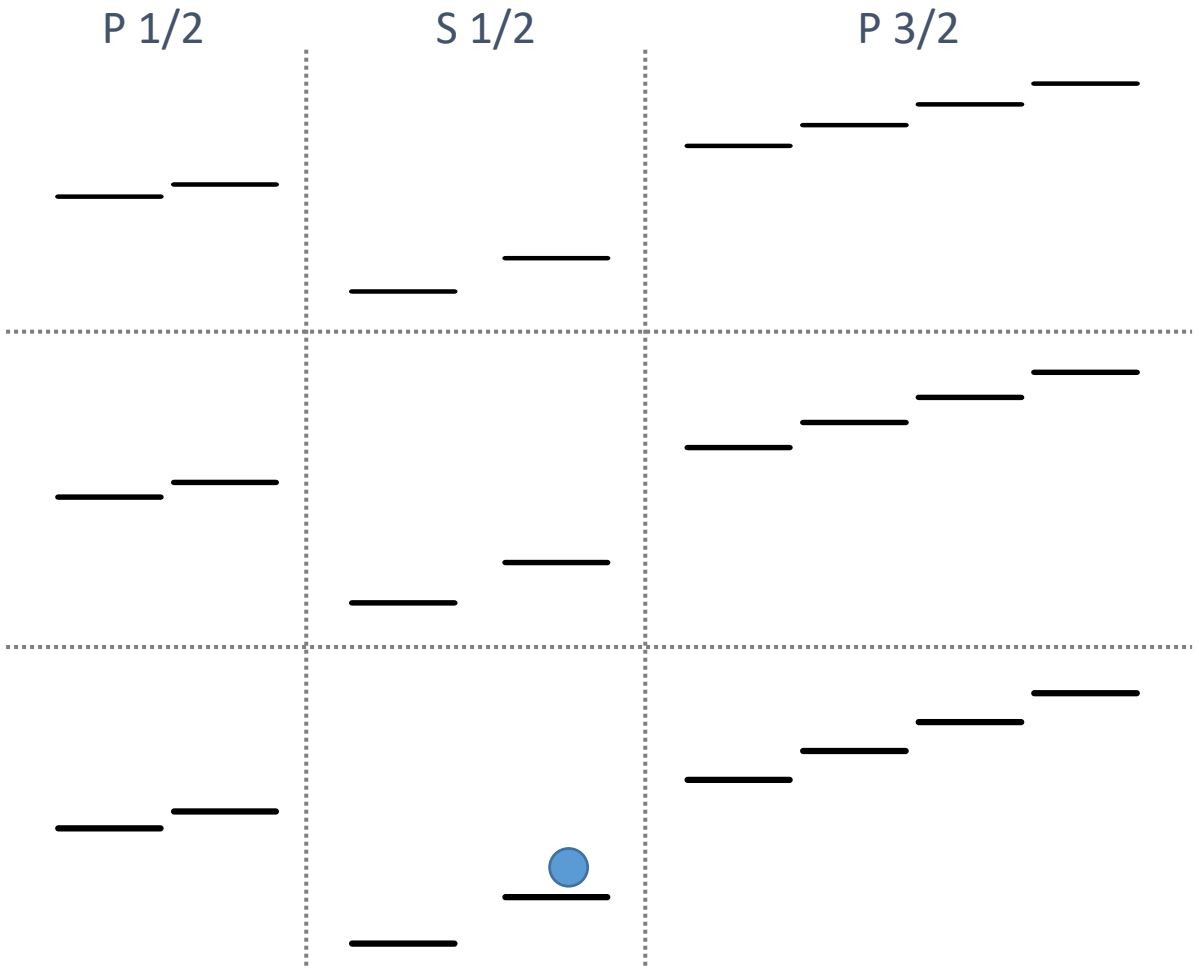




Rydberg synthetic lattices

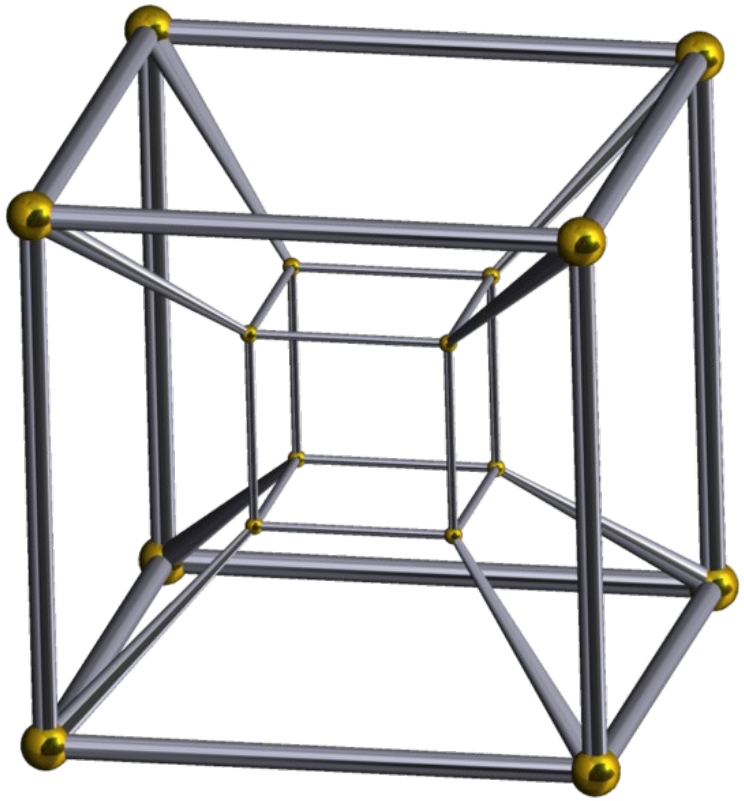
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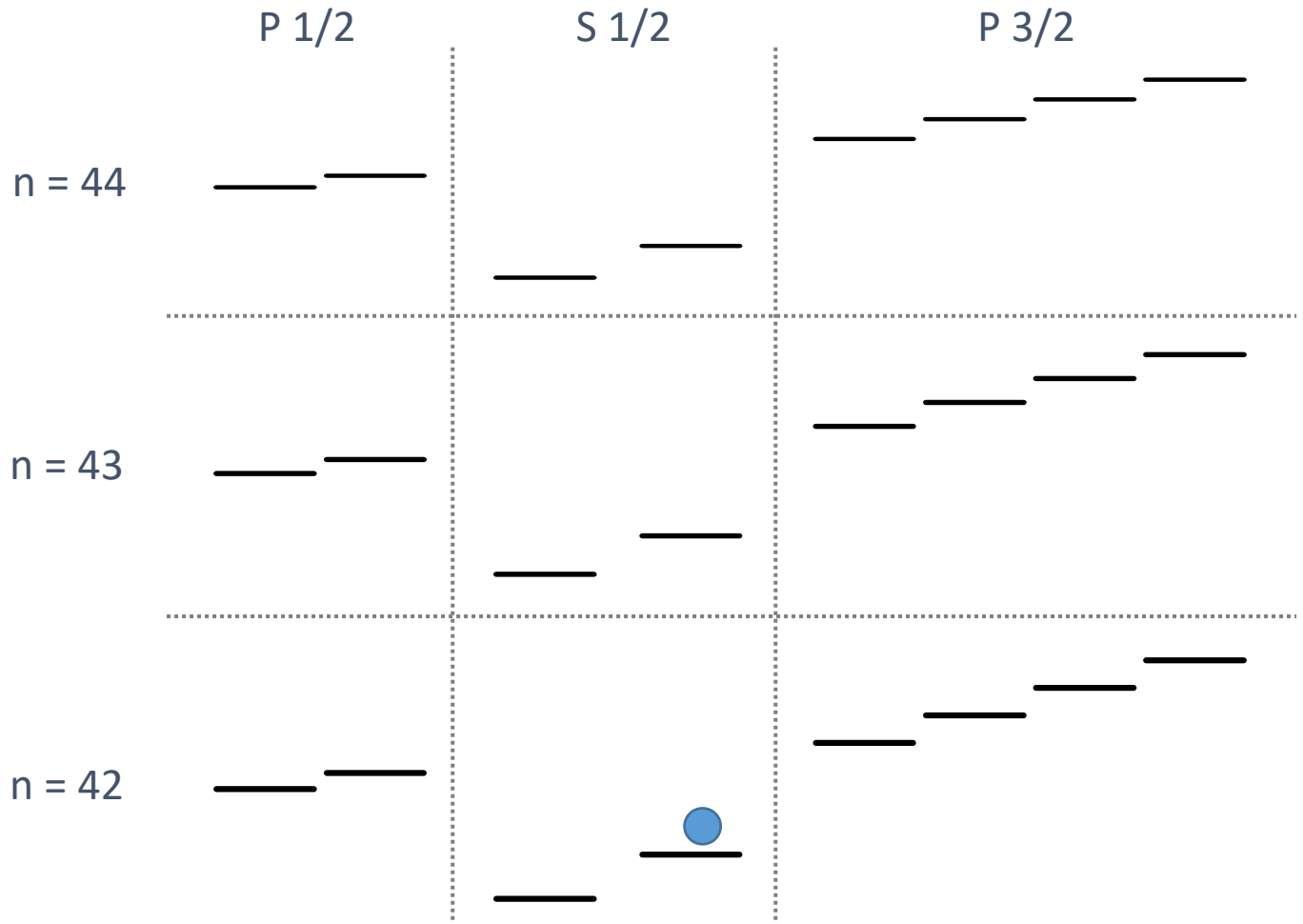


Rydberg synthetic lattices

see also work from Killian group:
S.K. Kanungo, *et al.* *Nat Commun* **13**, 972 (2022) ; Lu. *et al*. PRA **109**, 032801 (2024)



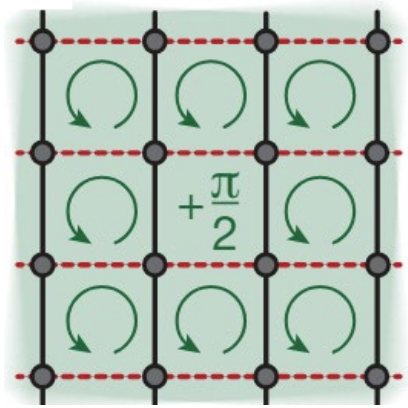
4D hypercube
Robert Webb, wikipedia



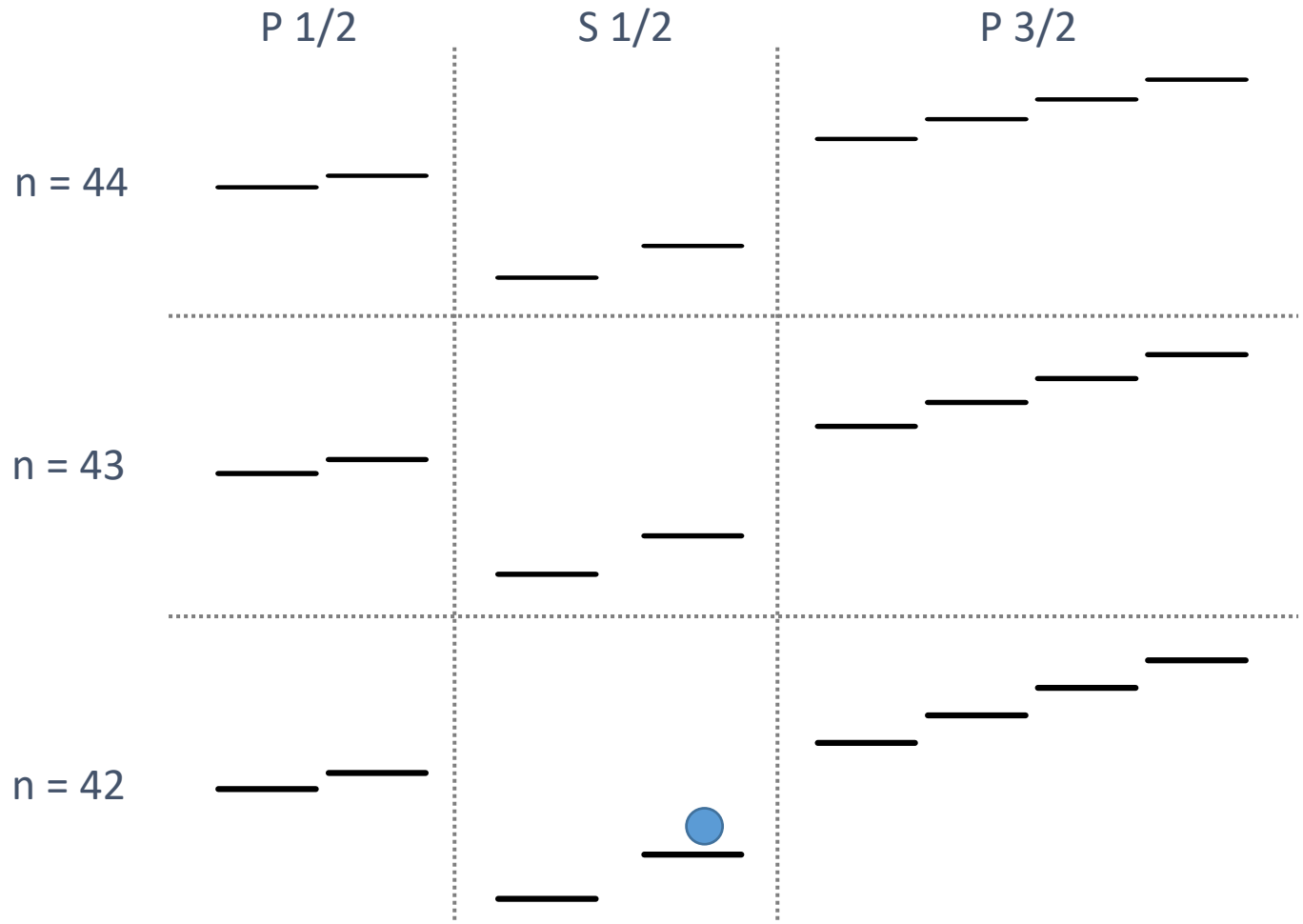
Rydberg synthetic lattices

see also work from Killian group:
S.K. Kanungo, *et al.* *Nat Commun* **13**, 972 (2022) ; Lu. *et al*. PRA **109**, 032801 (2024)

