CAS-Croucher Joint Laboratory on "Nonlinear Partial Differential Equations and Nonlinear Analysis": Project Summary

Principal Investigators:

From Chinese University of Hong Kong, Hong Kong:

- Zhouping Xin (William M. W. Professor of Mathematics)
- Juncheng Wei (Wei Lun Professor of Mathematics)

From Academy of Mathematics and System Sciences, CAS:

- Daomin Cao (Professor of Mathematics)
- Yanheng Ding (Professor of Mathematics)
- Feimin Huang (Professor of Mathematics)
- Liqun Zhang (Professor of Mathematics)

All PIs of the Lab have a cutting-edge expertise in PDE analysis and join strong competences particularly in nonlinear elliptic equations (Cao, Ding, Wei, Zhang), variational analysis (Cao, Ding, Wei), reaction-diffusion systems/mathematical biology (Cao, Wei, Zhang), hyperbolic conservation laws (Huang, Xin), fluids and kinetic theory (Huang, Xin), mathematical modeling (Huang, Wei, Xin). Several Members have been honored by prestigious awards, such as ICM Invited Speaker (Xin 2002), the Morningside Golden Medal (Xin, 2007), Morningside Silver Medal (Wei, 2010), SIAM Best Paper Award (Huang), Croucher Senior Fellowship (Wei 2005), First Class Natural Science Award of Ministry of Education (Wei, 2010), Outstanding Young Scholars (Cao, Huang, Wei, Zhang)

OBJECTIVES: The purpose of this Joint Laboratory is to bring together different groups of mathematicians working on nonlinear partial differential equations and nonlinear analysis in the CAS and CUHK. There are two main aims of this Laboratory. A primary goal is to position China (including Hong Kong) as a leading international centre in the field of nonlinear PDEs and nonlinear analysis, and to provide a forum on the interaction between theoretical researchers in nonlinear PDEs/nonlinear Analysis of various types with those researchers who are engaged in the mathematical modeling or experimental realization of nonlinear PDEs/Analysis in diverse applications. This interaction should stimulate new mathematical ideas, and also expose the mathematical community to the most updated advances in the field of nonlienar PDEs. A key feature of this Joint Laboratory is our intention to invite some world-known mathematicians who are working at the forefront of nonlinear PDEs/Analysis to visit the Joint Laboratory. The second main aim of the Joint Laboratory is to jointly supervise a limited number of postdoctoral fellows and graduate students. Another purpose of the Joint Laboratory is to enhance the existing collaborations between the researchers from CAS and CUHK to productive first-rate results in this area, in particular, in the area of mixed-type equations.

Overview: The interdisciplinary study of Partial Differential Equations (PDEs) and Nonlinear Analysis provides a major bridge between mathematics and many other disciplines in basic and applied sciences. Diverse real-life phenomena and devices are governed by and studied using PDE models, e.g. musical sound propagation versus shock waves, gentle water movement versus tsunami, competition and coexistence of species in population dynamics, mid and long term weather predictions, productivity of an enhanced oil recovery process, and cleaning-up of pollutant in ground water ecology. These examples are traditionally modeled by Euler and Navier-Stokes systems in continuum mechanics, Boltzmann equations in kinetic theory, reaction-diffusion systems in biology and chemical engineering, and Maxwell's equation in Magnato-Hydro-Dynamics. Some of the applications require more robust high resolution numerical methods supported by further theoretical understanding of the PDEs. On the other hand, the theoretical studies of such equations also have fundamental importance in modern mathematical physics and geometry. Indeed, solving Navier-Stokes system is listed as one of seven centennial open problems in mathematics by AMS. The field of research on PDEs, in particular on nonlinear PDEs, has been and continues to be one of the most exciting areas in both pure and applied sciences. We will concentrate on the following interlinking programs in nonlinear PDEs: Mathematical Theory of Continuum Mechanics, Critical Point Theory and Elliptic PDEs, PDEs in Biological Mathematics.

Nonlinear analysis is an area that was born and has natured from abundant science research. Theories, techniques and results in many different branches of mathematics have been combined in solving nonlinear problems. Main topics included are bifurcation, perturbation, transversality, Nash-Moser technique, Ky Fan's inequality, direct method, topological degree, compensation compactness method, concentration compactness, Ekeland variational principle, Morse theory, minimax method, index theory with group action, and Conley index theory. We are particularly concerned with developing new methods in critical point theory (CPT) oriented towards differential equations in infinite-dimensional Hamiltonian mechanics. CPT has been very successfully applied to studying lots of nonlinear problems arising from celestial mechanics (N-body problem), quantum mechanics (Schrödinger and Dirac equations), elastic mechanics, fluid mechanics, and field theory. These problems can be handled in the framework of Hamiltonian systems. Quantum Yang-Mills theory is now the foundation of most of elementary particle theory, and its predictions have been tested at many experimental laboratories, but its mathematical foundation is still unclear. Progress in establishing the existence of the Yang-Mills theory and a mass gap will require the introduction of fundamental new ideas both in physics and in mathematics. Due to the underlying variational structure it is expectable to explore CPT to promote the study on the Yang-Mills and Mass Gap (another centennial open problem listed by AMS).

Starting Date: September 1st 2012-August 31st 2015