2D Hofstadter models and more
(*i.e.*, put on a torus)



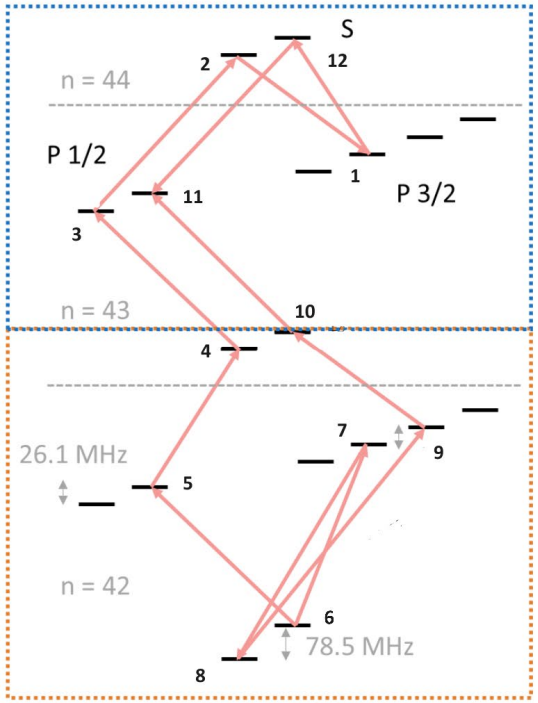
Aidelsburger, *et al* (2013)



interactions – in detail

- non-zero for states with $\Delta l = 1$ by dipole selection rules
- also quite local in n
- scaling with n, and structure based on Δm_l between states

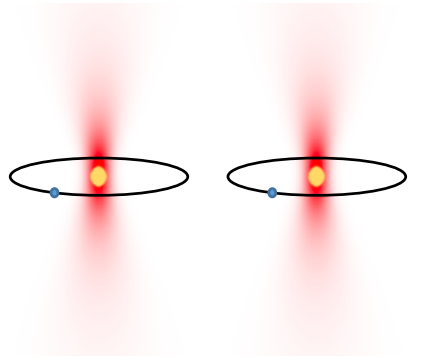
- there are also state-changing transitions – let's ignore those



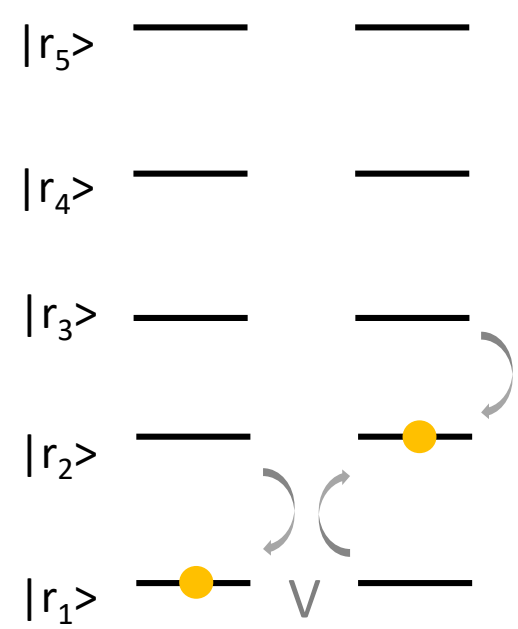
C3 values in units of MHz um^3

	43 P ½ - ½	43 P ½ + ½	43 P 3/2 - ½	43 P 3/2 + ½
44 S ½ + ½	705	-705	356	-1422
44 S ½ - ½	-705	705	-1422	356
43 S ½ + ½	834	-834	414	-1657
43 S ½ - ½	-834	834	-1657	414

	42 P ½ - ½	42 P ½ + ½	42 P 3/2 - ½	42 P 3/2 + ½
43 S ½ + ½	639	-639	322	-1289
43 S ½ - ½	-639	639	-1289	322
42 S ½ + ½	756	-756	376	-1502
42 S ½ - ½	-756	756	-1502	376



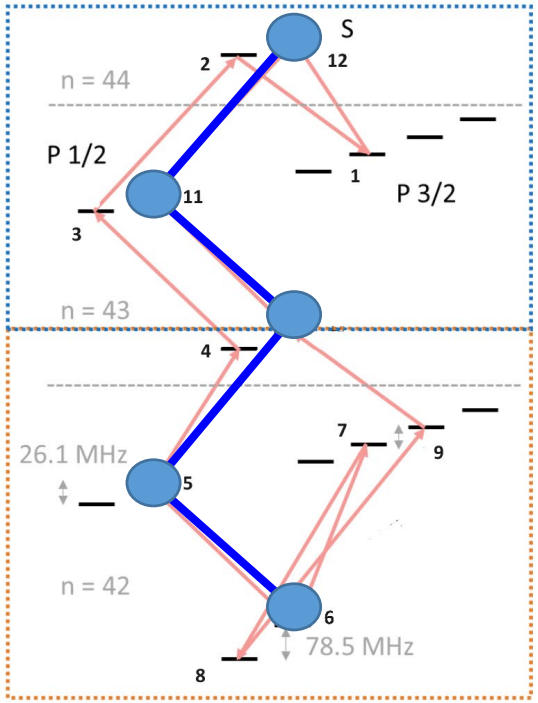
$$V_{i,j} \propto C_3^{i,j}$$



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42 S ½ + ½	756	-756	376	-1502
42 S ½ - ½	-756	756	-1502	376

for restricted "lattice" configurations, can achieve approximate translation invariance

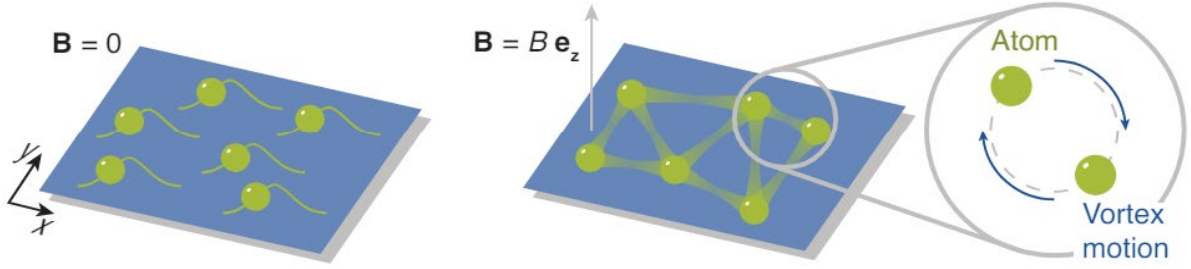
can achieve further tenability by state-dressing, e.g.

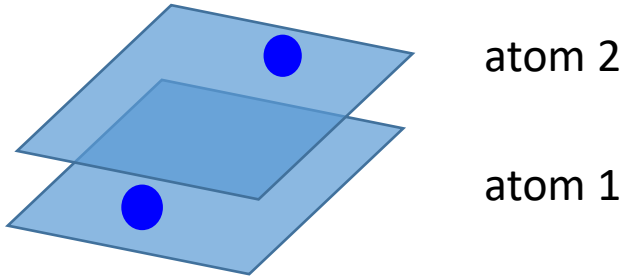
interactions – main takeaways

- the interactions are mostly “local” in the synthetic space
- the interactions are quite “random”, but entirely predictable (calibrations agree well with predictions)
- uniform (translation-invariant) interactions can be approximately achieved for some lattices

- for arrays, interactions decay with distance, but are long-ranged
- atoms occupy distinct “lattices” but still “feel” each other
  - relevant for few- and many-body physics

Léonard, *et al* (2023)





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Strategies for analog simulations

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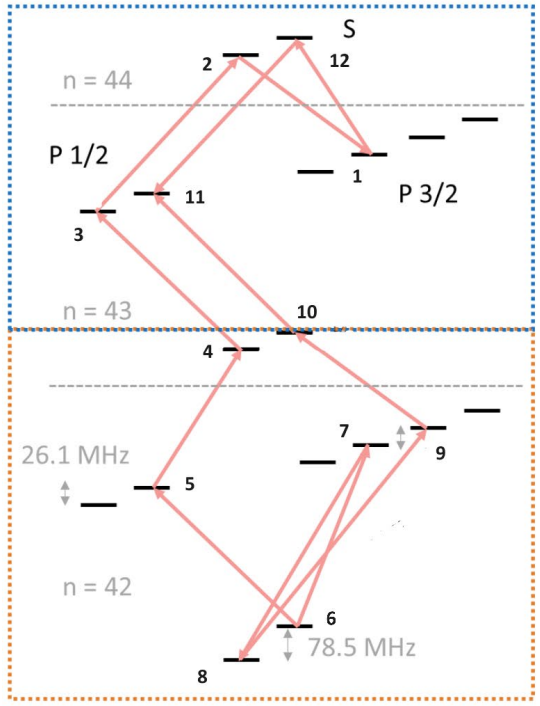
Strategies for analog simulations

- for larger lattices, choose problems that don’t depend on the details of interactions
- for some physics, use configurations that have approximately uniform interactions
- study exotic phenomena that arise **due to** “random” structure of interactions

interactions – main takeaways

Many-body localization due to random interactions

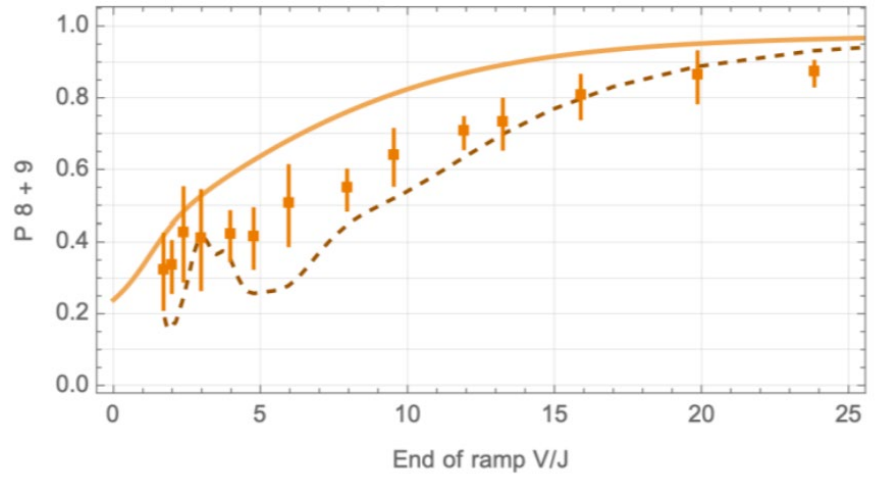
Piotr Sierant, Dominique Delande, and Jakub Zakrzewski
Phys. Rev. A **95**, 021601(R) – Published 9 February 2017



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- study exotic phenomena that arise **due to** "random" structure of interactions

Teaser: exploration of localized bound pairs

interactions – main takeaways

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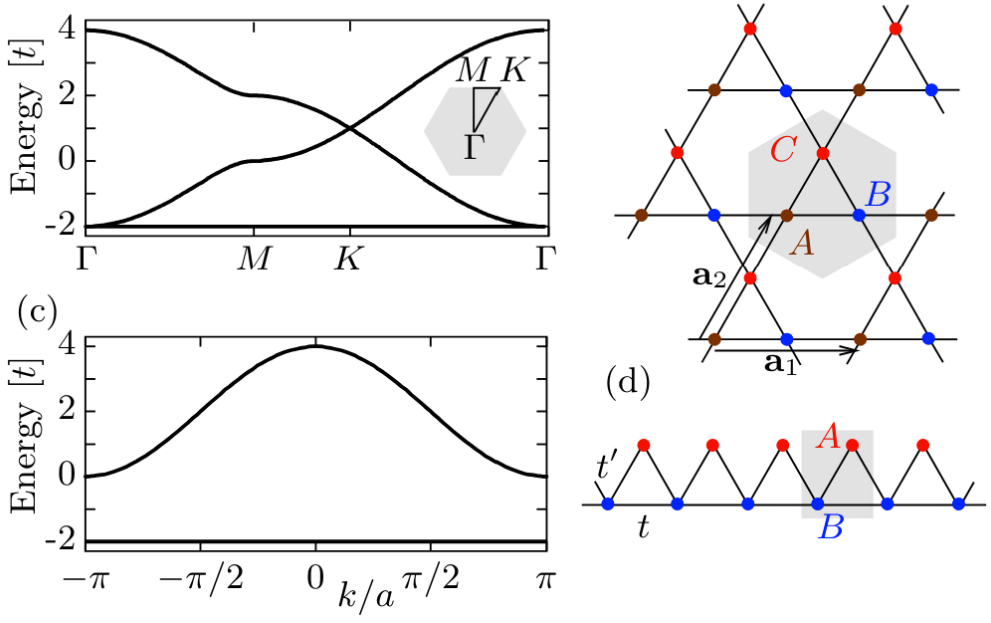
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interactions in flat-band lattices

- flattened bands arise from kinetic frustration, with perfectly flat bands found for certain lattices
- general interest in how interactions lead to emergent physics in flat bands (reorganize of eigenstates)
- even more interest in adding interactions to nearly-flat topological bands



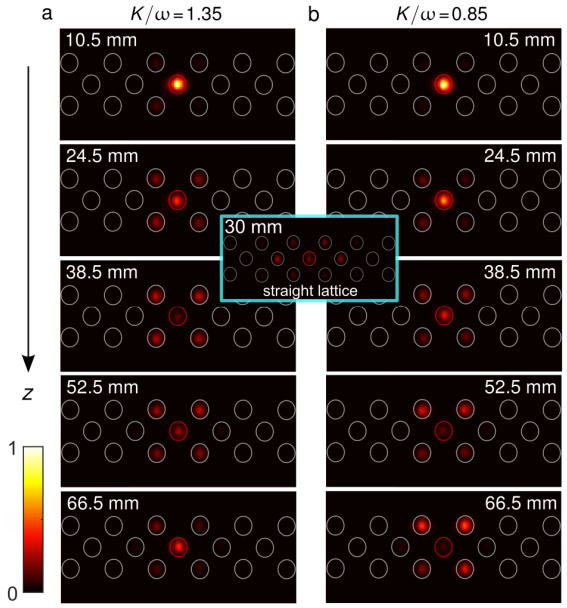
Huber & Altman
PRB 2010

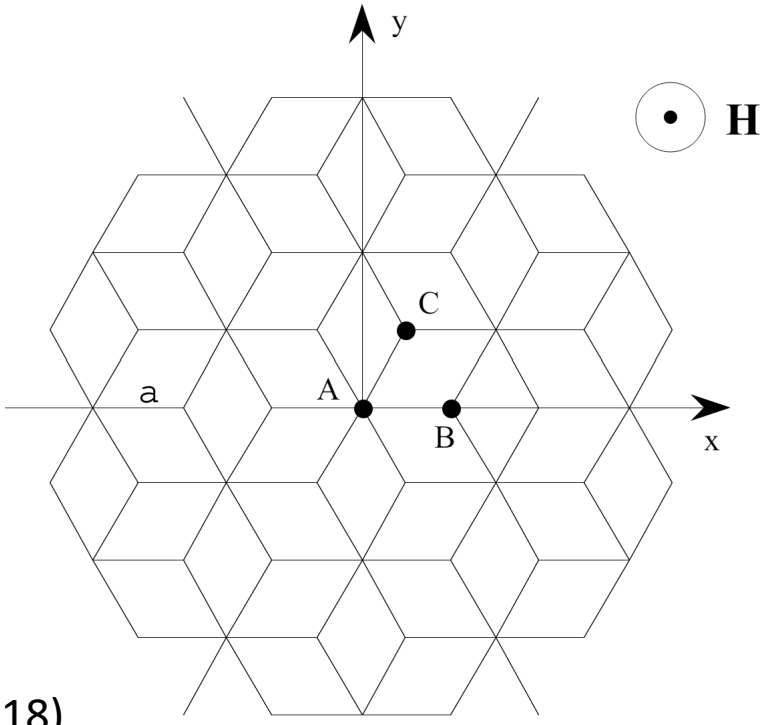
interactions in flat-band lattices

- Aharonov-Bohm caging, where all bands become perfectly flat, can occur with added gauge fields

Aharonov-Bohm Cages in Two-Dimensional Structures

Julien Vidal, Rémy Mosseri, and Benoit Douçot
Phys. Rev. Lett. **81**, 5888 – Published 28 December 1998





Mukherjee, et al. PRL (2018)

interactions in flat-band lattices

- Aharonov-Bohm caging, where all bands become perfectly flat, can occur with added gauge fields
- weak interactions lead to intra-band state-mixing, strong interactions can induce lattice dynamics between unit cells

Interaction Induced Delocalization for Two Particles in a Periodic Potential

Julien Vidal, Benoît Douçot, Rémy Mosseri, and Patrick Butaud
Phys. Rev. Lett. **85**, 3906 – Published 30 October 2000

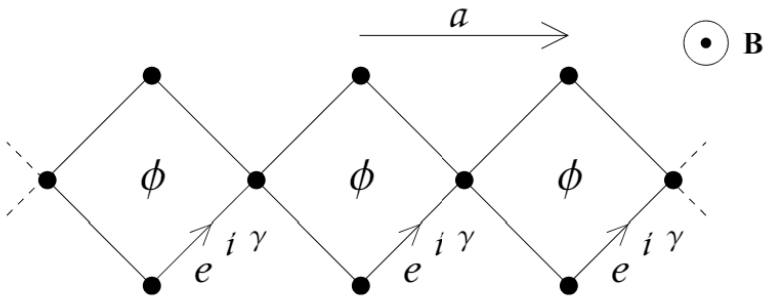
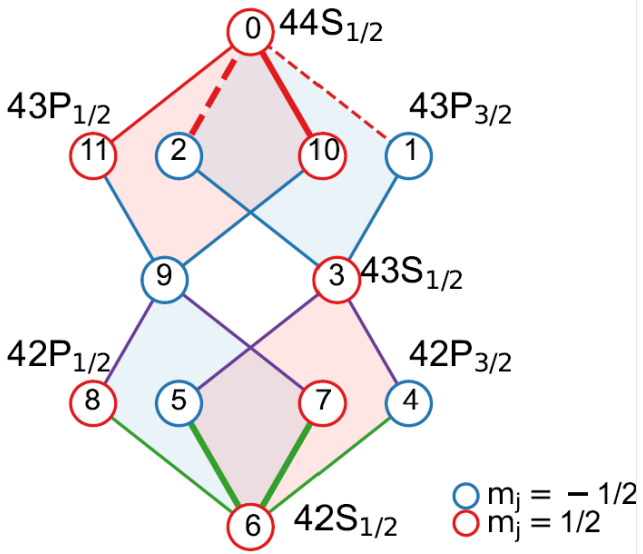
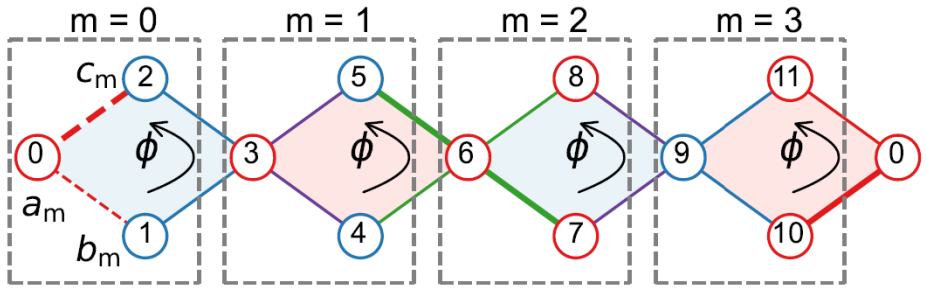


FIG. 1. Square chain under a uniform magnetic field.The magnetic flux per unit cell is denoted by ϕ and $\gamma = 2\pi\phi/\phi_0$.

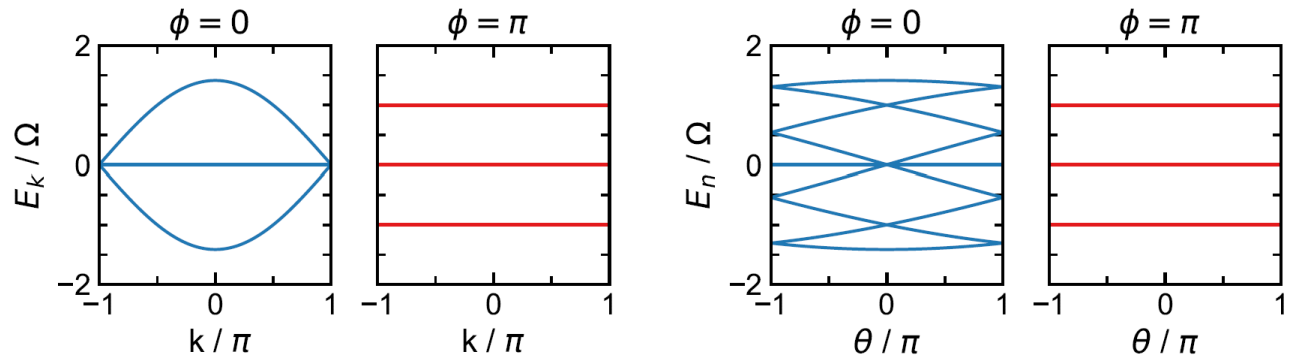
Aharonov-Bohm caging in a Rydberg atom



- Four-unit cell diamond lattice
- Twisted boundary conditions
- Tunable gauge fields and twist phase

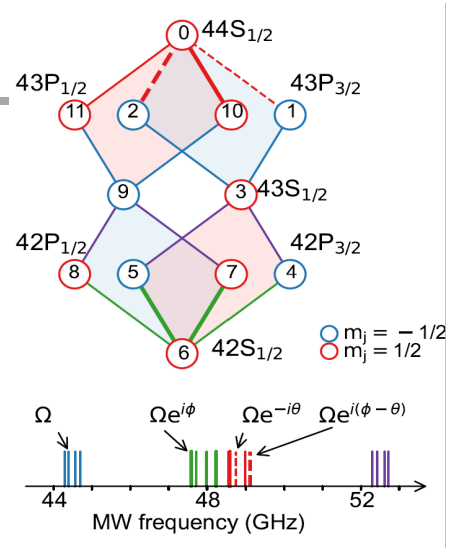


"side view" (unwrapped)

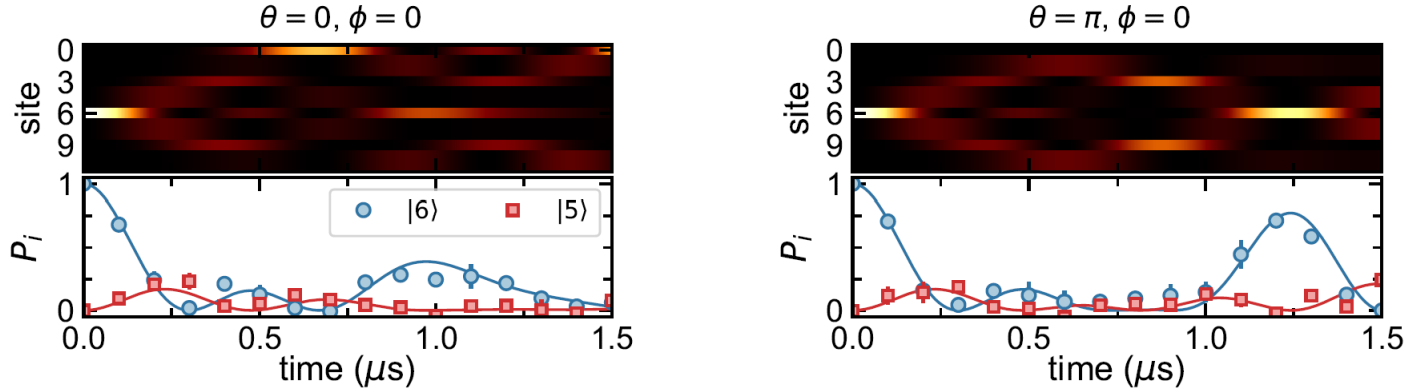


T. Chen*, C. Huang*, et al. (arXiv:2404.00737)
work w/ Hannah Price & Tomoki Ozawa

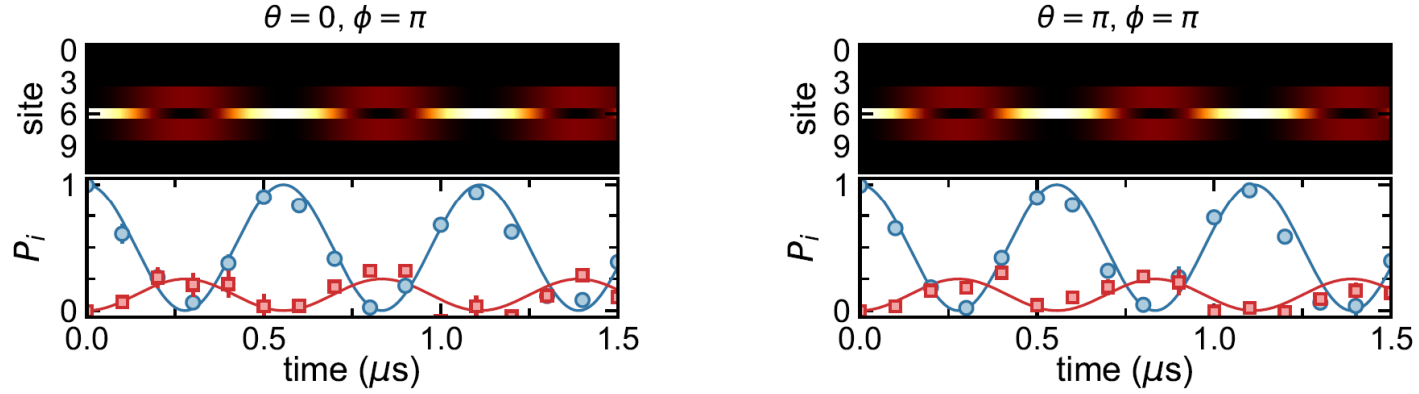
Aharonov-Bohm caging in a Rydberg atom

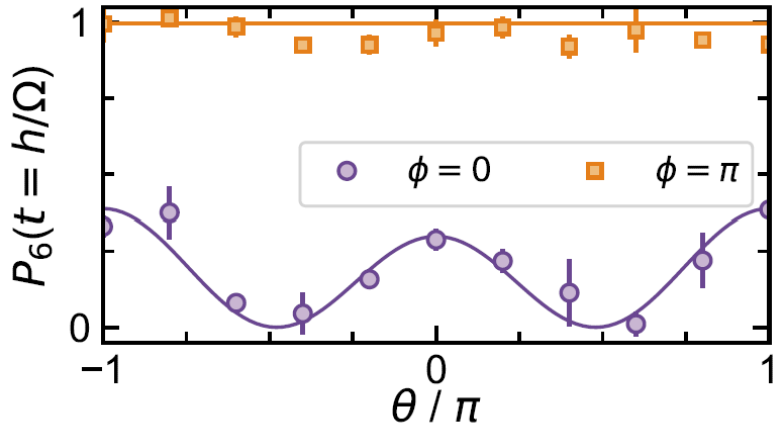


Non-caging condition – theory vs. experiment



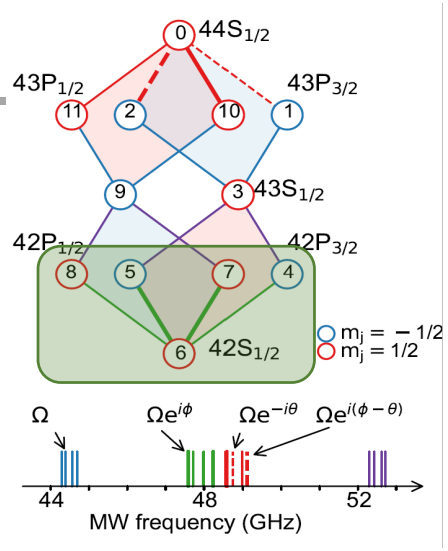
AB caging condition – theory vs. experiment



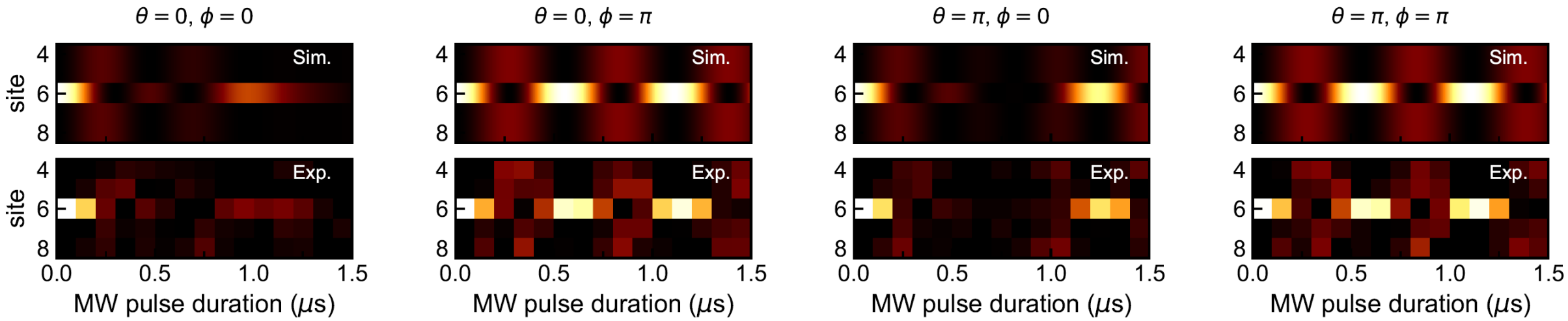


T. Chen*, C. Huang*, et al. (arXiv:2404.00737)
work w/ Hannah Price & Tomoki Ozawa

Aharonov-Bohm caging in a Rydberg atom

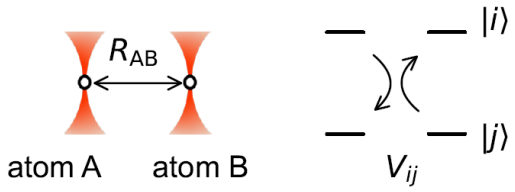


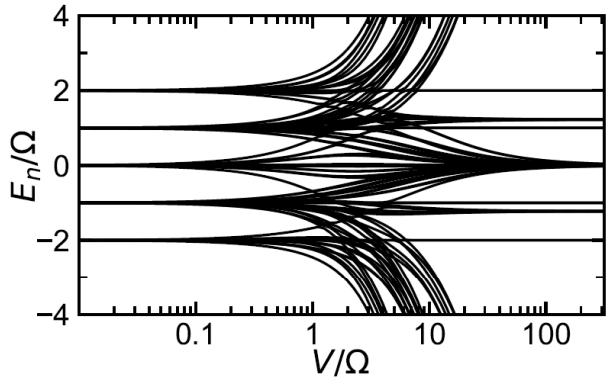
Theory vs. experiment for multiple state population dynamics

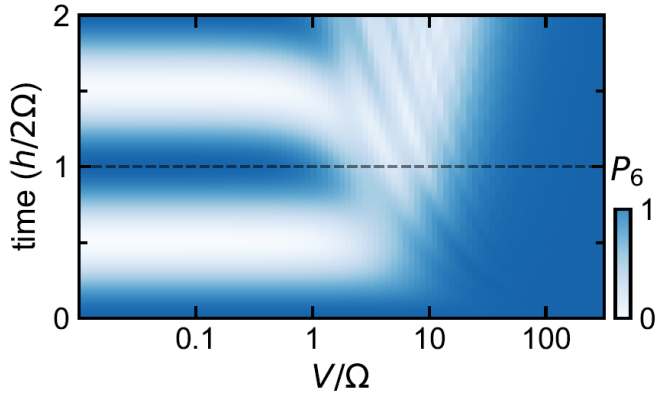


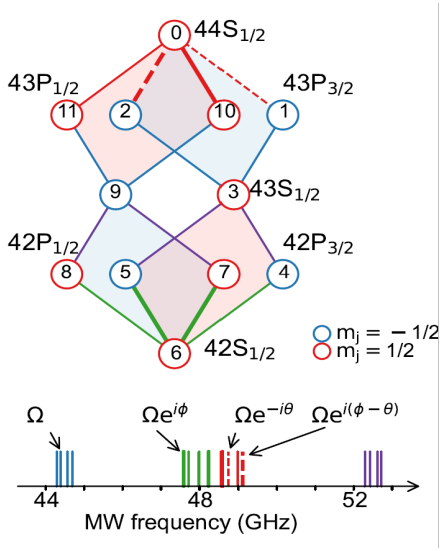
T. Chen*, C. Huang*, et al. (arXiv:2404.00737)
work w/ Hannah Price & Tomoki Ozawa

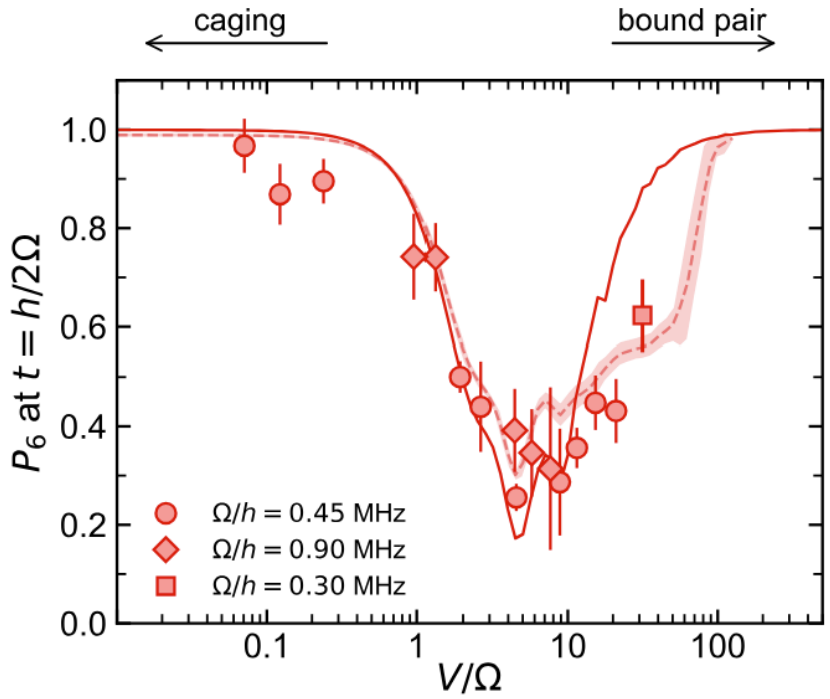
caging breakdown by interactions

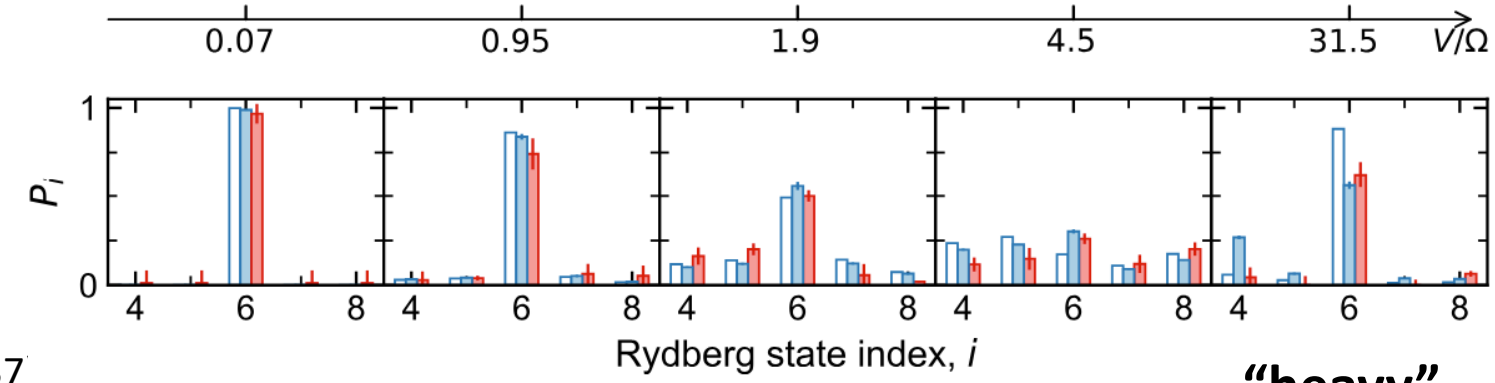












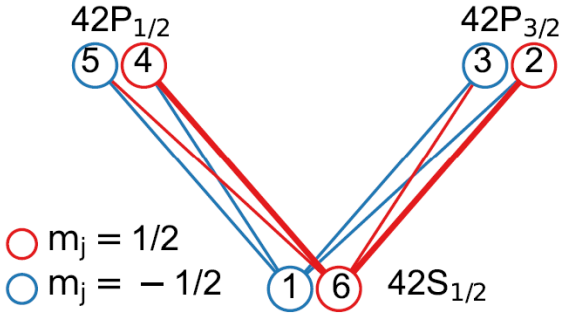
T. Chen*, C. Huang*, et al. (arXiv:2404.00737, work w/ Hannah Price & Tomoki Ozawa

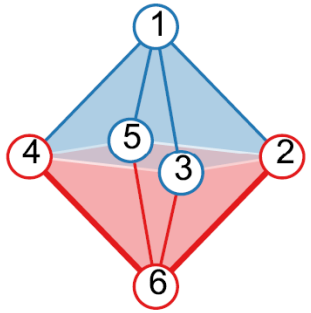
caged

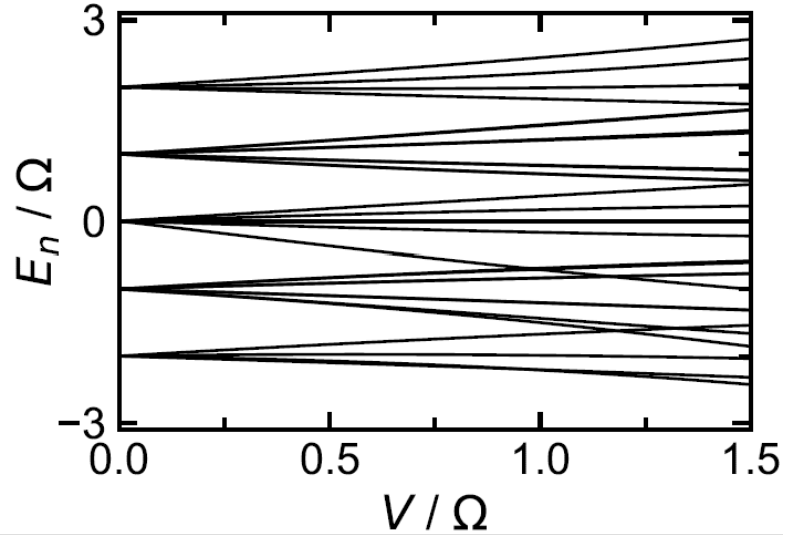
delocalized

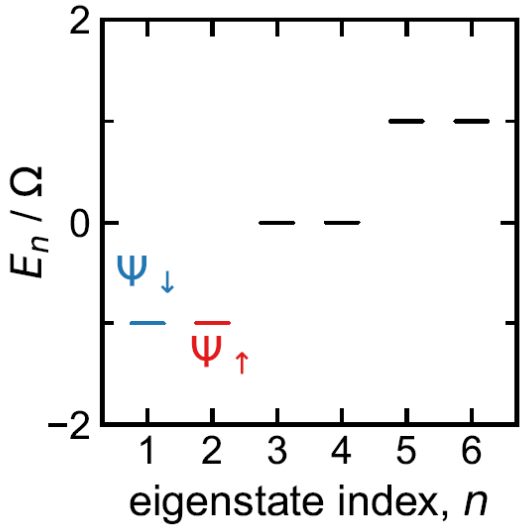
“heavy” bound state

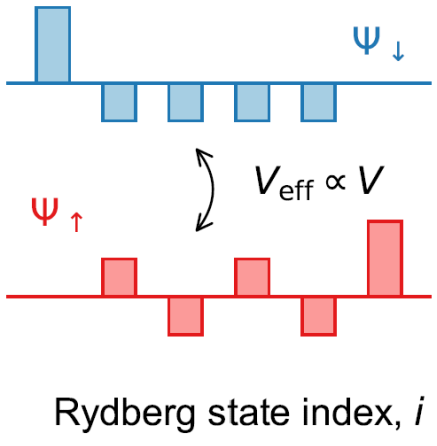
intra-band mixing due to weak interactions











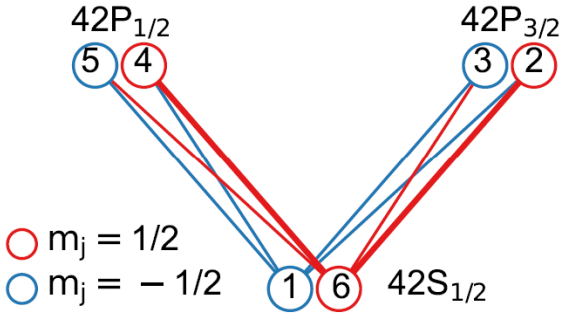
For small values of V (compared to the bandgap), interactions break the flat-band state degeneracy

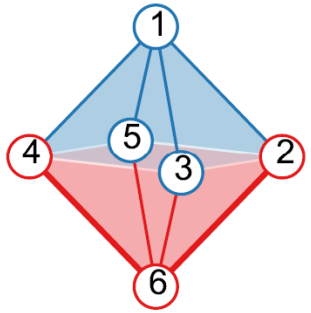
$$H_{\text{int}} = \frac{3V}{64}(\sigma_A^z\sigma_B^z + \mathbb{1}_A\mathbb{1}_B) + \frac{3V}{64}(\sigma_A^x\sigma_B^x + \sigma_A^y\sigma_B^y) + \frac{9V}{64}(\sigma_A^x\sigma_B^z + \sigma_A^z\sigma_B^x)$$

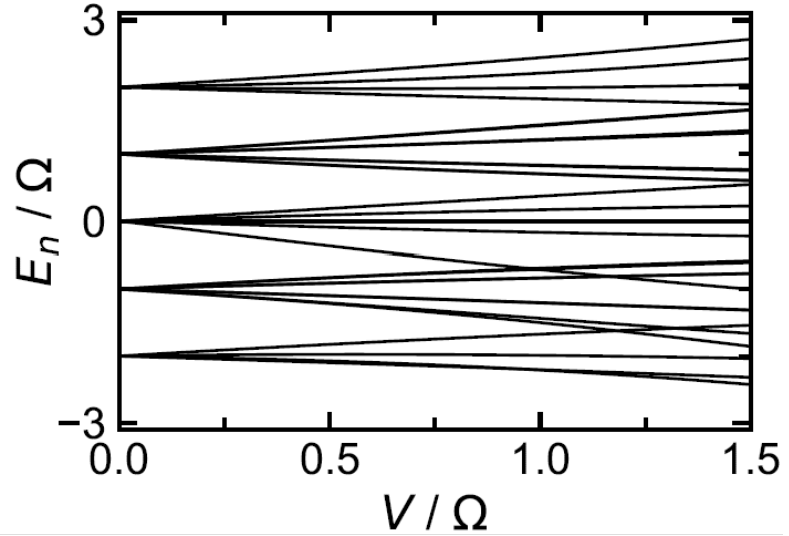
T. Chen*, C. Huang*, et al. (arXiv:2404.00737)
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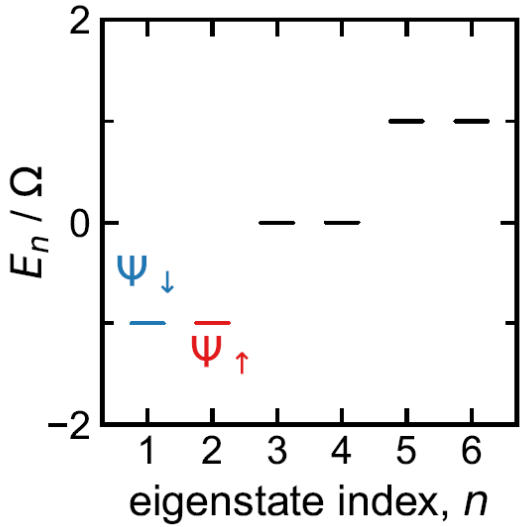
Emergent spin-1/2 magnetism for 2 unit cells

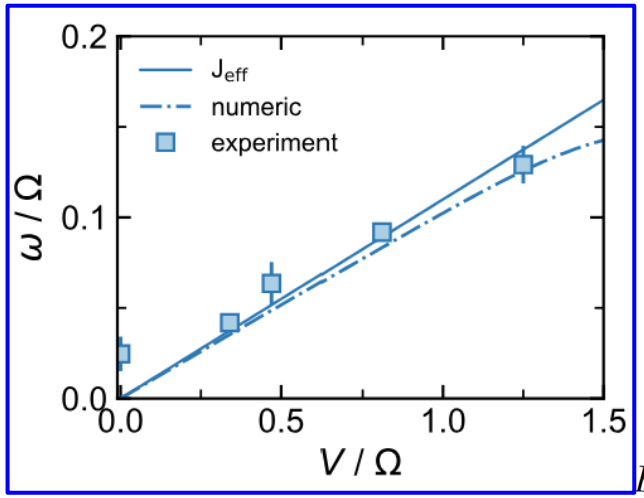
intra-band mixing due to weak interactions











For small values of V (compared to the bandgap), interactions break the flat-band state degeneracy

$$H_{\text{int}} = \frac{3V}{64}(\sigma_A^z\sigma_B^z + \mathbb{1}_A\mathbb{1}_B) + \frac{3V}{64}(\sigma_A^x\sigma_B^x + \sigma_A^y\sigma_B^y) + \frac{9V}{64}(\sigma_A^x\sigma_B^z + \sigma_A^z\sigma_B^x)$$

T. Chen*, C. Huang*, et al. (arXiv:2404.00737)
work w/ Hannah Price & Tomoki Ozawa

Emergent spin-1/2 magnetism for 2 unit cells

interactions – main takeaways

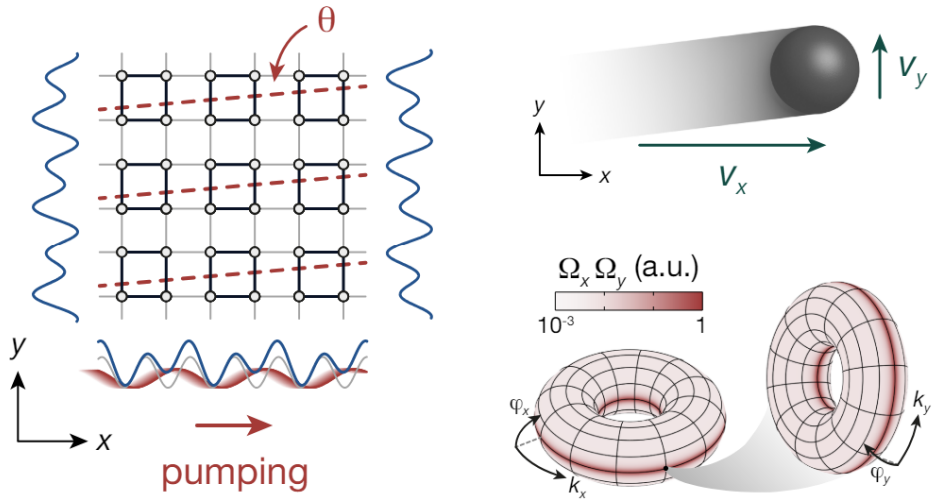
- the interactions are mostly “local” in the synthetic space
- the interactions are quite “random”, but entirely predictable (calibrations agree well with predictions)
- uniform (translation-invariant) interactions can be approximately achieved for some lattices

- for arrays, interactions decay with distance, but are long-ranged
- atoms occupy distinct “lattices” but still “feel” each other
  - relevant for few- and many-body physics

Strategies for analog simulations

- for larger lattices, choose problems that don’t depend on the details of interactions
- for some physics, use configurations that have approximately uniform interactions
- study exotic phenomena that arise **due to** “random” structure of interactions

interacting topological systems – let's start in 1D



Lohse, *et al*. Nature (2018)

the physics of 2D quantum Hall topology
gives rise to robust Thouless pumping phenomena

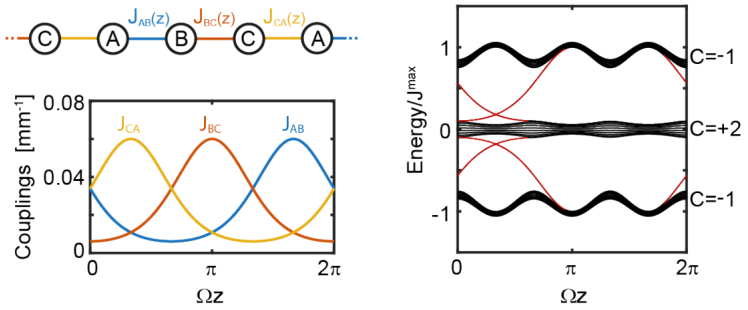
interacting topological systems – let's start in 1D

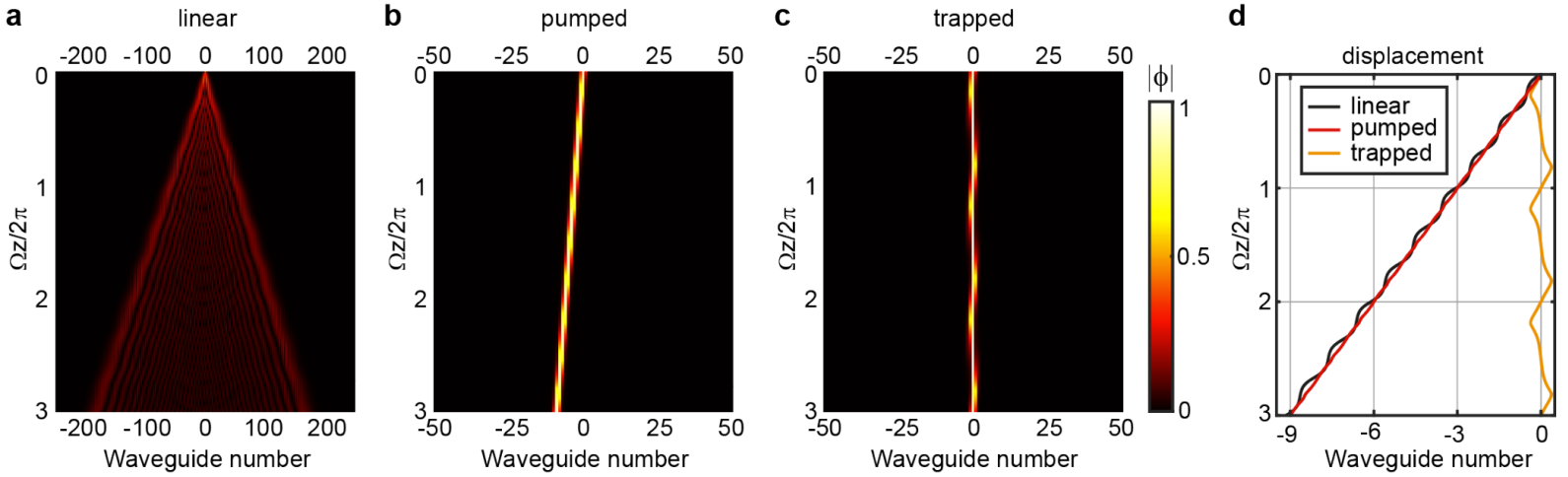
Nature, 2021

Quantized Nonlinear Thouless Pumping

Marius Jürgensen,* Sebabrata Mukherjee, and Mikael C. Rechtsman†
Department of Physics, The Pennsylvania State University, University Park, PA 16802, USA
(Dated: June 29, 2021)

increasing nonlinearity

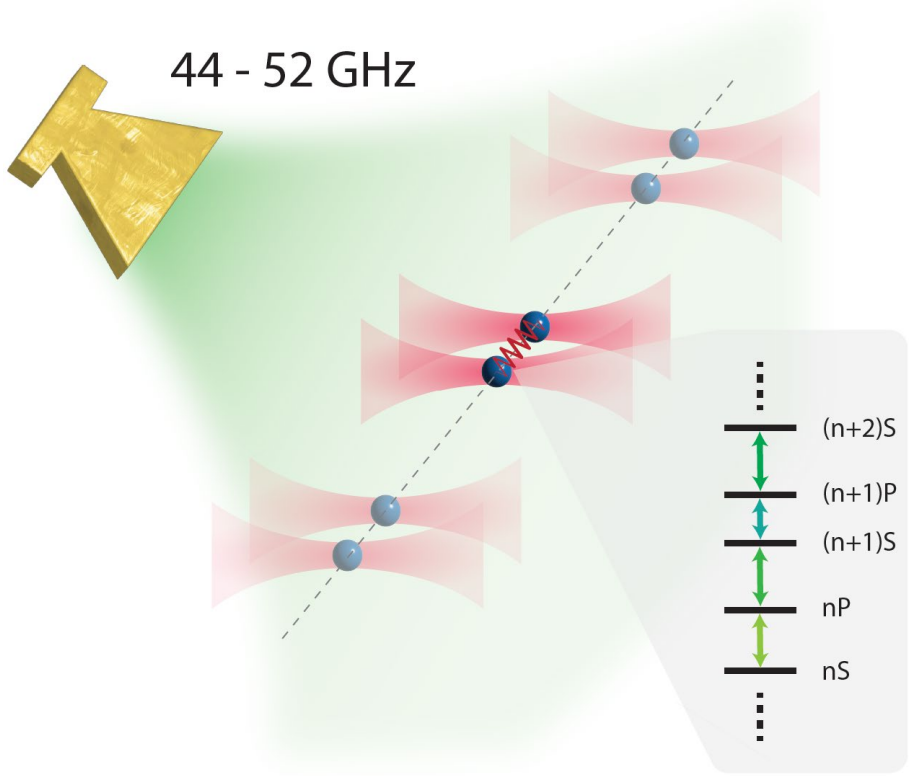


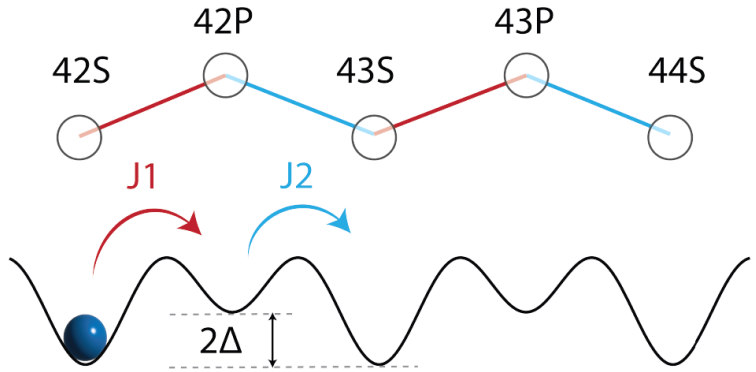


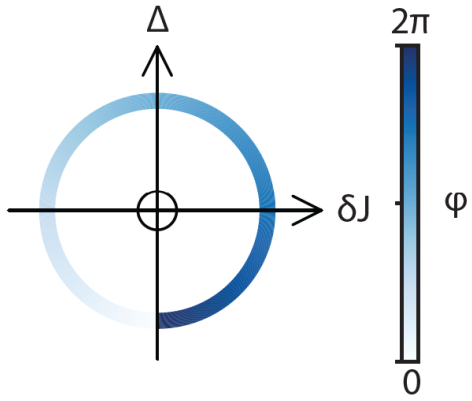
We also find "self-trapping" of 2-particle bound states
> can we obtain pumping of such localized states?

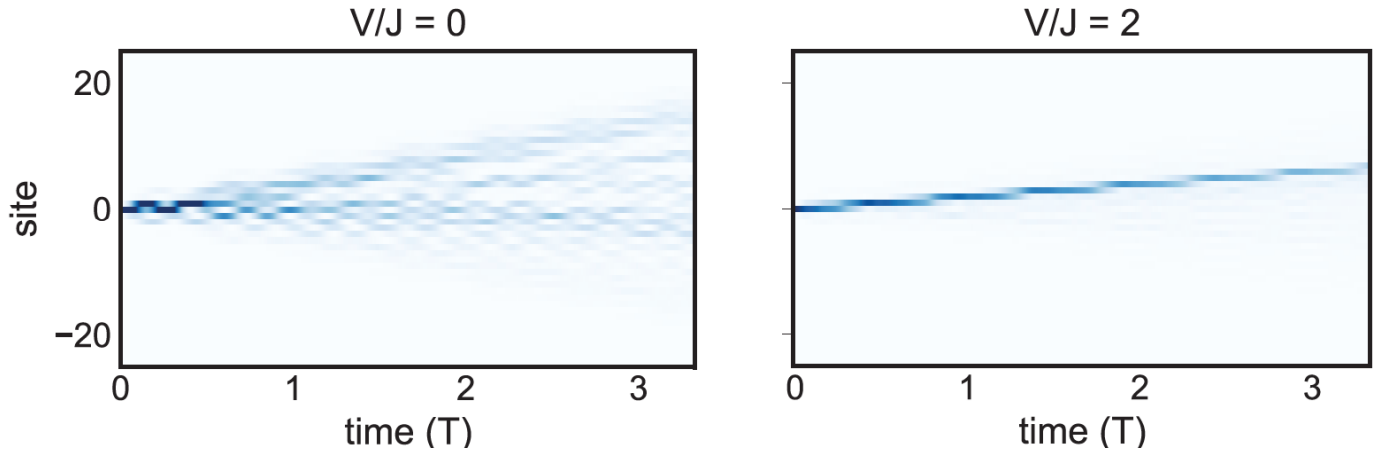
topological pumping of Rydberg pairs

C. Huang, *et al.* (in prep)

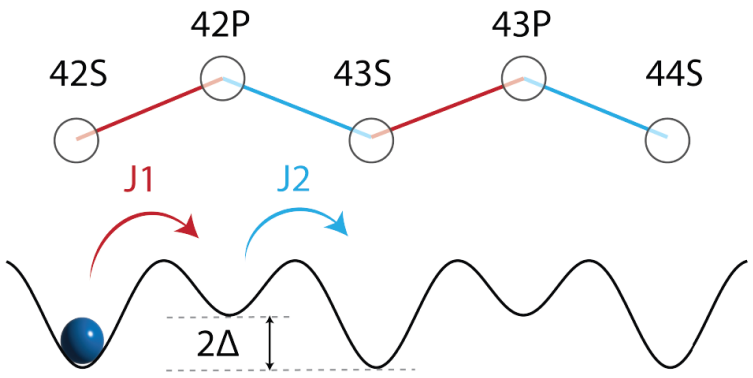




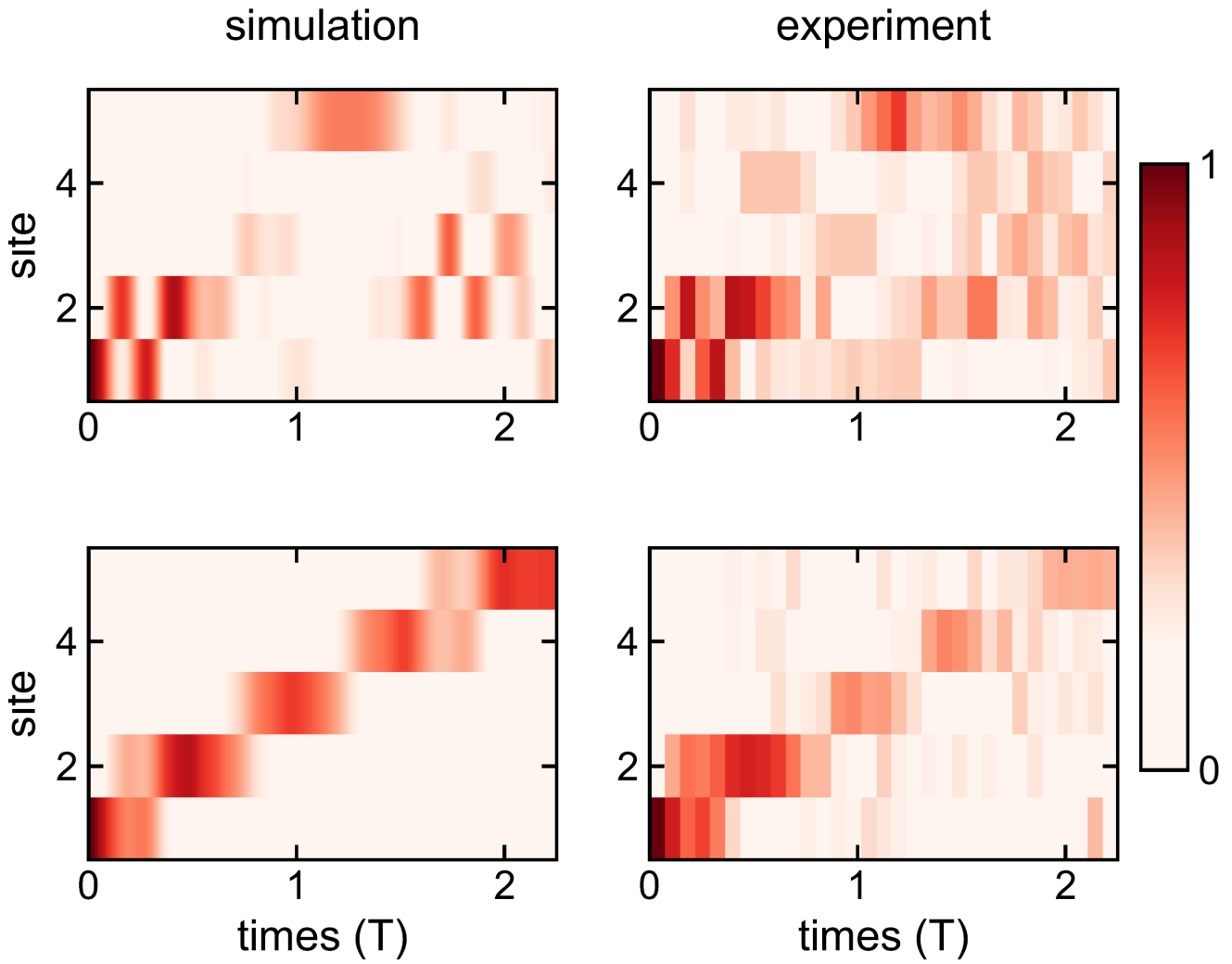




topological pumping of Rydberg pairs

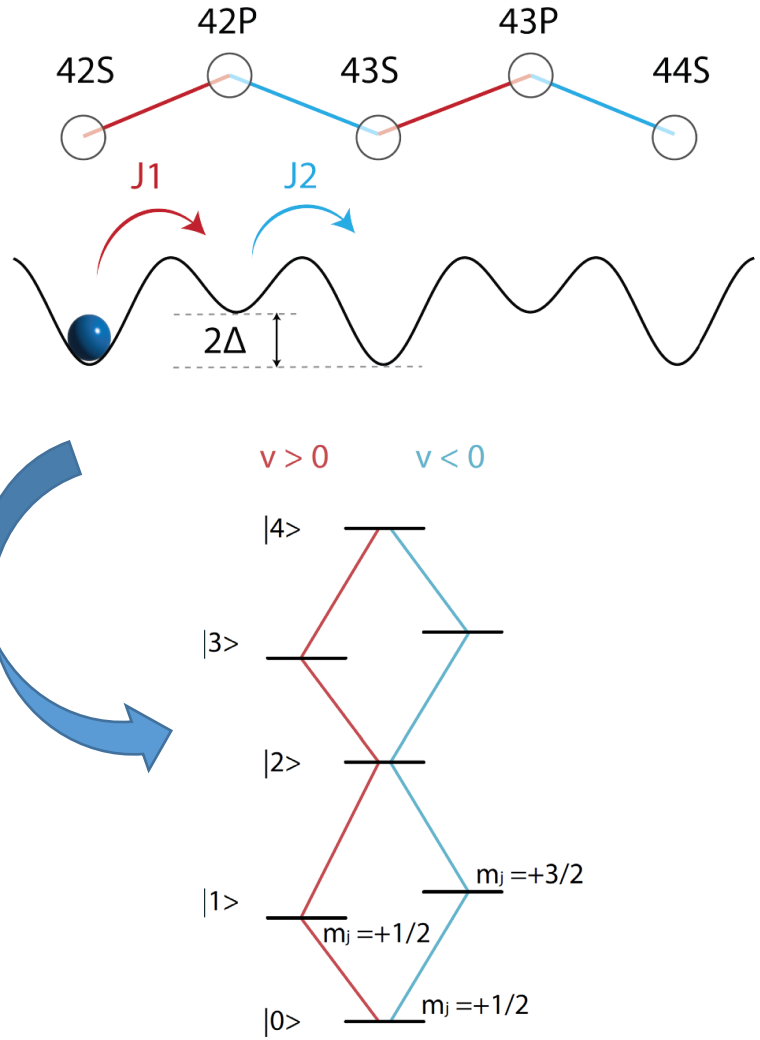


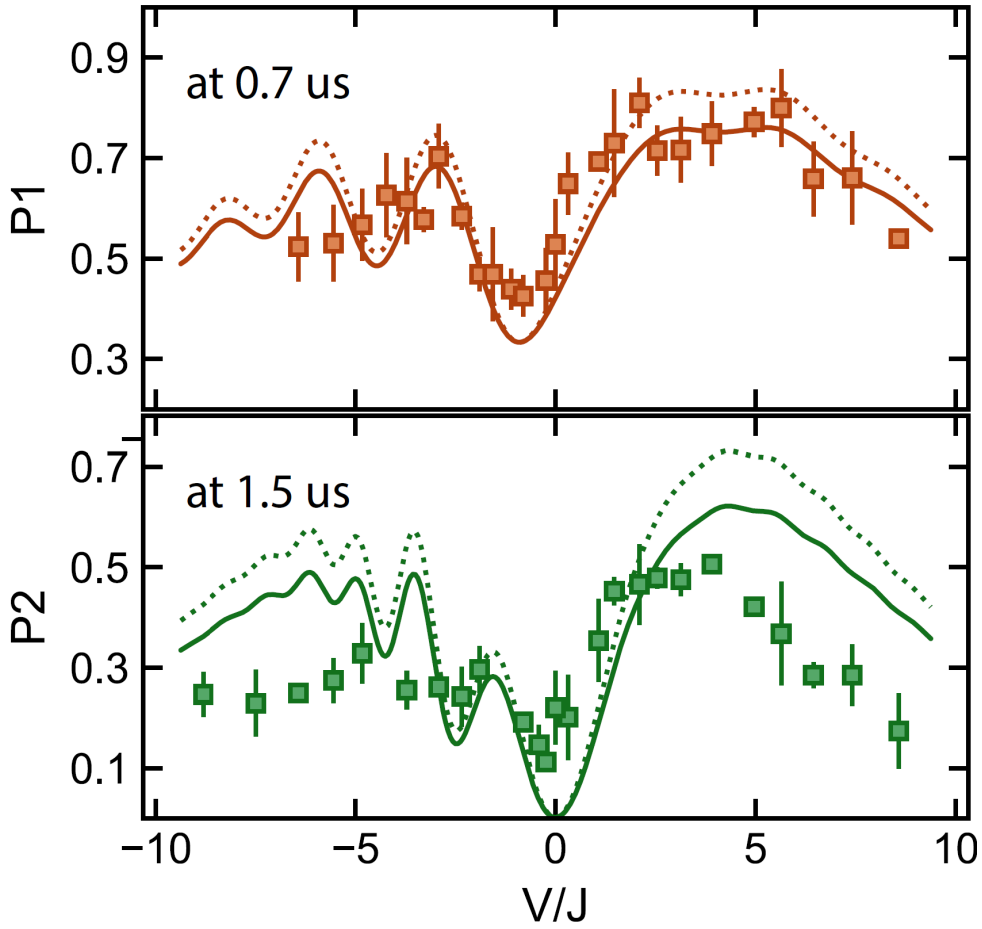
C. Huang, *et al.* (in prep)



interaction-dependence of “efficiency”

C. Huang, *et al.* (in prep)





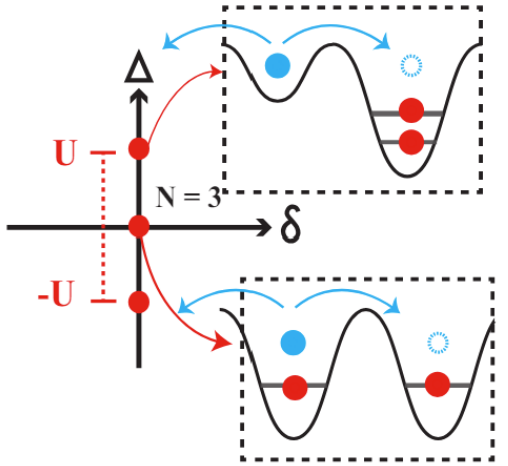
interaction-induced topological pumping?

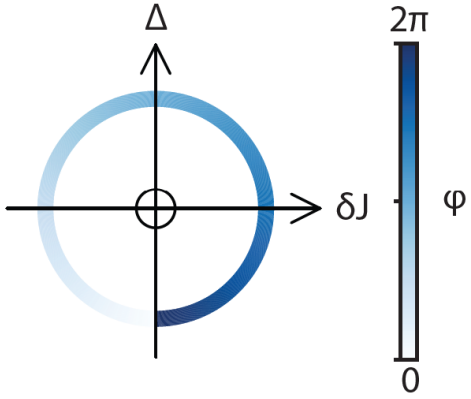
Interaction induced splitting of Dirac monopoles in the topological Thouless pumping of strongly interacting Bosons and SU(N) Fermions

Hei Lam[1] and Yangqian Yan[1,2,*]

[1] *Department of Physics, The Chinese University of Hong Kong, Hong Kong SAR, China*
[2] *The Chinese University of Hong Kong Shenzhen Research Institute, 518057 Shenzhen, China*

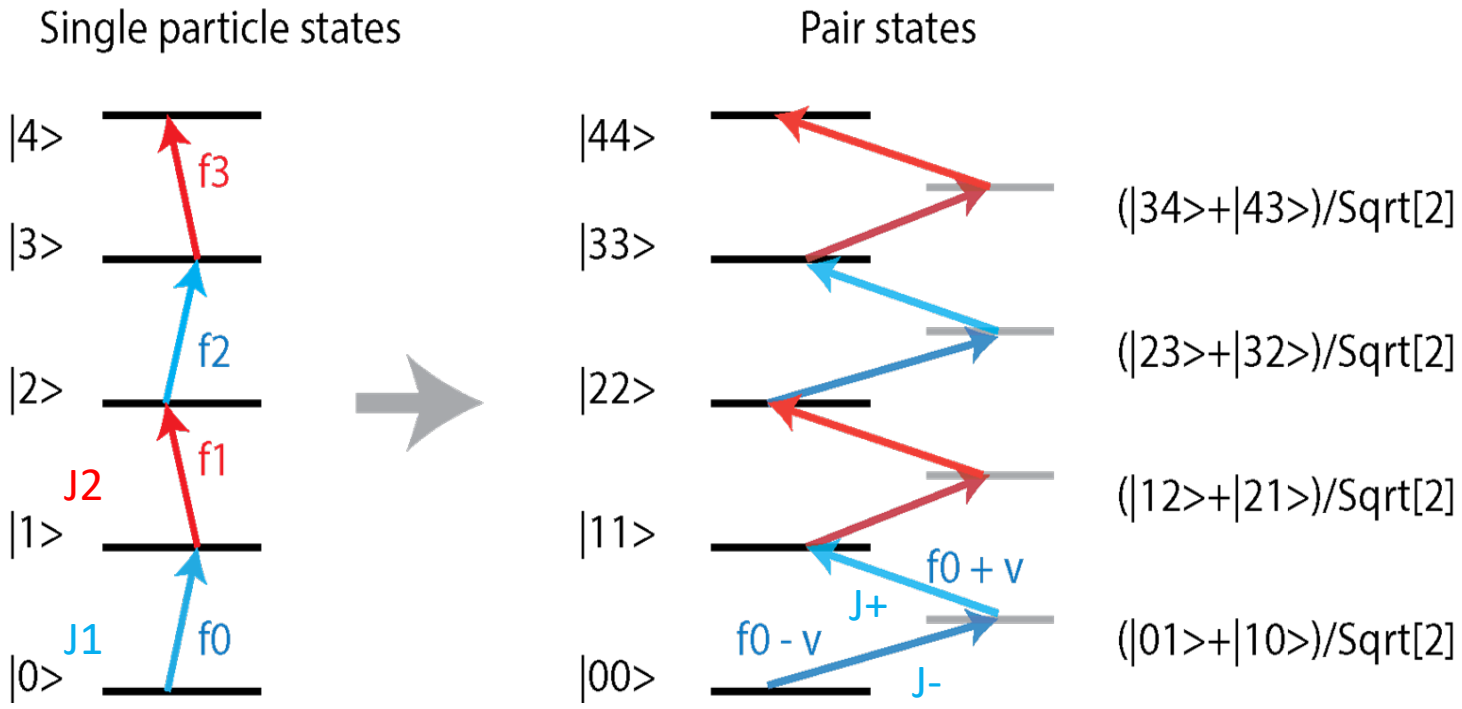
motivating experiments by Tilman Esslinger group





interaction-induced topological pumping?

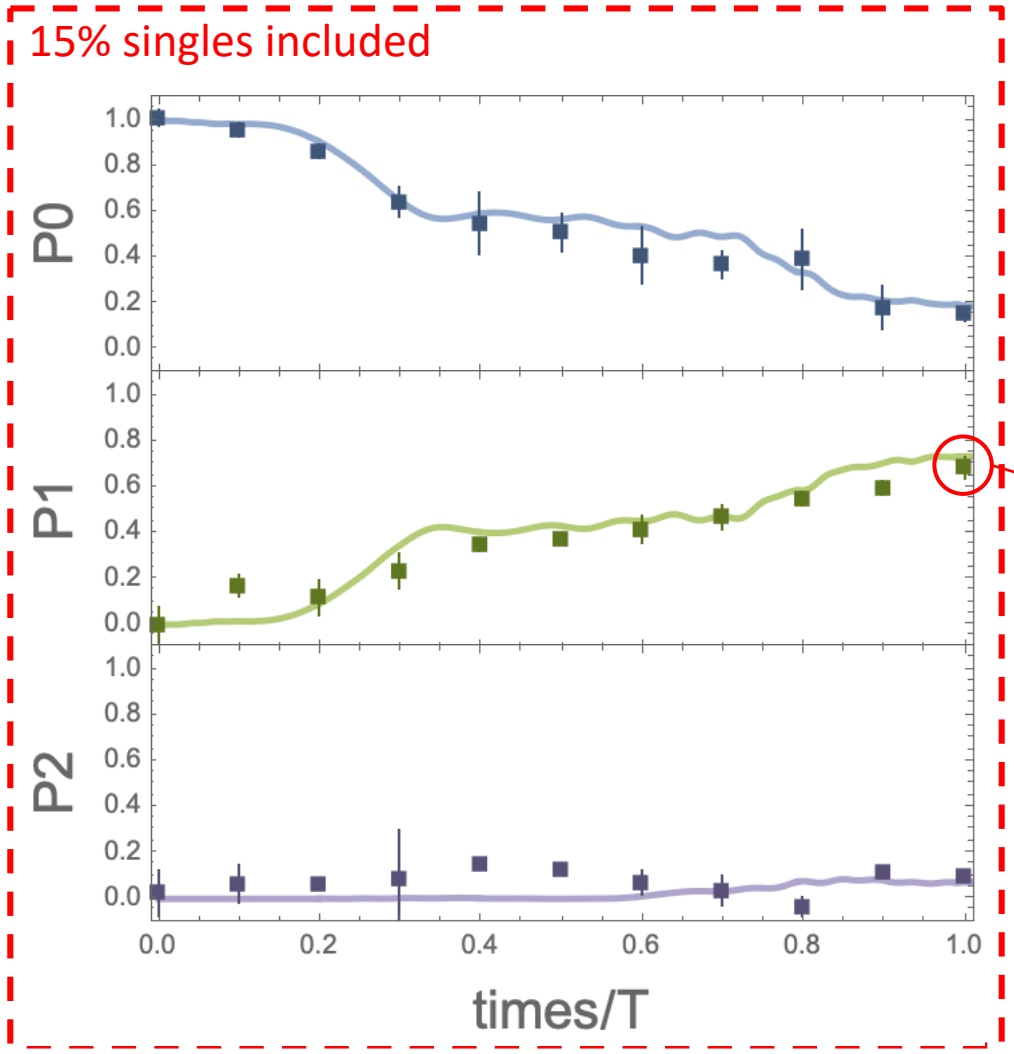
C. Huang, *et al.* (in prep)

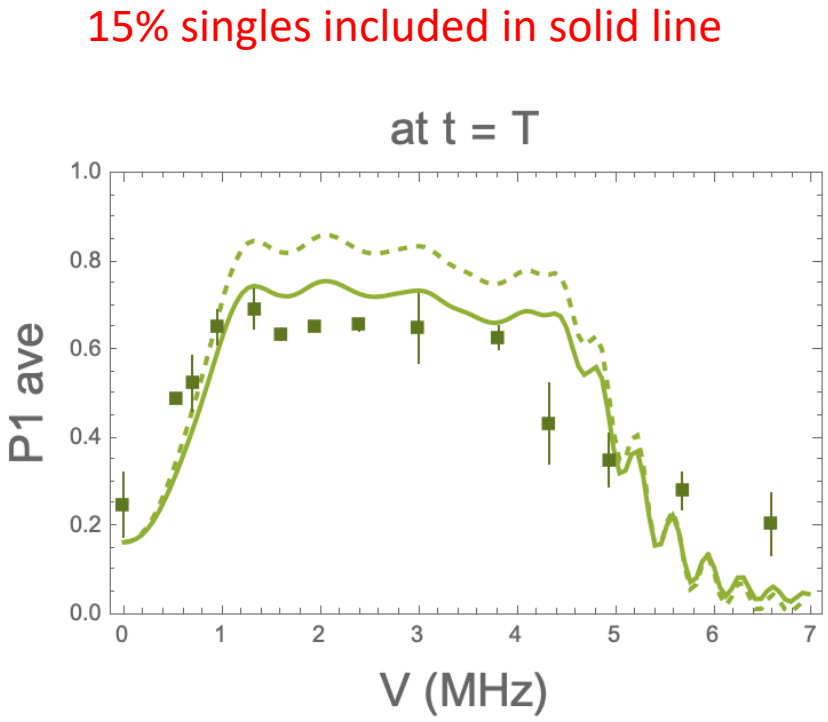


because our synthetic lattice is “tilted” naturally, we can separately access process that form and break bound states

interaction-induced topological pumping?

C. Huang, *et al.* (in prep)

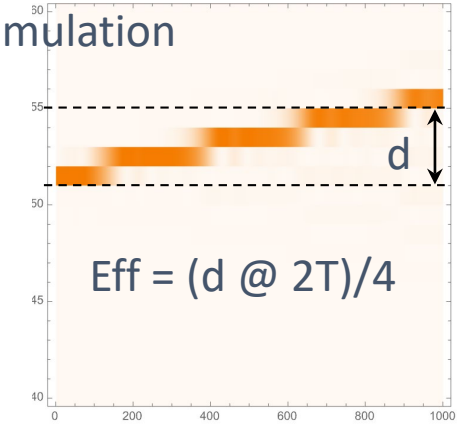


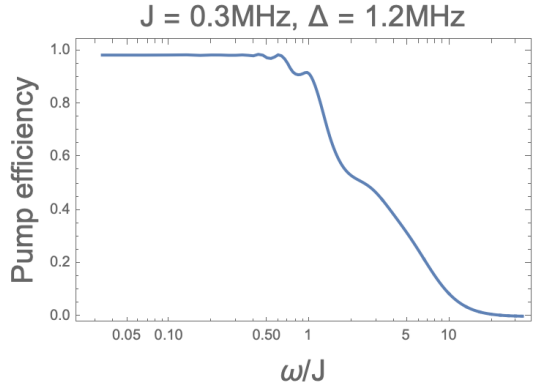


the pump is adiabatic

C. Huang, *et al.* (in prep)

101 site ideal simulation

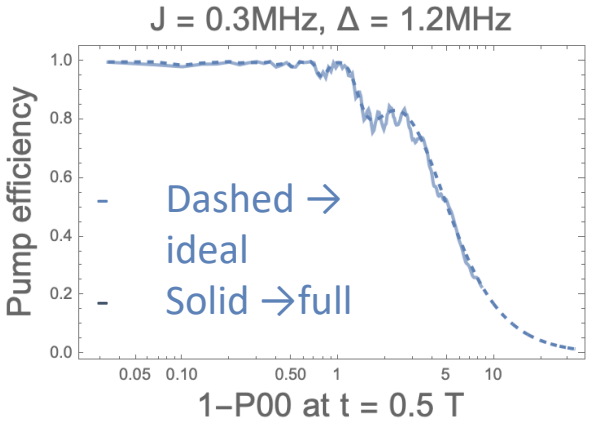




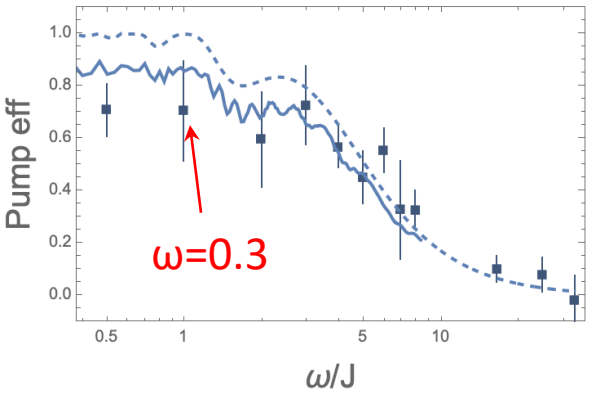
5 site simulation and data

Eff = (1-P00) @ T/2

Population leaves initial state or not

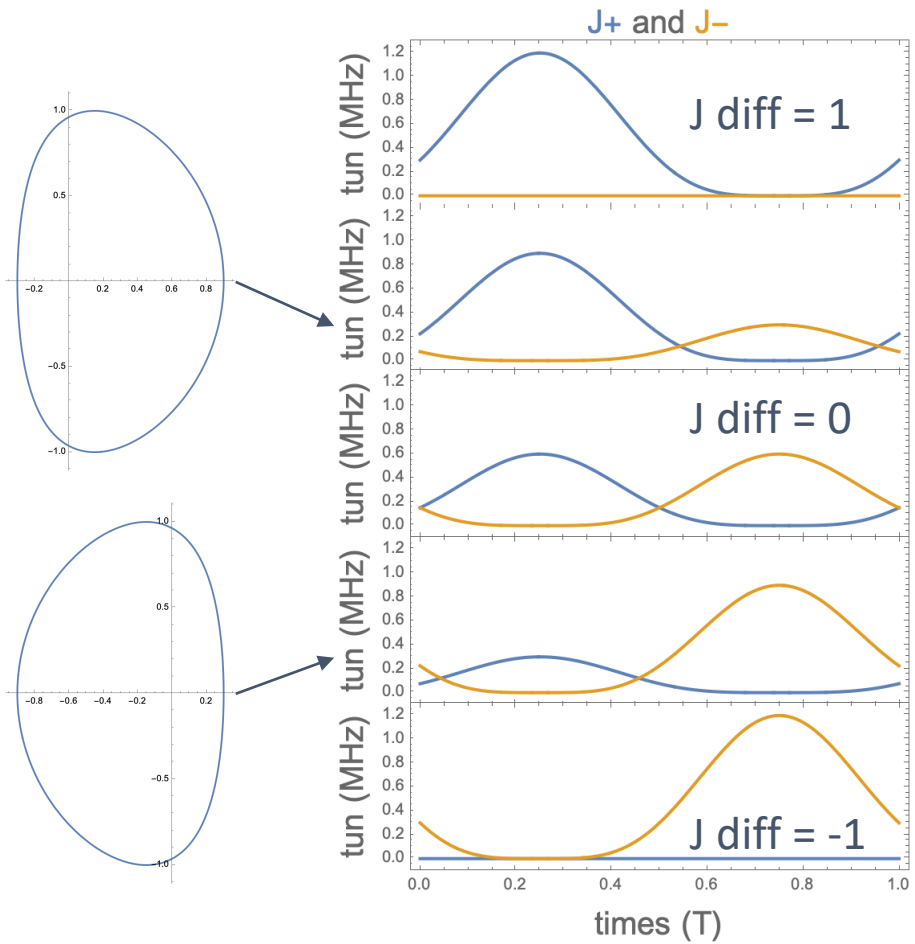


15% singles included in solid line

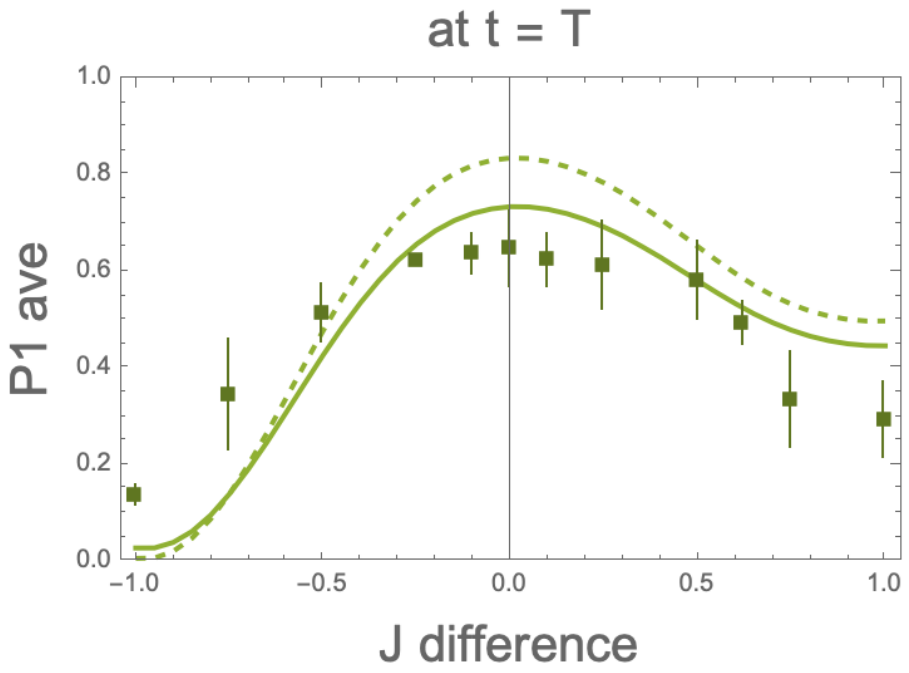


the pump is adiabatic, and robust to deformations

C. Huang, *et al.* (in prep)



15% singles included in solid line



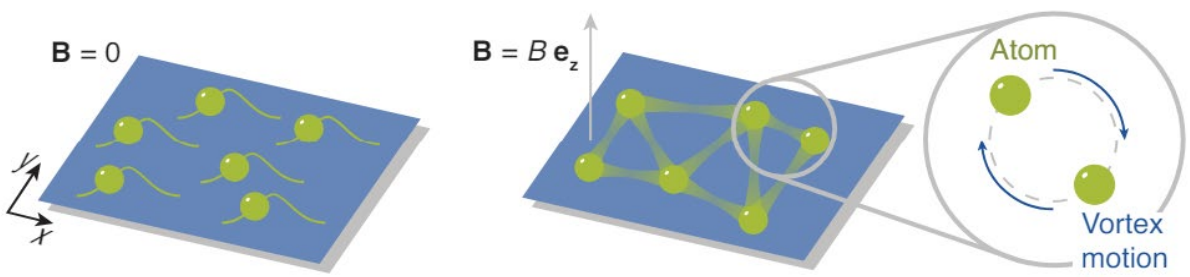
some challenges and prospects

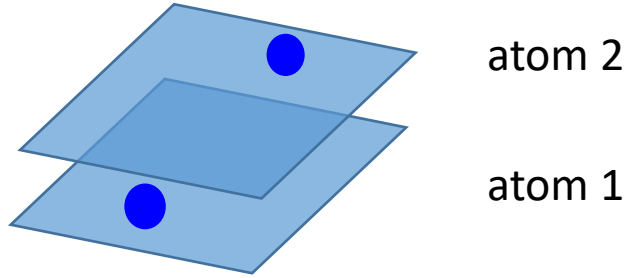
internal state synthetic lattices – challenge is in understanding / overcoming the structure of interactions along the internal dimension

momentum state synthetic lattices – challenge is in reaching the strongly correlated limit, such as by further confinement of atoms

Rydberg state synthetic lattices – understanding how the physics of some idealized models of many-body physics can be realized in this slightly awkward setting

Léonard, *et al* (2023)





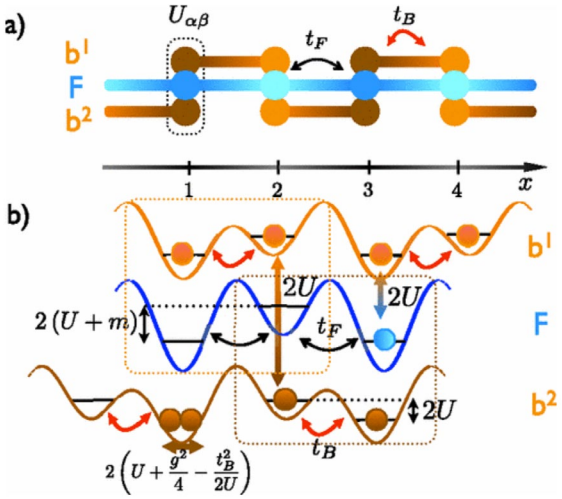
some challenges and prospects

one unique prospect – exotic, non-native interaction terms

- control of one-body terms is trivial in synthetic dimensions
- new opportunities for spectroscopic control of multi-body terms as well!

Atomic Quantum Simulation of Dynamical Gauge Fields Coupled to Fermionic Matter: From String Breaking to Evolution after a Quench

D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, U.-J. Wiese, and P. Zoller
Phys. Rev. Lett. **109**, 175302 – Published 23 October 2012



replace real-space fine-tuning of potentials / state-dependent light shifts
(hard)

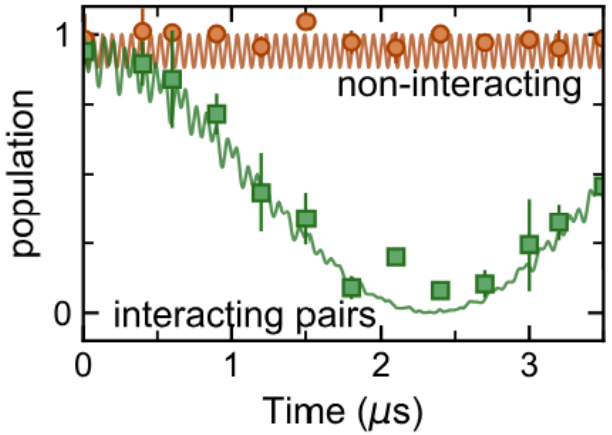
by fine-tuning of a few different state-to-state transitions
(easy)

some challenges and prospects

one unique prospect – exotic, non-native interaction terms

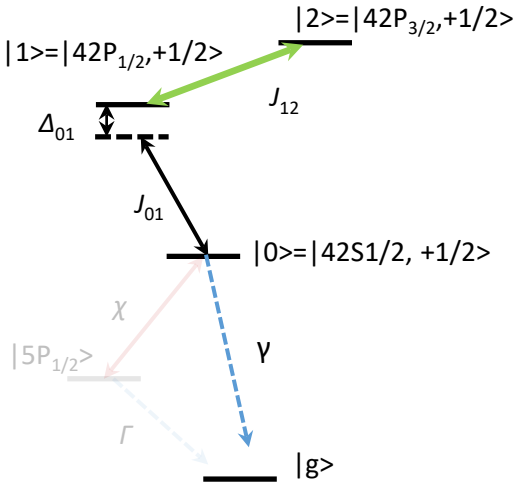
- control of one-body terms is trivial in synthetic dimensions
- new opportunities for spectroscopic control of multi-body terms as well!

engineered $\sigma_i^+\sigma_j^+ + \sigma_i^-\sigma_j^-$ terms

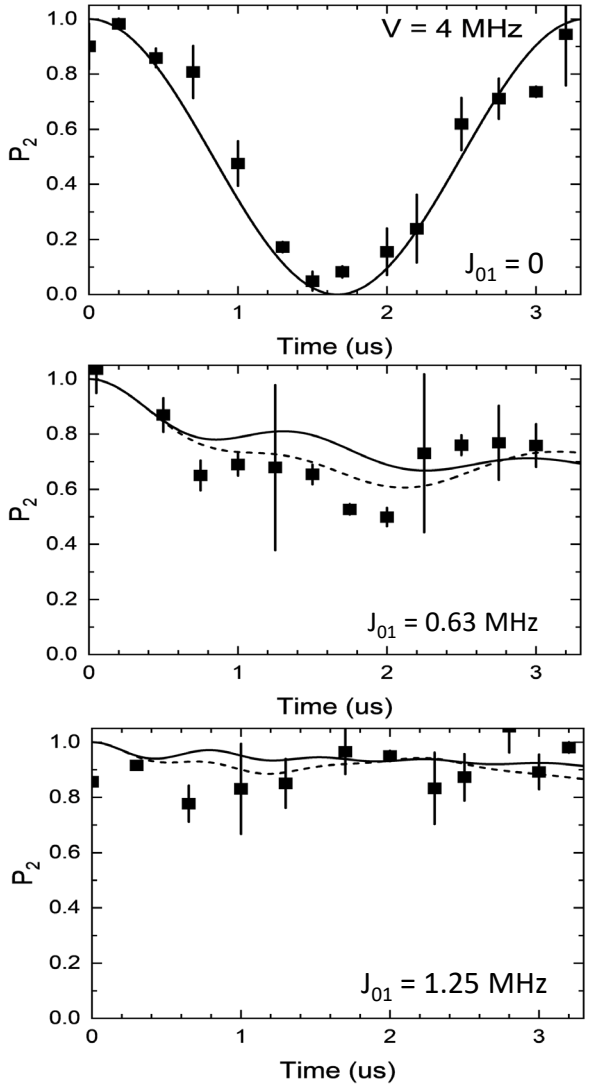


T. Chen, *et al.* arXiv:2404.00740

engineered pair-loss terms



T. Chen, *et al.* (in prep)



summary of Synthetic Dimensions

synthetic dimensions are easy to engineer, and offer some unique opportunities for exploring topology, disorder, and non-Hermitian physics

the community is scratching the surface for new phenomena and phases in synthetic dimensions driven by interactions

ideas from synthetic dimensions can also inspire new real-space approaches

Thanks for your attention and the great conversations!

Thank you! Questions?







collaborations w/ Brian DeMarco & Jake Covey

Penn State team

Matthew Krebs (grad student)
Jungi Lee (grad student)
Ethan Springhorn (grad student)

Many excellent theory collaborators!

Tabor Electronics – thanks for loaner AWG!